Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

7-Inverse-hyperbolic-functions/7.5-Inverse-hyperbolic-secant/200-7.5.1-u-a+b-arcsech-c-x-^n

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [190]. This is test number [200].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (190)	0.00 (0)
Mathematica	100.00 (190)	0.00 (0)
Maple	81.05 (154)	18.95 (36)
Fricas	68.42 (130)	31.58 (60)
Maxima	33.16 (63)	66.84 (127)
Mupad	27.37 (52)	72.63 (138)
Sympy	25.79 (49)	74.21 (141)
Giac	23.68 (45)	76.32 (145)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

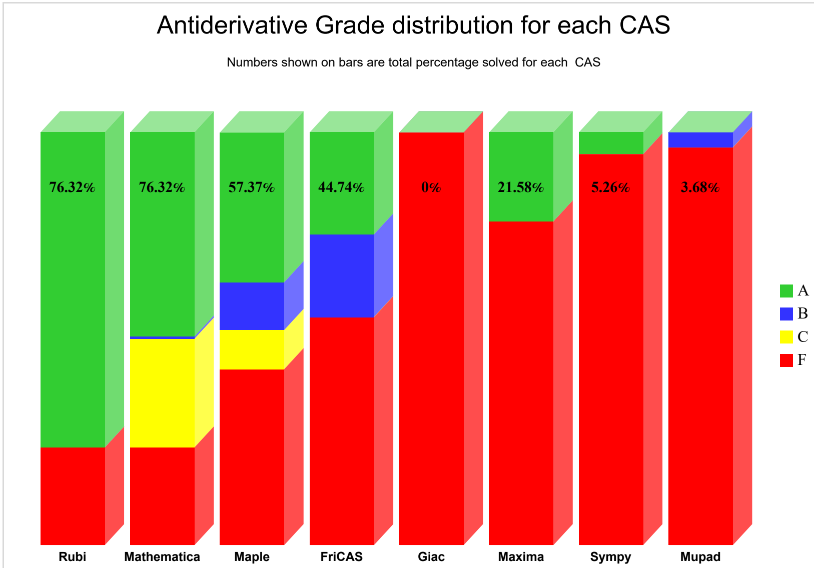
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

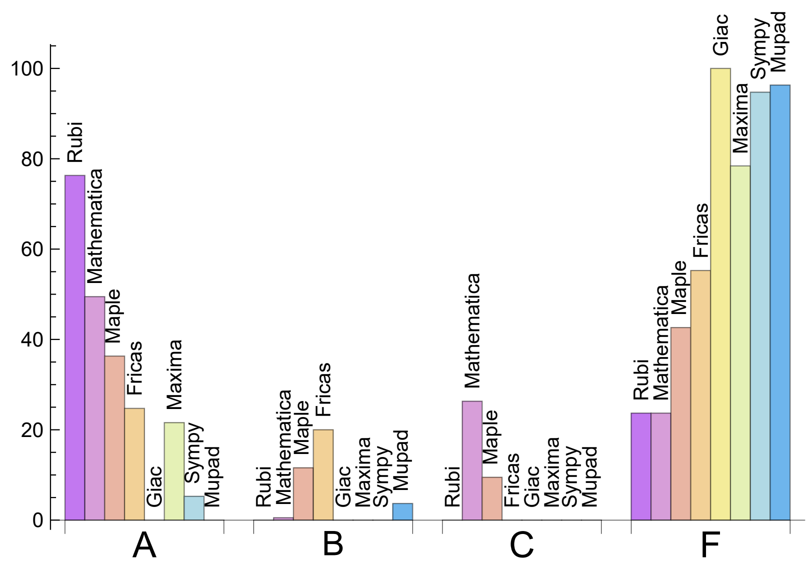
System	% A grade	% B grade	% C grade	% F grade
Rubi	65.263	0.000	11.053	23.684
Mathematica	49.474	0.526	26.316	23.684
Maple	36.316	11.579	9.474	42.632
Fricas	24.737	20.000	0.000	55.263
Maxima	21.579	0.000	0.000	78.421
Sympy	5.263	0.000	0.000	94.737
Giac	0.000	0.000	0.000	100.000
Mupad	0.000	3.684	0.000	96.316

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	36	100.00	0.00	0.00
Fricas	60	95.00	5.00	0.00
Maxima	127	53.54	0.79	45.67
Sympy	141	85.82	14.18	0.00
Mupad	138	0.00	100.00	0.00
Giac	145	98.62	0.00	1.38

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Giac	0.28
Fricas	0.30
Maxima	0.63
Rubi	0.63
Mupad	4.39
Maple	7.10
Mathematica	7.21
Sympy	9.38

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Giac	20.84	1.05	23.00	1.00
Mupad	30.00	1.23	27.00	1.17
Sympy	45.08	0.98	22.00	0.96
Rubi	195.55	0.99	132.00	1.00
Maxima	291.67	17.15	107.00	1.00
Maple	300.62	1.45	143.00	1.04
Fricas	307.53	2.09	135.50	1.50
Mathematica	382.47	1.59	139.50	1.10

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

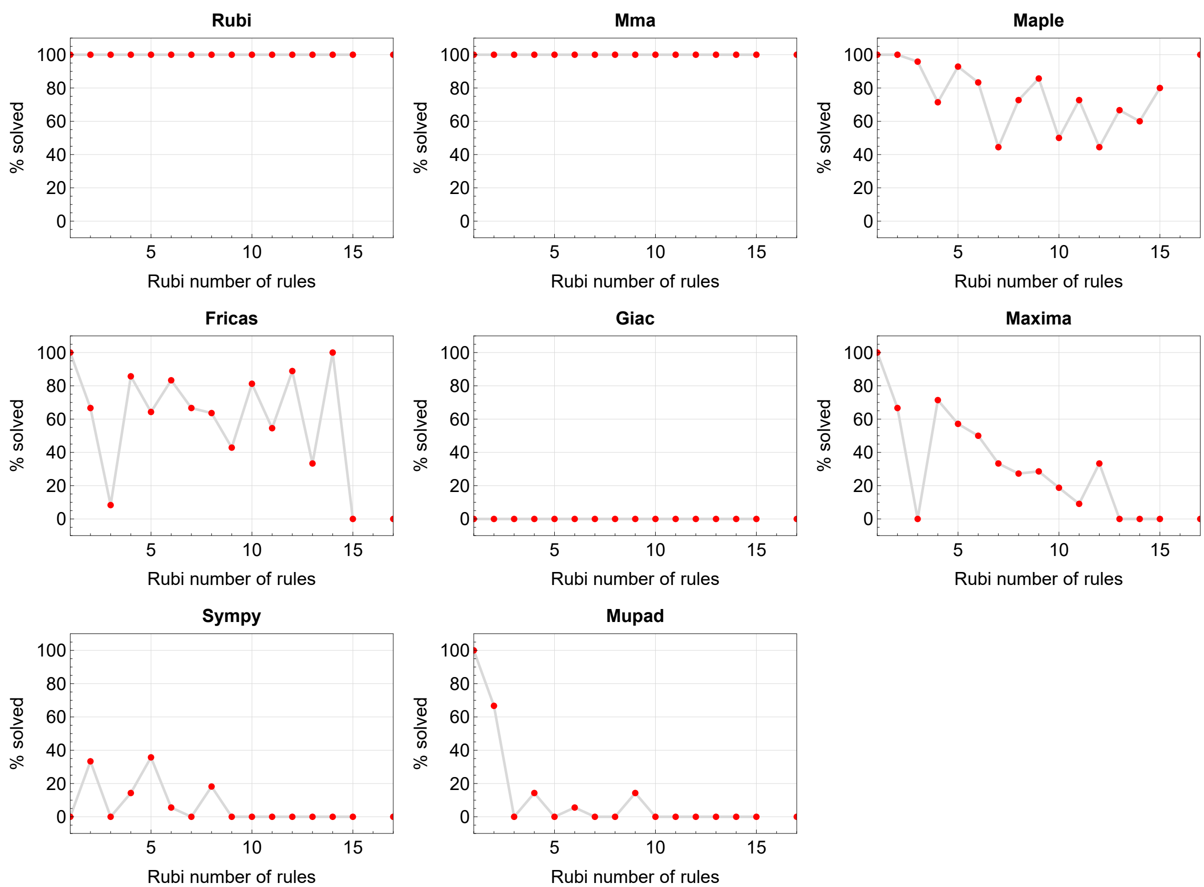


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

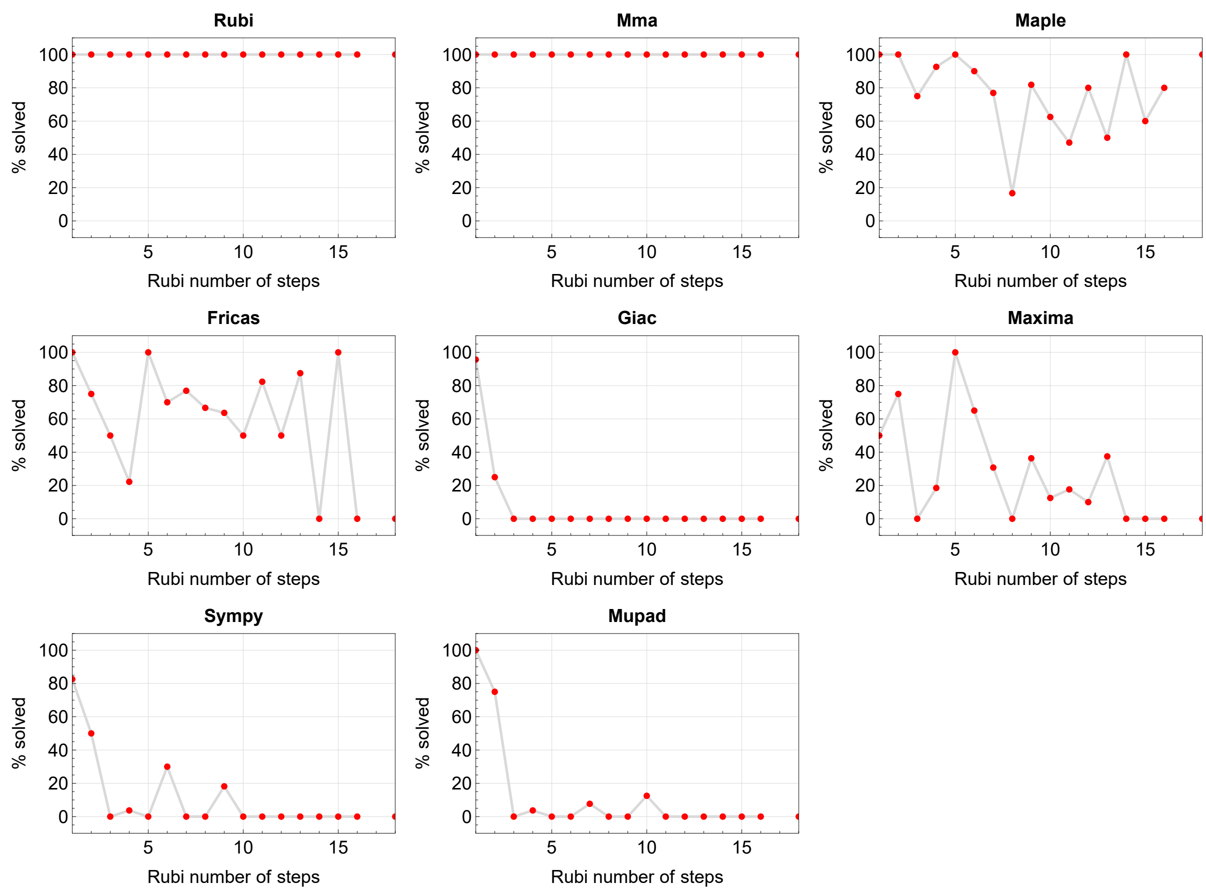


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

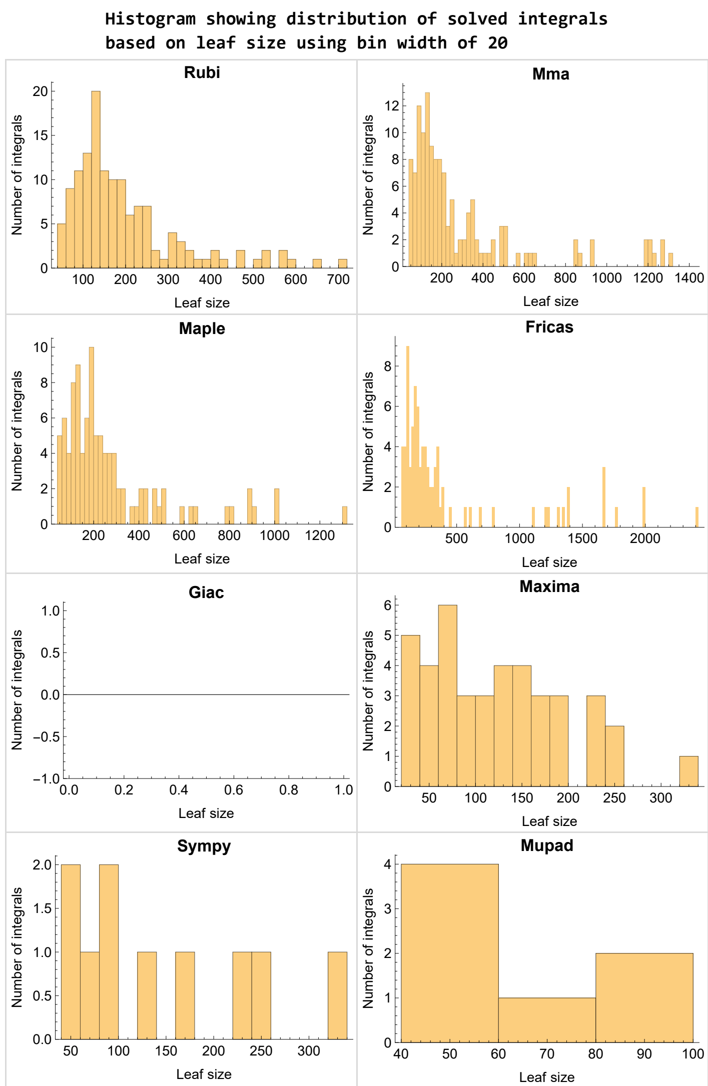


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

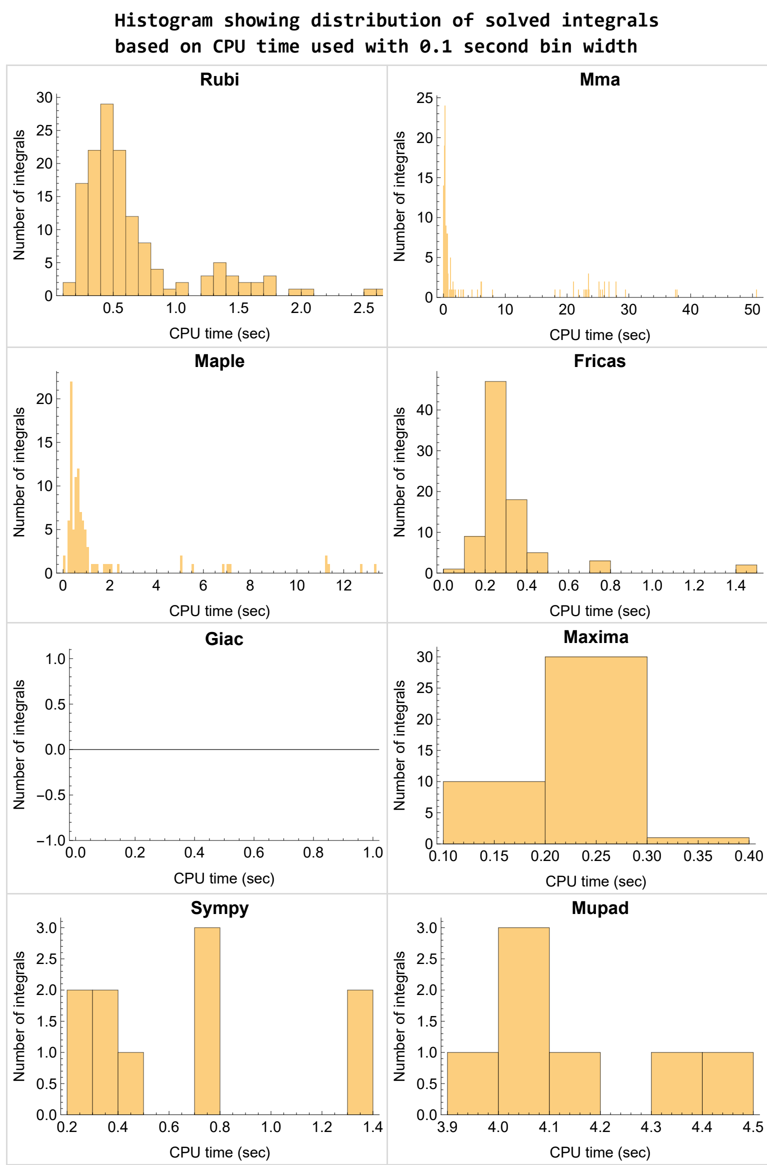


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

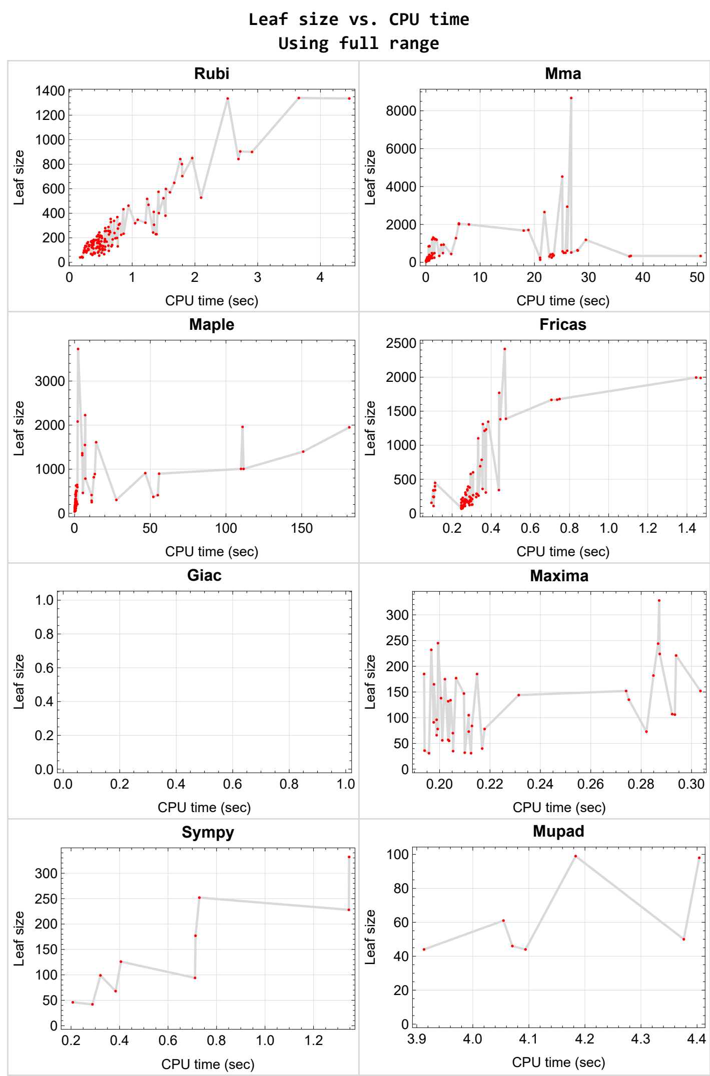


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{51, 52, 53, 57, 58, 59, 63, 64, 65, 69, 70, 72, 73, 87, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 147, 153, 154, 155, 156, 162, 163, 164, 165, 171, 172, 173, 174, 180, 181, 182, 183, 184, 185, 189, 190\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{26, 187\}$

Mathematica $\{81, 82, 83, 84, 85, 86, 115, 116, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129\}$

Maple $\{110, 111, 113, 115, 116, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

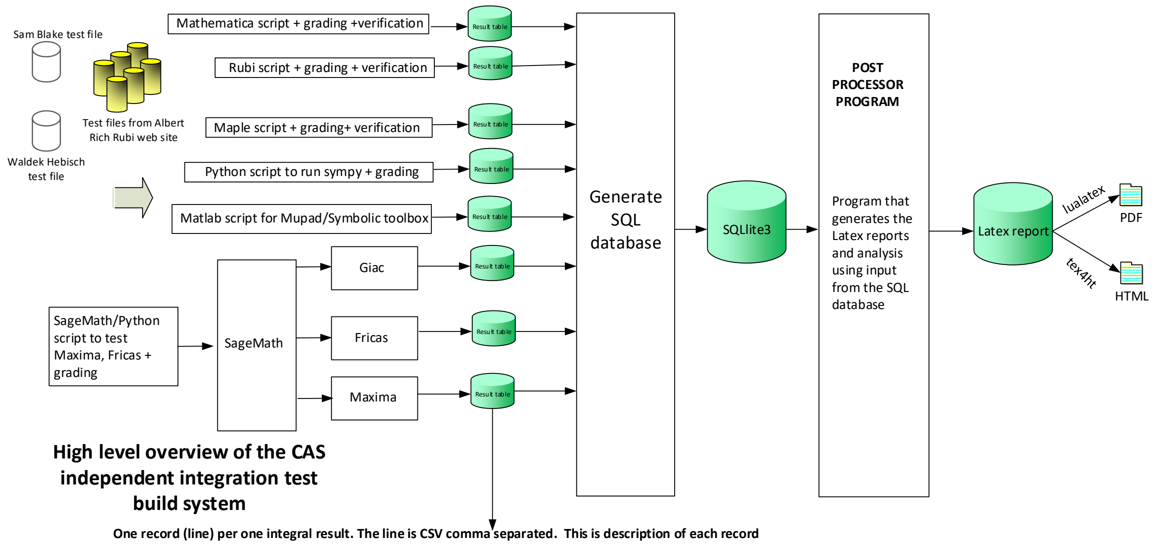
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

The following fields are present only in ***Rubi*** *Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 8, 9, 10, 12, 14, 17, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 43, 45, 48, 50, 56, 62, 68, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 186, 187, 188 }

B grade { }

C grade { 6, 7, 11, 13, 15, 16, 18, 26, 37, 38, 42, 44, 46, 47, 49, 54, 55, 60, 61, 66, 67 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 76, 77, 79, 91, 92, 93, 94, 95, 96, 97, 98, 99, 104, 105, 106, 107, 108, 109, 130, 131, 132, 140, 141, 150, 151, 152, 159, 160, 161, 168, 169, 170, 177, 178, 179, 186, 187, 188 }

B grade { 45 }

C grade { 19, 21, 23, 74, 75, 78, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 100, 101, 102, 103, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 138, 139, 148, 149, 157, 158, 166, 167, 175, 176 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 11, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 36, 37, 39, 40, 41, 42, 44, 54, 55, 56, 60, 74, 75, 76, 77, 79, 82, 83, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109 }

B grade { 4, 33, 35, 38, 46, 47, 48, 49, 50, 61, 62, 66, 67, 68, 80, 81, 84, 85, 86, 117, 124, 125 }

C grade { 78, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129 }

F normal fail { 10, 12, 14, 43, 45, 71, 130, 131, 132, 138, 139, 140, 141, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 186, 187, 188 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 2, 8, 9, 16, 17, 18, 19, 20, 22, 27, 28, 29, 30, 31, 32, 39, 40, 41, 48, 49, 50, 88, 92, 93, 94, 95, 96, 97, 100, 104, 105, 106, 107, 130, 131, 138, 139, 140, 148, 149, 150, 157, 158, 166, 167, 186, 187 }

B grade { 4, 7, 21, 23, 24, 25, 33, 35, 38, 47, 74, 75, 76, 77, 79, 80, 89, 90, 91, 101, 102, 103, 117, 124, 125, 132, 141, 151, 152, 159, 160, 161, 168, 169, 170, 175, 176, 188 }

C grade { }

F normal fail { 1, 3, 5, 6, 10, 11, 12, 13, 14, 15, 26, 34, 36, 37, 42, 43, 44, 45, 46, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 78, 84, 85, 86, 98, 99, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 177, 178, 179 }

F(-1) timedout fail { 81, 82, 83 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 4, 7, 16, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 35, 38, 47, 74, 75, 76, 77, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107 }

B grade { }

C grade { }

F normal fail { 2, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 26, 33, 34, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 78, 79, 98, 99, 108, 109, 111, 113, 115, 116, 117, 123, 126, 132, 141, 152, 161, 166, 169, 170, 175, 176, 177, 178, 179, 186, 187, 188 }

F(-1) timedout fail { 1 }

F(-2) exception fail { 80, 81, 82, 83, 84, 85, 86, 110, 112, 114, 118, 119, 120, 121, 122, 124, 125, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 167, 168, 171, 172, 173, 174 }

2.1.6 Giac

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 188 }

F(-1) timedout fail { }

F(-2) exception fail { 186, 187 }

2.1.7 Mupad

A grade { }

B grade { 24, 25, 27, 28, 76, 77, 91 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 186, 187, 188 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 4, 20, 22, 24, 35, 95, 96, 97, 106, 107 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 131, 132, 138, 139, 140, 141, 148, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 177, 178, 179, 187, 188 }

F(-1) timedout fail { 86, 123, 124, 125, 126, 127, 128, 149, 168, 169, 170, 171, 172, 173, 174, 175, 176, 181, 182, 186 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	168	182	280	0	0	0	0	0
N.S.	1	1.02	1.11	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.586	0.424	0.617	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	117	77	150	0	125	0	0	0
N.S.	1	1.12	0.74	1.44	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.469	0.117	0.391	0.000	0.263	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	116	138	230	0	0	0	0	0
N.S.	1	0.99	1.18	1.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.483	0.269	0.545	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	60	53	100	40	106	42	0	0
N.S.	1	1.13	1.00	1.89	0.75	2.00	0.79	0.00	0.00
time (sec)	N/A	0.365	0.073	0.359	0.217	0.252	0.288	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	64	90	183	0	0	0	0	0
N.S.	1	1.02	1.43	2.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.372	0.216	0.398	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	79	63	136	0	0	0	0	0
N.S.	1	1.23	0.98	2.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.464	0.042	0.260	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	69	42	61	35	97	0	0	0
N.S.	1	1.41	0.86	1.24	0.71	1.98	0.00	0.00	0.00
time (sec)	N/A	0.348	0.099	0.278	0.205	0.258	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	101	54	77	0	106	0	0	0
N.S.	1	1.12	0.60	0.86	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.310	0.055	0.270	0.000	0.248	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	121	73	112	0	116	0	0	0
N.S.	1	1.19	0.72	1.10	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.442	0.115	0.831	0.000	0.247	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	319	281	0	0	0	0	0	0
N.S.	1	1.07	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.047	0.654	0.000	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	199	188	236	0	0	0	0	0
N.S.	1	1.08	1.02	1.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.744	0.763	0.416	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	198	187	199	0	0	0	0	0	0
N.S.	1	0.94	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.686	0.556	0.000	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	114	101	149	0	0	0	0	0
N.S.	1	1.12	0.99	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.527	0.477	0.360	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	104	128	0	0	0	0	0	0
N.S.	1	0.94	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.491	0.140	0.000	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	106	84	181	0	0	0	0	0
N.S.	1	1.20	0.95	2.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.560	0.066	0.283	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	115	75	98	55	155	0	0	0
N.S.	1	1.39	0.90	1.18	0.66	1.87	0.00	0.00	0.00
time (sec)	N/A	0.427	0.093	0.293	0.204	0.247	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	158	147	126	0	174	0	0	0
N.S.	1	1.15	1.07	0.92	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.387	0.161	0.253	0.000	0.253	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	245	120	192	0	186	0	0	0
N.S.	1	1.37	0.67	1.07	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.639	0.133	0.829	0.000	0.257	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	154	143	134	135	183	0	0	0
N.S.	1	1.08	1.01	0.94	0.95	1.29	0.00	0.00	0.00
time (sec)	N/A	0.278	0.235	0.362	0.275	0.276	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	134	97	77	78	100	94	0	0
N.S.	1	1.23	0.89	0.71	0.72	0.92	0.86	0.00	0.00
time (sec)	N/A	0.260	0.110	0.312	0.199	0.255	0.711	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	117	123	114	106	174	0	0	0
N.S.	1	1.06	1.12	1.04	0.96	1.58	0.00	0.00	0.00
time (sec)	N/A	0.246	0.159	0.308	0.293	0.266	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	97	77	68	57	90	68	0	0
N.S.	1	1.26	1.00	0.88	0.74	1.17	0.88	0.00	0.00
time (sec)	N/A	0.227	0.100	0.315	0.203	0.253	0.384	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	80	103	92	73	162	0	0	0
N.S.	1	1.03	1.32	1.18	0.94	2.08	0.00	0.00	0.00
time (sec)	N/A	0.219	0.111	0.307	0.282	0.266	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	59	36	73	46	0	50
N.S.	1	1.00	1.27	1.31	0.80	1.62	1.02	0.00	1.11
time (sec)	N/A	0.193	0.075	0.305	0.194	0.253	0.207	0.000	4.376

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	80	42	31	119	0	0	44
N.S.	1	1.00	2.00	1.05	0.78	2.98	0.00	0.00	1.10
time (sec)	N/A	0.164	0.152	0.089	0.196	0.257	0.000	0.000	3.913

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	78	47	98	0	0	0	0	0
N.S.	1	1.39	0.84	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.499	0.070	0.408	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	42	54	32	66	0	0	46
N.S.	1	1.00	1.05	1.35	0.80	1.65	0.00	0.00	1.15
time (sec)	N/A	0.218	0.084	0.308	0.210	0.247	0.000	0.000	4.071

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	96	117	108	105	77	0	0	61
N.S.	1	1.02	1.24	1.15	1.12	0.82	0.00	0.00	0.65
time (sec)	N/A	0.257	0.091	0.316	0.212	0.249	0.000	0.000	4.055

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	97	74	73	56	79	0	0	0
N.S.	1	1.26	0.96	0.95	0.73	1.03	0.00	0.00	0.00
time (sec)	N/A	0.238	0.086	0.312	0.201	0.246	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	130	137	131	147	90	0	0	0
N.S.	1	1.03	1.09	1.04	1.17	0.71	0.00	0.00	0.00
time (sec)	N/A	0.276	0.120	0.305	0.210	0.255	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	131	94	81	73	89	0	0	0
N.S.	1	1.20	0.86	0.74	0.67	0.82	0.00	0.00	0.00
time (sec)	N/A	0.262	0.111	0.304	0.212	0.253	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	164	157	151	185	100	0	0	0
N.S.	1	1.04	0.99	0.96	1.17	0.63	0.00	0.00	0.00
time (sec)	N/A	0.294	0.181	0.314	0.215	0.249	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	132	212	224	0	244	0	0	0
N.S.	1	1.06	1.71	1.81	0.00	1.97	0.00	0.00	0.00
time (sec)	N/A	0.538	0.357	0.724	0.000	0.289	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	132	241	328	0	0	0	0	0
N.S.	1	0.94	1.72	2.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.557	1.554	0.863	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	72	112	165	84	205	99	0	0
N.S.	1	1.11	1.72	2.54	1.29	3.15	1.52	0.00	0.00
time (sec)	N/A	0.406	0.434	0.645	0.213	0.266	0.321	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	75	126	232	0	0	0	0	0
N.S.	1	0.96	1.62	2.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.418	0.334	0.540	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	99	116	241	0	0	0	0	0
N.S.	1	1.19	1.40	2.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.537	0.189	0.454	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	79	87	121	78	143	0	0	0
N.S.	1	1.30	1.43	1.98	1.28	2.34	0.00	0.00	0.00
time (sec)	N/A	0.388	0.251	0.449	0.218	0.263	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	120	183	191	0	165	0	0	0
N.S.	1	1.02	1.55	1.62	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.354	0.177	0.424	0.000	0.263	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	136	134	191	0	181	0	0	0
N.S.	1	1.11	1.10	1.57	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.516	0.271	0.809	0.000	0.263	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	158	268	263	0	204	0	0	0
N.S.	1	1.05	1.77	1.74	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.467	0.287	0.937	0.000	0.247	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	233	337	464	0	0	0	0	0
N.S.	1	1.04	1.51	2.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.886	2.476	0.924	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	224	440	0	0	0	0	0	0
N.S.	1	0.93	1.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.834	1.268	0.000	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	136	219	318	0	0	0	0	0
N.S.	1	1.08	1.74	2.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.605	1.157	0.785	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	127	282	0	0	0	0	0	0
N.S.	1	0.91	2.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.567	0.568	0.000	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	130	182	429	0	0	0	0	0
N.S.	1	1.14	1.60	3.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.678	0.242	0.578	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	130	165	225	144	228	0	0	0
N.S.	1	1.27	1.62	2.21	1.41	2.24	0.00	0.00	0.00
time (sec)	N/A	0.495	0.354	0.536	0.231	0.254	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	182	245	321	0	271	0	0	0
N.S.	1	1.12	1.50	1.97	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.459	0.529	0.502	0.000	0.272	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	275	256	387	0	305	0	0	0
N.S.	1	1.29	1.20	1.82	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.771	0.409	0.827	0.000	0.268	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	314	332	485	0	351	0	0	0
N.S.	1	1.30	1.37	2.00	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.801	0.742	0.986	0.000	0.278	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	10	14	18
N.S.	1	1.00	1.17	1.00	1.17	1.17	0.83	1.17	1.50
time (sec)	N/A	0.190	2.986	0.342	0.292	0.259	0.261	0.257	4.184

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	16
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.60
time (sec)	N/A	0.173	2.555	0.237	0.261	0.257	0.266	0.256	3.776

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	15	12	16	20
N.S.	1	1.00	1.14	1.00	1.14	1.07	0.86	1.14	1.43
time (sec)	N/A	0.197	0.350	0.214	0.321	0.251	0.671	0.262	3.868

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	54	43	54	0	0	0	0	0
N.S.	1	1.17	0.93	1.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	0.098	0.505	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	66	56	60	0	0	0	0	0
N.S.	1	1.05	0.89	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.546	0.089	0.684	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	110	91	110	0	0	0	0	0
N.S.	1	0.94	0.78	0.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.515	0.187	0.856	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	546	28	12	14	18
N.S.	1	1.00	1.17	1.00	45.50	2.33	1.00	1.17	1.50
time (sec)	N/A	0.188	19.987	0.366	0.607	0.263	0.569	0.260	4.319

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	535	26	12	12	16
N.S.	1	1.00	1.20	1.00	53.50	2.60	1.20	1.20	1.60
time (sec)	N/A	0.173	76.817	0.235	0.614	0.256	0.597	0.255	3.956

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	544	30	14	16	20
N.S.	1	1.00	1.14	1.00	38.86	2.14	1.00	1.14	1.43
time (sec)	N/A	0.198	7.066	0.225	0.512	0.277	1.181	0.266	3.969

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	103	82	164	0	0	0	0	0
N.S.	1	1.20	0.95	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.561	0.462	0.553	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	93	92	186	0	0	0	0	0
N.S.	1	1.09	1.08	2.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.637	0.590	0.760	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	180	250	420	0	0	0	0	0
N.S.	1	0.95	1.32	2.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.562	0.949	0.941	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	2818	42	12	14	18
N.S.	1	1.00	1.17	1.00	234.83	3.50	1.00	1.17	1.50
time (sec)	N/A	0.186	5.995	0.349	2.353	0.265	0.990	0.262	4.459

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	2771	40	12	12	16
N.S.	1	1.00	1.20	1.00	277.10	4.00	1.20	1.20	1.60
time (sec)	N/A	0.172	93.529	0.243	2.376	0.270	1.024	0.272	4.069

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	2638	45	14	16	20
N.S.	1	1.00	1.14	1.00	188.43	3.21	1.00	1.14	1.43
time (sec)	N/A	0.194	2.552	0.220	1.895	0.278	1.960	0.259	4.087

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	141	103	244	0	0	0	0	0
N.S.	1	1.24	0.90	2.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.691	0.288	0.612	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	130	122	277	0	0	0	0	0
N.S.	1	1.16	1.09	2.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.770	0.467	0.786	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	227	204	628	0	0	0	0	0
N.S.	1	0.95	0.85	2.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.630	0.575	1.005	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	1450	44	15	18	22
N.S.	1	1.00	1.12	1.00	90.62	2.75	0.94	1.12	1.38
time (sec)	N/A	0.202	5.025	0.652	10.450	0.276	5.199	0.276	4.586

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	704	30	15	18	22
N.S.	1	1.00	1.12	1.00	44.00	1.88	0.94	1.12	1.38
time (sec)	N/A	0.201	2.741	0.498	4.826	0.272	2.119	0.274	4.252

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	97	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.161	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.38
time (sec)	N/A	0.209	0.430	0.813	0.306	0.266	0.421	0.256	3.732

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	616	32	15	18	22
N.S.	1	1.00	1.12	1.00	38.50	2.00	0.94	1.12	1.38
time (sec)	N/A	0.207	0.933	0.716	0.895	0.282	1.133	0.257	3.824

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	196	190	265	221	358	0	0	0
N.S.	1	0.74	0.72	1.00	0.84	1.36	0.00	0.00	0.00
time (sec)	N/A	0.717	0.384	0.581	0.294	0.356	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	149	147	199	152	280	0	0	0
N.S.	1	0.74	0.73	0.99	0.76	1.39	0.00	0.00	0.00
time (sec)	N/A	0.480	0.215	0.593	0.304	0.329	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	105	142	107	70	177	0	0	99
N.S.	1	0.74	1.00	0.75	0.49	1.25	0.00	0.00	0.70
time (sec)	N/A	0.306	0.341	0.307	0.205	0.296	0.000	0.000	4.183

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	80	42	31	119	0	0	44
N.S.	1	1.00	2.00	1.05	0.78	2.98	0.00	0.00	1.10
time (sec)	N/A	0.164	0.054	0.089	0.212	0.289	0.000	0.000	4.094

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	393	511	0	0	0	0	0
N.S.	1	1.00	1.72	2.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.341	0.630	1.097	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	132	222	207	0	578	0	0	0
N.S.	1	0.90	1.51	1.41	0.00	3.93	0.00	0.00	0.00
time (sec)	N/A	0.368	0.244	1.914	0.000	0.295	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	244	342	593	0	1212	0	0	0
N.S.	1	0.80	1.12	1.94	0.00	3.96	0.00	0.00	0.00
time (sec)	N/A	0.478	0.629	1.882	0.000	0.366	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	343	306	2653	818	0	0	0	0	0
N.S.	1	0.89	7.73	2.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.806	21.844	12.724	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	279	256	2938	413	0	0	0	0	0
N.S.	1	0.92	10.53	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.618	26.038	11.261	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	187	170	1707	286	0	0	0	0	0
N.S.	1	0.91	9.13	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.471	18.887	11.285	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	105	123	1675	251	0	0	0	0	0
N.S.	1	1.17	15.95	2.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.361	18.020	11.347	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	278	254	4527	890	0	0	0	0	0
N.S.	1	0.91	16.28	3.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.564	25.129	13.388	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	609	433	8675	1612	0	0	0	0	0
N.S.	1	0.71	14.24	2.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.880	26.779	14.253	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	222	18	15	18	22
N.S.	1	1.00	1.12	1.00	13.88	1.12	0.94	1.12	1.38
time (sec)	N/A	0.254	21.678	0.396	0.844	0.287	13.563	0.266	4.339

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	164	162	213	244	259	0	0	0
N.S.	1	0.72	0.71	0.93	1.07	1.13	0.00	0.00	0.00
time (sec)	N/A	0.349	0.399	0.608	0.287	0.335	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	132	144	171	182	238	0	0	0
N.S.	1	0.76	0.83	0.98	1.05	1.37	0.00	0.00	0.00
time (sec)	N/A	0.310	0.267	0.592	0.285	0.323	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	96	189	120	107	209	0	0	0
N.S.	1	0.86	1.69	1.07	0.96	1.87	0.00	0.00	0.00
time (sec)	N/A	0.256	0.383	0.342	0.292	0.303	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	80	127	112	66	182	0	0	98
N.S.	1	0.83	1.32	1.17	0.69	1.90	0.00	0.00	1.02
time (sec)	N/A	0.271	0.268	0.346	0.199	0.290	0.000	0.000	4.404

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	110	76	110	91	106	0	0	0
N.S.	1	0.87	0.60	0.87	0.72	0.84	0.00	0.00	0.00
time (sec)	N/A	0.289	0.142	0.346	0.198	0.266	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	141	101	129	132	128	0	0	0
N.S.	1	0.77	0.55	0.70	0.72	0.70	0.00	0.00	0.00
time (sec)	N/A	0.315	0.202	0.346	0.203	0.303	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	170	117	147	165	149	0	0	0
N.S.	1	0.71	0.49	0.62	0.69	0.63	0.00	0.00	0.00
time (sec)	N/A	0.340	0.254	0.359	0.198	0.267	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	172	126	139	177	168	228	0	0
N.S.	1	0.74	0.54	0.60	0.76	0.72	0.98	0.00	0.00
time (sec)	N/A	0.395	0.254	0.579	0.207	0.288	1.346	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	141	106	121	138	147	177	0	0
N.S.	1	0.78	0.59	0.67	0.77	0.82	0.98	0.00	0.00
time (sec)	N/A	0.354	0.217	0.605	0.201	0.285	0.713	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	126	85	194	96	125	126	0	0
N.S.	1	0.77	0.52	1.18	0.59	0.76	0.77	0.00	0.00
time (sec)	N/A	0.461	0.169	0.621	0.199	0.301	0.406	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	323	108	161	0	0	0	0	0
N.S.	1	1.09	0.36	0.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.217	0.221	0.974	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	347	170	168	0	0	0	0	0
N.S.	1	1.12	0.55	0.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.124	0.246	1.049	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	212	207	286	328	341	0	0	0
N.S.	1	0.77	0.75	1.04	1.19	1.24	0.00	0.00	0.00
time (sec)	N/A	0.448	0.464	0.738	0.287	0.439	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	175	174	208	224	305	0	0	0
N.S.	1	0.86	0.85	1.02	1.10	1.50	0.00	0.00	0.00
time (sec)	N/A	0.367	0.281	0.620	0.287	0.372	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	142	158	184	152	287	0	0	0
N.S.	1	0.80	0.89	1.04	0.86	1.62	0.00	0.00	0.00
time (sec)	N/A	0.367	0.239	0.617	0.274	0.325	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	143	149	201	134	267	0	0	0
N.S.	1	0.81	0.85	1.14	0.76	1.52	0.00	0.00	0.00
time (sec)	N/A	0.379	0.271	0.620	0.204	0.307	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	181	134	177	175	167	0	0	0
N.S.	1	0.85	0.63	0.83	0.82	0.78	0.00	0.00	0.00
time (sec)	N/A	0.409	0.277	0.612	0.202	0.278	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	212	160	209	232	199	0	0	0
N.S.	1	0.75	0.57	0.74	0.83	0.71	0.00	0.00	0.00
time (sec)	N/A	0.429	0.347	0.626	0.197	0.272	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	218	168	198	245	227	332	0	0
N.S.	1	0.78	0.60	0.71	0.88	0.82	1.19	0.00	0.00
time (sec)	N/A	0.497	0.275	0.735	0.199	0.296	1.347	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	171	139	283	185	192	252	0	0
N.S.	1	0.74	0.60	1.23	0.80	0.83	1.10	0.00	0.00
time (sec)	N/A	0.521	0.255	0.737	0.194	0.291	0.729	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	401	176	276	0	0	0	0	0
N.S.	1	1.08	0.48	0.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.426	0.417	1.229	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	411	223	250	0	0	0	0	0
N.S.	1	1.10	0.60	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.357	1.110	1.388	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	519	571	921	411	0	0	0	0	0
N.S.	1	1.10	1.77	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.580	2.836	54.985	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	459	523	860	506	0	0	0	0	0
N.S.	1	1.14	1.87	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.501	0.667	1.454	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	517	849	302	0	0	0	0	0
N.S.	1	1.10	1.81	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.238	0.671	27.625	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	417	469	841	2081	0	0	0	0	0
N.S.	1	1.12	2.02	4.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.291	0.520	2.056	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	523	575	933	372	0	0	0	0	0
N.S.	1	1.10	1.78	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.442	3.283	52.023	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	631	703	1278	786	0	0	0	0	0
N.S.	1	1.11	2.03	1.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.824	5.543	7.178	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	580	648	1208	644	0	0	0	0	0
N.S.	1	1.12	2.08	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.701	1.157	1.710	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	127	345	462	0	602	0	0	0
N.S.	1	0.86	2.35	3.14	0.00	4.10	0.00	0.00	0.00
time (sec)	N/A	0.465	1.102	5.515	0.000	0.307	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	542	598	1189	2226	0	0	0	0	0
N.S.	1	1.10	2.19	4.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.516	1.985	7.004	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	840	900	1270	1006	0	0	0	0	0
N.S.	1	1.07	1.51	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.864	1.465	111.627	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	786	842	1226	910	0	0	0	0	0
N.S.	1	1.07	1.56	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.748	1.790	46.703	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	786	842	1216	898	0	0	0	0	0
N.S.	1	1.07	1.55	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.555	1.641	55.842	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	844	904	1305	1007	0	0	0	0	0
N.S.	1	1.07	1.55	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.817	1.354	109.899	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	778	850	2000	1549	0	0	0	0	0
N.S.	1	1.09	2.57	1.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.953	7.932	6.870	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	153	486	1362	0	1346	0	0	0
N.S.	1	0.88	2.81	7.87	0.00	7.78	0.00	0.00	0.00
time (sec)	N/A	0.364	1.581	5.088	0.000	0.384	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	198	486	1318	0	1232	0	0	0
N.S.	1	0.91	2.24	6.07	0.00	5.68	0.00	0.00	0.00
time (sec)	N/A	0.529	1.114	5.096	0.000	0.374	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	741	801	2054	3727	0	0	0	0	0
N.S.	1	1.08	2.77	5.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.826	6.088	2.332	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1272	1336	2022	1960	0	0	0	0	0
N.S.	1	1.05	1.59	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.525	6.110	110.957	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1276	1340	2030	1398	0	0	0	0	0
N.S.	1	1.05	1.59	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.677	6.105	150.993	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1272	1336	2015	1950	0	0	0	0	0
N.S.	1	1.05	1.58	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.451	6.075	181.431	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	447	380	340	0	0	1995	0	0	0
N.S.	1	0.85	0.76	0.00	0.00	4.46	0.00	0.00	0.00
time (sec)	N/A	1.552	37.746	0.000	0.000	1.447	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	329	279	365	0	0	1669	0	0	0
N.S.	1	0.85	1.11	0.00	0.00	5.07	0.00	0.00	0.00
time (sec)	N/A	0.576	23.449	0.000	0.000	0.737	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	190	307	0	0	1382	0	0	0
N.S.	1	0.86	1.39	0.00	0.00	6.25	0.00	0.00	0.00
time (sec)	N/A	0.570	22.759	0.000	0.000	0.446	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.87	1.00	1.17
time (sec)	N/A	0.276	8.491	0.250	0.000	0.263	4.151	0.297	4.789

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.282	9.094	0.242	0.000	0.266	4.037	0.292	4.544

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.271	19.221	0.214	0.000	0.257	7.943	0.281	4.563

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.212	6.935	0.210	0.000	0.250	2.041	0.290	4.304

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.256	2.615	0.235	0.000	0.253	2.602	0.279	4.589

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	312	256	576	0	0	242	0	0	0
N.S.	1	0.82	1.85	0.00	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.585	25.184	0.000	0.000	0.104	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	446	369	641	0	0	340	0	0	0
N.S.	1	0.83	1.44	0.00	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.794	27.925	0.000	0.000	0.113	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	418	354	313	0	0	1989	0	0	0
N.S.	1	0.85	0.75	0.00	0.00	4.76	0.00	0.00	0.00
time (sec)	N/A	0.663	37.502	0.000	0.000	1.470	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	254	342	0	0	1667	0	0	0
N.S.	1	0.86	1.15	0.00	0.00	5.61	0.00	0.00	0.00
time (sec)	N/A	0.638	23.438	0.000	0.000	0.708	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.87	1.00	1.17
time (sec)	N/A	0.295	10.530	0.250	0.000	0.259	23.893	0.306	4.458

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.302	8.691	0.720	0.000	0.261	19.948	0.268	4.627

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	43	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.87	0.96	1.00	1.17
time (sec)	N/A	0.291	19.894	0.830	0.000	0.265	69.279	0.292	4.963

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	37	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.85	0.95	1.00	1.20
time (sec)	N/A	0.224	8.307	0.807	0.000	0.261	20.249	0.291	4.390

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.278	14.907	0.257	0.000	0.252	18.369	0.288	4.800

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.277	18.912	0.313	0.000	0.268	20.322	0.302	4.607

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	409	338	620	0	0	338	0	0	0
N.S.	1	0.83	1.52	0.00	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	0.717	27.983	0.000	0.000	0.104	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	556	462	1187	0	0	447	0	0	0
N.S.	1	0.83	2.13	0.00	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.957	29.473	0.000	0.000	0.113	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	356	306	366	0	0	1679	0	0	0
N.S.	1	0.86	1.03	0.00	0.00	4.72	0.00	0.00	0.00
time (sec)	N/A	1.357	23.560	0.000	0.000	0.749	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	251	216	406	0	0	1389	0	0	0
N.S.	1	0.86	1.62	0.00	0.00	5.53	0.00	0.00	0.00
time (sec)	N/A	0.497	22.963	0.000	0.000	0.475	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	137	239	0	0	1102	0	0	0
N.S.	1	0.90	1.56	0.00	0.00	7.20	0.00	0.00	0.00
time (sec)	N/A	0.501	21.037	0.000	0.000	0.333	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	31	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.35	0.87	1.00	1.17
time (sec)	N/A	0.272	3.539	0.292	0.000	0.260	2.478	0.275	4.373

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	33	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.43	0.96	1.00	1.17
time (sec)	N/A	0.288	8.116	0.241	0.000	0.252	7.323	0.284	4.482

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.264	13.722	0.204	0.000	0.253	3.462	0.292	4.561

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.213	1.115	0.214	0.000	0.261	1.163	0.280	4.731

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	186	501	0	0	154	0	0	0
N.S.	1	0.84	2.27	0.00	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.458	25.414	0.000	0.000	0.095	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	346	288	612	0	0	244	0	0	0
N.S.	1	0.83	1.77	0.00	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.635	26.043	0.000	0.000	0.106	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	278	243	436	0	0	1771	0	0	0
N.S.	1	0.87	1.57	0.00	0.00	6.37	0.00	0.00	0.00
time (sec)	N/A	1.388	23.343	0.000	0.000	0.441	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	161	249	0	0	1311	0	0	0
N.S.	1	0.91	1.41	0.00	0.00	7.41	0.00	0.00	0.00
time (sec)	N/A	0.451	23.174	0.000	0.000	0.357	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	135	0	0	379	0	0	0
N.S.	1	1.00	1.55	0.00	0.00	4.36	0.00	0.00	0.00
time (sec)	N/A	0.486	21.060	0.000	0.000	0.290	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	42	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.83	0.87	1.00	1.17
time (sec)	N/A	0.288	11.494	0.261	0.000	0.257	18.227	0.264	4.952

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	44	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.91	0.96	1.00	1.17
time (sec)	N/A	0.306	15.275	0.725	0.000	0.251	77.562	0.272	4.750

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.283	19.490	0.811	0.000	0.248	40.456	0.295	4.661

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.275	7.967	0.873	0.000	0.252	9.141	0.283	4.514

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	334	0	0	107	0	0	0
N.S.	1	1.00	3.63	0.00	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.283	50.615	0.000	0.000	0.105	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	249	214	501	0	0	238	0	0	0
N.S.	1	0.86	2.01	0.00	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.524	25.727	0.000	0.000	0.107	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	272	229	348	0	0	2415	0	0	0
N.S.	1	0.84	1.28	0.00	0.00	8.88	0.00	0.00	0.00
time (sec)	N/A	1.400	23.415	0.000	0.000	0.469	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	157	218	0	0	786	0	0	0
N.S.	1	0.88	1.22	0.00	0.00	4.39	0.00	0.00	0.00
time (sec)	N/A	0.447	0.698	0.000	0.000	0.351	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	135	204	0	0	692	0	0	0
N.S.	1	0.88	1.32	0.00	0.00	4.49	0.00	0.00	0.00
time (sec)	N/A	0.514	0.620	0.000	0.000	0.344	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	53	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.30	0.00	1.00	1.17
time (sec)	N/A	0.310	20.346	0.268	0.000	0.265	0.000	0.277	4.527

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	55	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.39	0.00	1.00	1.17
time (sec)	N/A	0.321	24.001	0.839	0.000	0.256	0.000	0.276	4.820

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.299	20.714	0.851	0.000	0.255	0.000	0.289	5.141

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.287	19.657	0.900	0.000	0.260	0.000	0.284	4.580

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	246	205	488	0	0	335	0	0	0
N.S.	1	0.83	1.98	0.00	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.486	3.139	0.000	0.000	0.105	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	266	224	517	0	0	397	0	0	0
N.S.	1	0.84	1.94	0.00	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.457	26.786	0.000	0.000	0.112	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	596	527	441	0	0	0	0	0	0
N.S.	1	0.88	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.110	4.667	0.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	372	328	322	0	0	0	0	0	0
N.S.	1	0.88	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.674	0.797	0.000	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	206	184	190	0	0	0	0	0	0
N.S.	1	0.89	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.387	0.533	0.000	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	1.09	1.26
time (sec)	N/A	0.255	3.766	1.323	0.321	0.264	8.615	0.258	3.915

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	36	0	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.57	0.00	1.09	1.26
time (sec)	N/A	0.257	7.643	1.289	0.350	0.266	0.000	0.271	4.034

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	42	0	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.68	0.00	1.00	1.16
time (sec)	N/A	0.291	1.478	1.315	0.320	0.272	0.000	0.324	4.118

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.282	0.223	0.497	0.310	0.272	8.730	0.299	4.037

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.271	1.537	0.456	0.326	0.274	3.271	0.285	4.102

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	45	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.80	0.96	1.00	1.16
time (sec)	N/A	0.286	1.933	1.340	0.325	0.261	22.929	0.283	4.664

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	473	239	213	0	0	393	0	0	0
N.S.	1	0.51	0.45	0.00	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	0.724	0.633	0.000	0.000	0.282	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-2)	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	316	168	178	0	0	336	0	0	0
N.S.	1	0.53	0.56	0.00	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.541	0.524	0.000	0.000	0.278	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	159	113	140	0	0	279	0	0	0
N.S.	1	0.71	0.88	0.00	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.378	0.487	0.000	0.000	0.269	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	100	37	34	26	30
N.S.	1	1.00	1.08	0.92	3.85	1.42	1.31	1.00	1.15
time (sec)	N/A	0.268	0.854	0.258	0.789	0.252	2.718	0.268	5.322

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	124	39	36	26	30
N.S.	1	1.00	1.08	0.92	4.77	1.50	1.38	1.00	1.15
time (sec)	N/A	0.284	9.050	0.536	0.858	0.243	24.428	0.272	4.827

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [10] had the largest ratio of [1.50000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	11	10	1.02	10	1.000
2	A	11	10	1.12	10	1.000
3	A	9	8	0.99	10	0.800
4	A	9	8	1.13	8	1.000
5	A	7	6	1.02	6	1.000
6	C	9	8	1.23	10	0.800
7	C	11	10	1.41	10	1.000
8	A	7	6	1.12	10	0.600
9	A	11	10	1.19	10	1.000
10	A	16	15	1.07	10	1.500
11	C	16	15	1.08	10	1.500
12	A	11	10	0.94	10	1.000
13	C	12	11	1.12	8	1.375
14	A	8	7	0.94	6	1.167
15	C	10	9	1.20	10	0.900
16	C	13	12	1.39	10	1.200
17	A	12	11	1.15	10	1.100
18	C	15	14	1.37	10	1.400
19	A	9	9	1.08	12	0.750
20	A	6	6	1.23	12	0.500
21	A	7	7	1.06	12	0.583
22	A	4	4	1.26	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	5	5	1.03	12	0.417
24	A	2	2	1.00	10	0.200
25	A	1	1	1.00	8	0.125
26	C	10	9	1.39	12	0.750
27	A	2	2	1.00	12	0.167
28	A	7	6	1.02	12	0.500
29	A	4	4	1.26	12	0.333
30	A	9	8	1.03	12	0.667
31	A	6	6	1.20	12	0.500
32	A	11	10	1.04	12	0.833
33	A	11	10	1.06	14	0.714
34	A	9	8	0.94	14	0.571
35	A	9	8	1.11	12	0.667
36	A	7	6	0.96	10	0.600
37	C	9	8	1.19	14	0.571
38	C	11	10	1.30	14	0.714
39	A	7	6	1.02	14	0.429
40	A	11	10	1.11	14	0.714
41	A	8	7	1.05	14	0.500
42	C	16	15	1.04	14	1.071
43	A	11	10	0.93	14	0.714
44	C	12	11	1.08	12	0.917
45	A	8	7	0.91	10	0.700
46	C	10	9	1.14	14	0.643
47	C	13	12	1.27	14	0.857
48	A	12	11	1.12	14	0.786
49	C	15	14	1.29	14	1.000
50	A	15	14	1.30	14	1.000
51	N/A	1	0	1.00	12	0.000
52	N/A	1	0	1.00	10	0.000
53	N/A	1	0	1.00	14	0.000
54	C	10	9	1.17	14	0.643
55	C	12	11	1.05	14	0.786

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	4	3	0.94	14	0.214
57	N/A	1	0	1.00	12	0.000
58	N/A	1	0	1.00	10	0.000
59	N/A	1	0	1.00	14	0.000
60	C	12	11	1.20	14	0.786
61	C	14	13	1.09	14	0.929
62	A	4	3	0.95	14	0.214
63	N/A	1	0	1.00	12	0.000
64	N/A	1	0	1.00	10	0.000
65	N/A	1	0	1.00	14	0.000
66	C	16	15	1.24	14	1.071
67	C	18	17	1.16	14	1.214
68	A	4	3	0.95	14	0.214
69	N/A	1	0	1.00	16	0.000
70	N/A	1	0	1.00	16	0.000
71	A	3	3	1.00	14	0.214
72	N/A	1	0	1.00	16	0.000
73	N/A	1	0	1.00	16	0.000
74	A	13	12	0.74	16	0.750
75	A	12	11	0.74	16	0.688
76	A	10	9	0.74	14	0.643
77	A	1	1	1.00	8	0.125
78	A	2	2	1.00	16	0.125
79	A	3	3	0.90	16	0.188
80	A	3	3	0.80	16	0.188
81	A	14	13	0.89	18	0.722
82	A	12	11	0.92	18	0.611
83	A	3	3	0.91	18	0.167
84	A	6	5	1.17	18	0.278
85	A	13	12	0.91	18	0.667
86	A	16	15	0.71	18	0.833
87	N/A	2	0	1.00	16	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	6	6	0.72	19	0.316
89	A	5	5	0.76	19	0.263
90	A	4	4	0.86	16	0.250
91	A	4	4	0.83	19	0.211
92	A	4	4	0.87	19	0.211
93	A	5	5	0.77	19	0.263
94	A	6	6	0.71	19	0.316
95	A	6	5	0.74	19	0.263
96	A	6	5	0.78	19	0.263
97	A	6	5	0.77	17	0.294
98	A	6	5	1.09	19	0.263
99	A	6	5	1.12	19	0.263
100	A	7	7	0.77	21	0.333
101	A	6	6	0.86	18	0.333
102	A	6	6	0.80	21	0.286
103	A	6	6	0.81	21	0.286
104	A	6	6	0.85	21	0.286
105	A	7	7	0.75	21	0.333
106	A	6	5	0.78	21	0.238
107	A	6	5	0.74	19	0.263
108	A	6	5	1.08	21	0.238
109	A	6	5	1.10	21	0.238
110	A	4	3	1.10	21	0.143
111	A	4	3	1.14	19	0.158
112	A	4	3	1.10	18	0.167
113	A	4	3	1.12	21	0.143
114	A	4	3	1.10	21	0.143
115	A	4	3	1.11	21	0.143
116	A	4	3	1.12	21	0.143
117	A	7	6	0.86	19	0.316
118	A	4	3	1.10	21	0.143
119	A	4	3	1.07	21	0.143
120	A	4	3	1.07	21	0.143
121	A	4	3	1.07	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
122	A	4	3	1.07	21	0.143
123	A	4	3	1.09	21	0.143
124	A	7	6	0.88	21	0.286
125	A	9	8	0.91	19	0.421
126	A	4	3	1.08	21	0.143
127	A	4	3	1.05	21	0.143
128	A	4	3	1.05	21	0.143
129	A	4	3	1.05	18	0.167
130	A	15	14	0.85	23	0.609
131	A	13	12	0.85	23	0.522
132	A	11	10	0.86	21	0.476
133	N/A	1	0	1.00	23	0.000
134	N/A	1	0	1.00	23	0.000
135	N/A	1	0	1.00	23	0.000
136	N/A	1	0	1.00	20	0.000
137	N/A	1	0	1.00	23	0.000
138	A	11	11	0.82	23	0.478
139	A	12	12	0.83	23	0.522
140	A	15	14	0.85	23	0.609
141	A	13	12	0.86	21	0.571
142	N/A	1	0	1.00	23	0.000
143	N/A	1	0	1.00	23	0.000
144	N/A	1	0	1.00	23	0.000
145	N/A	1	0	1.00	20	0.000
146	N/A	1	0	1.00	23	0.000
147	N/A	1	0	1.00	23	0.000
148	A	12	12	0.83	23	0.522
149	A	13	13	0.83	23	0.565
150	A	13	12	0.86	23	0.522
151	A	11	10	0.86	23	0.435
152	A	10	9	0.90	21	0.429
153	N/A	1	0	1.00	23	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
154	N/A	1	0	1.00	23	0.000
155	N/A	1	0	1.00	23	0.000
156	N/A	1	0	1.00	20	0.000
157	A	10	10	0.84	23	0.435
158	A	11	11	0.83	23	0.478
159	A	11	10	0.87	23	0.435
160	A	9	8	0.91	23	0.348
161	A	6	5	1.00	21	0.238
162	N/A	1	0	1.00	23	0.000
163	N/A	1	0	1.00	23	0.000
164	N/A	1	0	1.00	23	0.000
165	N/A	1	0	1.00	23	0.000
166	A	4	4	1.00	20	0.200
167	A	11	11	0.86	23	0.478
168	A	11	10	0.84	23	0.435
169	A	8	7	0.88	23	0.304
170	A	7	6	0.88	21	0.286
171	N/A	1	0	1.00	23	0.000
172	N/A	1	0	1.00	23	0.000
173	N/A	1	0	1.00	23	0.000
174	N/A	1	0	1.00	23	0.000
175	A	8	8	0.83	23	0.348
176	A	10	10	0.84	20	0.500
177	A	7	7	0.88	23	0.304
178	A	6	6	0.88	23	0.261
179	A	4	4	0.89	21	0.190
180	N/A	1	0	1.00	23	0.000
181	N/A	1	0	1.00	23	0.000
182	N/A	1	0	1.00	25	0.000
183	N/A	1	0	1.00	25	0.000
184	N/A	1	0	1.00	25	0.000
185	N/A	1	0	1.00	25	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
186	A	7	6	0.51	26	0.231
187	A	9	8	0.53	26	0.308
188	A	8	7	0.71	26	0.269
189	N/A	1	0	1.00	26	0.000
190	N/A	1	0	1.00	26	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^4 \mathrm{sech}^{-1}(ax)^2 \, dx$

3.1.1 Optimal result

Integrand size = 10, antiderivative size = 164

$$\int x^4 \mathrm{sech}^{-1}(ax)^2 \, dx = -\frac{3x}{20a^4} - \frac{x^3}{30a^2} - \frac{3x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\mathrm{sech}^{-1}(ax)}{20a^4} - \frac{x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\mathrm{sech}^{-1}(ax)}{10a^2} + \frac{1}{5}x^5\mathrm{sech}^{-1}(ax)^2 - \frac{3\mathrm{sech}^{-1}(ax)\arctan\left(e^{\mathrm{sech}^{-1}(ax)}\right)}{10a^5} + \frac{3i\,\mathrm{PolyLog}\left(2,-ie^{\mathrm{sech}^{-1}(ax)}\right)}{20a^5} - \frac{3i\,\mathrm{PolyLog}\left(2,ie^{\mathrm{sech}^{-1}(ax)}\right)}{20a^5}$$

output
```
-3/20*x/a^4-1/30*x^3/a^2+1/5*x^5*arcsech(a*x)^2-3/10*arcsech(a*x)*arctan(1
/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/a^5+3/20*I*polylog(2,-I*(1/a/x+(1/a/
x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^5-3/20*I*polylog(2,I*(1/a/x+(1/a/x-1)^(1/2)
*(1+1/a/x)^(1/2)))/a^5-3/20*x*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2
)/a^4-1/10*x^3*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)/a^2
```

3.1.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.11

$$\int x^4 \text{sech}^{-1}(ax)^2 \, dx$$
$$= \frac{-9ax - 2a^3x^3 - 9ax\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax) - 6a^3x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax) + 12a^5x^5\text{sech}^{-1}(ax)^2 +}{}$$

input
```
Integrate[x^4*ArcSech[a*x]^2,x]
```

output
```
(-9*a*x - 2*a^3*x^3 - 9*a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*
x] - 6*a^3*x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] + 12*a^5*x
^5*ArcSech[a*x]^2 + (9*I)*ArcSech[a*x]*Log[1 - I/E^ArcSech[a*x]] - (9*I)*A
rcSech[a*x]*Log[1 + I/E^ArcSech[a*x]] + (9*I)*PolyLog[2, (-I)/E^ArcSech[a*
x]] - (9*I)*PolyLog[2, I/E^ArcSech[a*x]])/(60*a^5)
```

3.1.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6839, 5941, 3042, 4673, 3042, 4673, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{sech}^{-1}(ax)^2 \, dx$$
$$\Big\downarrow \text{ 6839}$$
$$-\frac{\int a^5x^5\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2 d\text{sech}^{-1}(ax)}{a^5}$$
$$\Big\downarrow \text{ 5941}$$
$$-\frac{\frac{2}{5}\int a^5x^5\text{sech}^{-1}(ax)d\text{sech}^{-1}(ax) - \frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^2}{a^5}$$
$$\Big\downarrow \text{ 3042}$$
$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^2 + \frac{2}{5}\int \text{sech}^{-1}(ax)\csc\left(i\text{sech}^{-1}(ax) + \frac{\pi}{2}\right)^5 d\text{sech}^{-1}(ax)}{a^5}$$

$$\Big\downarrow \text{4673}$$

$$-\frac{\frac{2}{5}\left(\frac{3}{4}\int a^3x^3\text{sech}^{-1}(ax)d\text{sech}^{-1}(ax)+\frac{a^3x^3}{12}+\frac{1}{4}a^3x^3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)\right)-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^2}{a^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^2+\frac{2}{5}\left(\frac{3}{4}\int \text{sech}^{-1}(ax)\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^3d\text{sech}^{-1}(ax)+\frac{a^3x^3}{12}+\frac{1}{4}a^3x^3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sec}}{a^5}$$

$$\Big\downarrow \text{4673}$$

$$-\frac{\frac{2}{5}\left(\frac{3}{4}\left(\frac{1}{2}\int ax\text{sech}^{-1}(ax)d\text{sech}^{-1}(ax)+\frac{ax}{2}+\frac{1}{2}ax\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)\right)+\frac{a^3x^3}{12}+\frac{1}{4}a^3x^3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}}{a^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^2+\frac{2}{5}\left(\frac{3}{4}\left(\frac{1}{2}\int \text{sech}^{-1}(ax)\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)+\frac{ax}{2}+\frac{1}{2}ax\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-}}{a^5}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^2+\frac{2}{5}\left(\frac{3}{4}\left(\frac{1}{2}\left(-i\int\log\left(1-ie^{\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)+i\int\log\left(1+ie^{\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)-\right.\right.\right.}{}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^2+\frac{2}{5}\left(\frac{3}{4}\left(\frac{1}{2}\left(-i\int e^{-\text{sech}^{-1}(ax)}\log\left(1-ie^{\text{sech}^{-1}(ax)}\right)de^{\text{sech}^{-1}(ax)}+i\int e^{-\text{sech}^{-1}(ax)}\log\left(1+ie^{\text{se}}\right.\right.\right.\right.}{}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^2+\frac{2}{5}\left(\frac{a^3x^3}{12}+\frac{1}{4}a^3x^3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)+\frac{3}{4}\left(\frac{1}{2}\left(2\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)-i\,\text{I}\right.\right.\right.}{a^5}$$

input
```
Int[x^4*ArcSech[a*x]^2,x]
```

output
```
-((-1/5*(a^5*x^5*ArcSech[a*x]^2) + (2*((a^3*x^3)/12 + (a^3*x^3*Sqrt[(1 - a
*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/4 + (3*((a*x)/2 + (a*x*Sqrt[(1 - a*
x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/2 + (2*ArcSech[a*x]*ArcTan[E^ArcSech
[a*x]] - I*PolyLog[2, (-I)*E^ArcSech[a*x]] + I*PolyLog[2, I*E^ArcSech[a*x]
])/2))/4))/5)/a^5)
```

3.1.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.1.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.71

method	result
derivativedivides	$\frac{\left(-6\,\mathrm{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,a^3x^3+12a^4x^4\,\mathrm{arcsech}(ax)^2-9\,\mathrm{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax-2a^2x^2-9\right)ax}{60}+\frac{3i\,\mathrm{arcsech}(ax)}{}$
default	$\frac{\left(-6\,\mathrm{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,a^3x^3+12a^4x^4\,\mathrm{arcsech}(ax)^2-9\,\mathrm{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax-2a^2x^2-9\right)ax}{60}+\frac{3i\,\mathrm{arcsech}(ax)}{}$

input
```
int(x^4*arcsech(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/60*(-6*arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*a^3*
x^3+12*a^4*x^4*arcsech(a*x)^2-9*arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x+1)
/a/x)^(1/2)*a*x-2*a^2*x^2-9)*a*x+3/20*I*arcsech(a*x)*ln(1+I*(1/a/x+(1/a/x-
1)^(1/2)*(1+1/a/x)^(1/2)))-3/20*I*arcsech(a*x)*ln(1-I*(1/a/x+(1/a/x-1)^(1/
2)*(1+1/a/x)^(1/2)))+3/20*I*dilog(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/
2)))-3/20*I*dilog(1-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))))
```

3.1.5 Fricas [F]

$$\int x^4 \mathrm{sech}^{-1}(ax)^2\,dx = \int x^4\,\mathrm{arsech}\,(ax)^2\,dx$$

input
```
integrate(x^4*arcsech(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^4*arcsech(a*x)^2, x)
```

3.1.6 Sympy [F]

$$\int x^4 \mathrm{sech}^{-1}(ax)^2\,dx = \int x^4\,\mathrm{asech}^2\,(ax)\,dx$$

input
```
integrate(x**4*asech(a*x)**2,x)
```

output
```
Integral(x**4*asech(a*x)**2, x)
```

3.1.7 Maxima [F(-1)]

Timed out.

$$\int x^4 \mathrm{sech}^{-1}(ax)^2\,dx = \text{Timed out}$$

input
```
integrate(x^4*arcsech(a*x)^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.1.8 Giac [F]

$$\int x^4 \mathrm{sech}^{-1}(ax)^2\,dx = \int x^4\,\mathrm{arsech}\,(ax)^2\,dx$$

input
```
integrate(x^4*arcsech(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^4*arcsech(a*x)^2, x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \text{sech}^{-1}(ax)^2\,dx = \int x^4\,\text{acosh}\left(\frac{1}{a\,x}\right)^2\,dx$$

input
```
int(x^4*acosh(1/(a*x))^2,x)
```

output
```
int(x^4*acosh(1/(a*x))^2, x)
```

3.2 $\int x^3 \text{sech}^{-1}(ax)^2 \, dx$

3.2.1 Optimal result

Integrand size = 10, antiderivative size = 104

$$\int x^3 \text{sech}^{-1}(ax)^2 \, dx = -\frac{x^2}{12a^2} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{3a^4} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{6a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^2 - \frac{\log(x)}{3a^4}$$

output
```
-1/12*x^2/a^2+1/4*x^4*arcsech(a*x)^2-1/3*ln(x)/a^4-1/3*(a*x+1)*arcsech(a*x
)*((-a*x+1)/(a*x+1))^(1/2)/a^4-1/6*x^2*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x
+1))^(1/2)/a^2
```

3.2.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.74

$$\int x^3 \text{sech}^{-1}(ax)^2 \, dx = -\frac{a^2x^2 + 2\sqrt{\frac{1-ax}{1+ax}}(2+2ax+a^2x^2+a^3x^3)\,\text{sech}^{-1}(ax) - 3a^4x^4\text{sech}^{-1}(ax)^2 + 4\log(x)}{12a^4}$$

input
```
Integrate[x^3*ArcSech[a*x]^2,x]
```

output
```
-1/12*(a^2*x^2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(2 + 2*a*x + a^2*x^2 + a^3*x^
3)*ArcSech[a*x] - 3*a^4*x^4*ArcSech[a*x]^2 + 4*Log[x])/a^4
```

3.2.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6839, 5941, 3042, 4673, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{sech}^{-1}(ax)^2\,dx$$

$$\Big\downarrow \ 6839$$

$$-\frac{\int a^4x^4\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2d\text{sech}^{-1}(ax)}{a^4}$$

$$\Big\downarrow \ 5941$$

$$-\frac{\frac{1}{2}\int a^4x^4\text{sech}^{-1}(ax)d\text{sech}^{-1}(ax)-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^2}{a^4}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^2+\frac{1}{2}\int \text{sech}^{-1}(ax)\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^4d\text{sech}^{-1}(ax)}{a^4}$$

$$\Big\downarrow \ 4673$$

$$-\frac{\frac{1}{2}\left(\frac{2}{3}\int a^2x^2\text{sech}^{-1}(ax)d\text{sech}^{-1}(ax)+\frac{a^2x^2}{6}+\frac{1}{3}a^2x^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)\right)-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^2}{a^4}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^2+\frac{1}{2}\left(\frac{2}{3}\int \text{sech}^{-1}(ax)\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^2d\text{sech}^{-1}(ax)+\frac{a^2x^2}{6}+\frac{1}{3}a^2x^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sec}\right.}{a^4}$$

$$\Big\downarrow \ 4672$$

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^2+\frac{1}{2}\left(\frac{2}{3}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)-i\int -i\sqrt{\frac{1-ax}{ax+1}}(ax+1)d\text{sech}^{-1}(ax)\right)+\frac{a^2x^2}{6}+\frac{1}{3}a^2x^2\sqrt{\right.}{a^4}$$

$$\Big\downarrow \ 26$$

$$-\frac{\frac{1}{2}\left(\frac{2}{3}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)-\int \sqrt{\frac{1-ax}{ax+1}}(ax+1)d\text{sech}^{-1}(ax)\right)+\frac{a^2x^2}{6}+\frac{1}{3}a^2x^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)\right)}{a^4}$$

$$\downarrow 3042$$

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^2+\frac{1}{2}\left(\frac{2}{3}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)-\int -i\tan\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)\right)+\frac{a^2x^2}{6}+\frac{1}{3}a^2x^2}{a^4}$$

$$\downarrow 26$$

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^2+\frac{1}{2}\left(\frac{2}{3}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)+i\int\tan\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)\right)+\frac{a^2x^2}{6}+\frac{1}{3}a^2x^2\sqrt{}}{a^4}$$

$$\downarrow 3956$$

$$-\frac{\frac{1}{2}\left(\frac{a^2x^2}{6}+\frac{1}{3}a^2x^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)+\frac{2}{3}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)-\log\left(\frac{1}{ax}\right)\right)\right)-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^2}{a^4}$$

input
```
Int[x^3*ArcSech[a*x]^2,x]
```

output
```
-((-1/4*(a^4*x^4*ArcSech[a*x]^2) + ((a^2*x^2)/6 + (a^2*x^2*Sqrt[(1 - a*x)/
(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/3 + (2*(Sqrt[(1 - a*x)/(1 + a*x)]*(1 +
a*x)*ArcSech[a*x] - Log[1/(a*x)]))/3)/2)/a^4)
```

3.2.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.2.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{-\frac{\operatorname{arcsech}(ax)}{3}+\frac{a^4x^4\operatorname{arcsech}(ax)^2}{4}-\frac{\operatorname{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,a^3x^3}{6}-\frac{\operatorname{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,ax}{3}-\frac{a^2x^2}{12}+\frac{\ln\left(1+\left(\frac{1}{ax}+\cdots\right)\right)}{\cdots}}{a^4}$
default	$\frac{-\frac{\operatorname{arcsech}(ax)}{3}+\frac{a^4x^4\operatorname{arcsech}(ax)^2}{4}-\frac{\operatorname{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,a^3x^3}{6}-\frac{\operatorname{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,ax}{3}-\frac{a^2x^2}{12}+\frac{\ln\left(1+\left(\frac{1}{ax}+\cdots\right)\right)}{\cdots}}{a^4}$

input
```
int(x^3*arcsech(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(-1/3*arcsech(a*x)+1/4*a^4*x^4*arcsech(a*x)^2-1/6*arcsech(a*x)*(-(a*
x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*a^3*x^3-1/3*arcsech(a*x)*(-(a*x-1)/a/x
)^(1/2)*((a*x+1)/a/x)^(1/2)*a*x-1/12*a^2*x^2+1/3*ln(1+(1/a/x+(1/a/x-1)^(1/
2)*(1+1/a/x)^(1/2))^2))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.20

$$\int x^3\text{sech}^{-1}(ax)^2\,dx$$
$$= \frac{3\,a^4x^4\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2 - a^2x^2 - 2\,(a^3x^3+2\,ax)\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right) - 4\,\log(x)}{12\,a^4}$$

input
```
integrate(x^3*arcsech(a*x)^2,x, algorithm="fricas")
```

output
```
1/12*(3*a^4*x^4*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - a^
2*x^2 - 2*(a^3*x^3 + 2*a*x)*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-
(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)) - 4*log(x))/a^4
```

3.2.6 Sympy [F]

$$\int x^3\text{sech}^{-1}(ax)^2\,dx = \int x^3\,\text{asech}^2\,(ax)\,dx$$

input
```
integrate(x**3*asech(a*x)**2,x)
```

output
```
Integral(x**3*asech(a*x)**2, x)
```

3.2.7 Maxima [F]

$$\int x^3\text{sech}^{-1}(ax)^2\,dx = \int x^3\,\text{arsech}\,(ax)^2\,dx$$

input
```
integrate(x^3*arcsech(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*arcsech(a*x)^2, x)
```

3.2.8 Giac [F]

$$\int x^3\text{sech}^{-1}(ax)^2\,dx = \int x^3\,\text{arsech}\,(ax)^2\,dx$$

input
```
integrate(x^3*arcsech(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arcsech(a*x)^2, x)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int x^3\text{sech}^{-1}(ax)^2\,dx = \int x^3\,\text{acosh}\left(\frac{1}{a\,x}\right)^2\,dx$$

input
```
int(x^3*acosh(1/(a*x))^2,x)
```

output
```
int(x^3*acosh(1/(a*x))^2, x)
```

3.3 $\int x^2 \operatorname{sech}^{-1}(ax)^2 \, dx$

3.3.1 Optimal result

Integrand size = 10, antiderivative size = 117

$$\int x^2 \operatorname{sech}^{-1}(ax)^2 \, dx = -\frac{x}{3a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{3a^2} + \frac{1}{3}x^3\operatorname{sech}^{-1}(ax)^2 - \frac{2\operatorname{sech}^{-1}(ax)\arctan\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{3a^3} + \frac{i\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{3a^3} - \frac{i\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(ax)}\right)}{3a^3}$$

output
```
-1/3*x/a^2+1/3*x^3*arcsech(a*x)^2-2/3*arcsech(a*x)*arctan(1/a/x+(1/a/x-1)^
(1/2)*(1+1/a/x)^(1/2))/a^3+1/3*I*polylog(2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/
a/x)^(1/2)))/a^3-1/3*I*polylog(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))
)/a^3-1/3*x*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)/a^2
```

3.3.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

$$\int x^2 \operatorname{sech}^{-1}(ax)^2 \, dx$$
$$= \frac{-ax - ax\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax) + a^3x^3\operatorname{sech}^{-1}(ax)^2 + i\operatorname{sech}^{-1}(ax)\log\left(1-ie^{-\operatorname{sech}^{-1}(ax)}\right) - i\operatorname{sech}^{-1}(a}{3a^3}$$

input
```
Integrate[x^2*ArcSech[a*x]^2,x]
```

output
```
(-(a*x) - a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] + a^3*x^3*A
rcSech[a*x]^2 + I*ArcSech[a*x]*Log[1 - I/E^ArcSech[a*x]] - I*ArcSech[a*x]*
Log[1 + I/E^ArcSech[a*x]] + I*PolyLog[2, (-I)/E^ArcSech[a*x]] - I*PolyLog[
2, I/E^ArcSech[a*x]])/(3*a^3)
```

3.3.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6839, 5941, 3042, 4673, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{sech}^{-1}(ax)^2\,dx$$

$$\Big\downarrow \text{ 6839}$$

$$-\frac{\int a^3x^3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2d\text{sech}^{-1}(ax)}{a^3}$$

$$\Big\downarrow \text{ 5941}$$

$$-\frac{\frac{2}{3}\int a^3x^3\text{sech}^{-1}(ax)d\text{sech}^{-1}(ax)-\frac{1}{3}a^3x^3\text{sech}^{-1}(ax)^2}{a^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{3}a^3x^3\text{sech}^{-1}(ax)^2+\frac{2}{3}\int \text{sech}^{-1}(ax)\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^3d\text{sech}^{-1}(ax)}{a^3}$$

$$\Big\downarrow \text{ 4673}$$

$$-\frac{\frac{2}{3}\left(\frac{1}{2}\int ax\text{sech}^{-1}(ax)d\text{sech}^{-1}(ax)+\frac{ax}{2}+\frac{1}{2}ax\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)\right)-\frac{1}{3}a^3x^3\text{sech}^{-1}(ax)^2}{a^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{3}a^3x^3\text{sech}^{-1}(ax)^2+\frac{2}{3}\left(\frac{1}{2}\int \text{sech}^{-1}(ax)\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)+\frac{ax}{2}+\frac{1}{2}ax\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(\right.}{a^3}$$

$$\Big\downarrow \text{ 4668}$$

$$-\frac{-\frac{1}{3}a^3x^3\text{sech}^{-1}(ax)^2+\frac{2}{3}\left(\frac{1}{2}\left(-i\int\log\left(1-ie^{\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)+i\int\log\left(1+ie^{\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)+2\right.\right.}{a^3}$$

$\big\downarrow$ 2715

$$-\frac{-\frac{1}{3}a^3x^3\text{sech}^{-1}(ax)^2+\frac{2}{3}\left(\frac{1}{2}\left(-i\int e^{-\text{sech}^{-1}(ax)}\log\left(1-ie^{\text{sech}^{-1}(ax)}\right)de^{\text{sech}^{-1}(ax)}+i\int e^{-\text{sech}^{-1}(ax)}\log\left(1+ie^{\text{sech}^{-1}}\right.\right.\right.}{a^3}$$

$\big\downarrow$ 2838

$$-\frac{-\frac{1}{3}a^3x^3\text{sech}^{-1}(ax)^2+\frac{2}{3}\left(\frac{1}{2}\left(2\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)-i\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)+i\,\text{PolyLog}\left(2,ie^{\text{se}}\right.\right.\right.}{a^3}$$

input
```
Int[x^2*ArcSech[a*x]^2,x]
```

output
```
-((-1/3*(a^3*x^3*ArcSech[a*x]^2) + (2*((a*x)/2 + (a*x*Sqrt[(1 - a*x)/(1 +
a*x)]*(1 + a*x)*ArcSech[a*x])/2 + (2*ArcSech[a*x]*ArcTan[E^ArcSech[a*x]] -
 I*PolyLog[2, (-I)*E^ArcSech[a*x]] + I*PolyLog[2, I*E^ArcSech[a*x]])/2))/3
)/a^3)
```

3.3.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.3.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.97

method	result
derivativedivides	$\frac{\frac{\left(a^2x^2\,\mathrm{arcsech}(ax)^2-\mathrm{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax-1\right)ax}{3}+\frac{i\,\mathrm{arcsech}(ax)\ln\left(1+i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)\right)}{3}-\frac{i\,\mathrm{arcsech}(ax)\ln\left(1\right.}{}}{a^3}$
default	$\frac{\frac{\left(a^2x^2\,\mathrm{arcsech}(ax)^2-\mathrm{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax-1\right)ax}{3}+\frac{i\,\mathrm{arcsech}(ax)\ln\left(1+i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)\right)}{3}-\frac{i\,\mathrm{arcsech}(ax)\ln\left(1\right.}{}}{a^3}$

input
```
int(x^2*arcsech(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/3*(a^2*x^2*arcsech(a*x)^2-arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x
+1)/a/x)^(1/2)*a*x-1)*a*x+1/3*I*arcsech(a*x)*ln(1+I*(1/a/x+(1/a/x-1)^(1/2)
*(1+1/a/x)^(1/2)))-1/3*I*arcsech(a*x)*ln(1-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a
/x)^(1/2)))+1/3*I*dilog(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))-1/3*I
*dilog(1-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))))
```

3.3.5 Fricas [F]

$$\int x^2\text{sech}^{-1}(ax)^2\,dx = \int x^2\,\text{arsech}\,(ax)^2\,dx$$

input
```
integrate(x^2*arcsech(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^2*arcsech(a*x)^2, x)
```

3.3.6 Sympy [F]

$$\int x^2\text{sech}^{-1}(ax)^2\,dx = \int x^2\,\text{asech}^2\,(ax)\,dx$$

input
```
integrate(x**2*asech(a*x)**2,x)
```

output
```
Integral(x**2*asech(a*x)**2, x)
```

3.3.7 Maxima [F]

$$\int x^2\text{sech}^{-1}(ax)^2\,dx = \int x^2\,\text{arsech}\,(ax)^2\,dx$$

input
```
integrate(x^2*arcsech(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2*arcsech(a*x)^2, x)
```

3.3.8 Giac [F]

$$\int x^2\text{sech}^{-1}(ax)^2\,dx = \int x^2\,\text{arsech}\,(ax)^2\,\,dx$$

input
```
integrate(x^2*arcsech(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arcsech(a*x)^2, x)
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int x^2\text{sech}^{-1}(ax)^2\,dx = \int x^2\,\text{acosh}\left(\frac{1}{a\,x}\right)^2\,dx$$

input
```
int(x^2*acosh(1/(a*x))^2,x)
```

output
```
int(x^2*acosh(1/(a*x))^2, x)
```

3.4 $\int x\text{sech}^{-1}(ax)^2\,dx$

3.4.1 Optimal result

Integrand size = 8, antiderivative size = 53

$$\int x\text{sech}^{-1}(ax)^2\,dx=-\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{a^2}+\frac{1}{2}x^2\text{sech}^{-1}(ax)^2-\frac{\log(x)}{a^2}$$

output
```
1/2*x^2*arcsech(a*x)^2-ln(x)/a^2-(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(
1/2)/a^2
```

3.4.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int x\text{sech}^{-1}(ax)^2\,dx=-\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{a^2}+\frac{1}{2}x^2\text{sech}^{-1}(ax)^2-\frac{\log(x)}{a^2}$$

input
```
Integrate[x*ArcSech[a*x]^2,x]
```

output
```
-((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/a^2) + (x^2*ArcSech[a
*x]^2)/2 - Log[x]/a^2
```

3.4.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6839, 5941, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{sech}^{-1}(ax)^2\,dx$$

$$\Big\downarrow \text{6839}$$

$$-\frac{\int a^2x^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2d\text{sech}^{-1}(ax)}{a^2}$$

$$\Big\downarrow \text{5941}$$

$$-\frac{\int a^2x^2\text{sech}^{-1}(ax)d\text{sech}^{-1}(ax)-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^2}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^2+\int \text{sech}^{-1}(ax)\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^2d\text{sech}^{-1}(ax)}{a^2}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{-i\int -i\sqrt{\frac{1-ax}{ax+1}}(ax+1)d\text{sech}^{-1}(ax)-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^2+\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{a^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-\int\sqrt{\frac{1-ax}{ax+1}}(ax+1)d\text{sech}^{-1}(ax)-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^2+\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\int -i\tan\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^2+\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{a^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int\tan\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^2+\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{a^2}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{-\frac{1}{2}a^2x^2\operatorname{sech}^{-1}(ax)^2 - \log\left(\frac{1}{ax}\right) + \sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{a^2}$$

input
```
Int[x*ArcSech[a*x]^2,x]
```

output
```
-((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] - (a^2*x^2*ArcSech[a*x
]^2)/2 - Log[1/(a*x)])/a^2)
```

3.4.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.4.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(49) = 98.

Time = 0.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.89

method	result	size
derivativedivides	$\frac{-2\,\operatorname{arcsech}(ax)+\frac{\operatorname{arcsech}(ax)\left(\operatorname{arcsech}(ax)a^{2}x^{2}-2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax+2\right)}{2}+\ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)^{2}\right)}{a^{2}}$	100
default	$\frac{-2\,\operatorname{arcsech}(ax)+\frac{\operatorname{arcsech}(ax)\left(\operatorname{arcsech}(ax)a^{2}x^{2}-2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax+2\right)}{2}+\ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)^{2}\right)}{a^{2}}$	100

input
```
int(x*arcsech(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-2*arcsech(a*x)+1/2*arcsech(a*x)*(arcsech(a*x)*a^2*x^2-2*(-(a*x-1)/
a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*a*x+2)+ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x
)^(1/2))^2))
```

3.4.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(49) = 98.

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.00

$$\int x\operatorname{sech}^{-1}(ax)^{2}\,dx$$
$$=\frac{a^{2}x^{2}\log\left(\frac{ax\sqrt{-\frac{a^{2}x^{2}-1}{a^{2}x^{2}}}+1}{ax}\right)^{2}-2\,ax\sqrt{-\frac{a^{2}x^{2}-1}{a^{2}x^{2}}}\log\left(\frac{ax\sqrt{-\frac{a^{2}x^{2}-1}{a^{2}x^{2}}}+1}{ax}\right)-2\,\log\left(x\right)}{2\,a^{2}}$$

input
```
integrate(x*arcsech(a*x)^2,x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - 2*a*x
*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) +
1)/(a*x)) - 2*log(x))/a^2
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int x\text{sech}^{-1}(ax)^2\,dx = \begin{cases} \frac{x^2\,\text{asech}^2\,(ax)}{2} - \frac{\sqrt{-a^2x^2+1}\,\text{asech}\,(ax)}{a^2} - \frac{\log\,(x)}{a^2} & \text{for } a \neq 0 \\ \infty x^2 & \text{otherwise} \end{cases}$$

input
```
integrate(x*asech(a*x)**2,x)
```

output
```
Piecewise((x**2*asech(a*x)**2/2 - sqrt(-a**2*x**2 + 1)*asech(a*x)/a**2 - l
og(x)/a**2, Ne(a, 0)), (oo*x**2, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int x\text{sech}^{-1}(ax)^2\,dx = \frac{1}{2}\,x^2\,\text{arsech}\,(ax)^2 - \frac{x\sqrt{\frac{1}{a^2x^2}-1}\,\text{arsech}\,(ax)}{a} - \frac{\log\,(x)}{a^2}$$

input
```
integrate(x*arcsech(a*x)^2,x, algorithm="maxima")
```

output
```
1/2*x^2*arcsech(a*x)^2 - x*sqrt(1/(a^2*x^2) - 1)*arcsech(a*x)/a - log(x)/a
^2
```

3.4.8 Giac [F]

$$\int x\text{sech}^{-1}(ax)^2\,dx = \int x\,\text{arsech}\,(ax)^2\,dx$$

input
```
integrate(x*arcsech(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x*arcsech(a*x)^2, x)
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int x\text{sech}^{-1}(ax)^2\,dx = \int x\,\text{acosh}\left(\frac{1}{a\,x}\right)^2\,dx$$

input
```
int(x*acosh(1/(a*x))^2,x)
```

output
```
int(x*acosh(1/(a*x))^2, x)
```

3.5 $\int \text{sech}^{-1}(ax)^2\,dx$

3.5.1 Optimal result

Integrand size = 6, antiderivative size = 63

$$\int \text{sech}^{-1}(ax)^2\,dx = x\text{sech}^{-1}(ax)^2 - \frac{4\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{a} + \frac{2i\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{a} - \frac{2i\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{a}$$

output
```
x*arcsech(a*x)^2-4*arcsech(a*x)*arctan(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/
2))/a+2*I*polylog(2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a-2*I*poly
log(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a
```

3.5.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \text{sech}^{-1}(ax)^2\,dx = \frac{i\left(\text{sech}^{-1}(ax)\left(-iax\text{sech}^{-1}(ax) + 2\log\left(1-ie^{-\text{sech}^{-1}(ax)}\right) - 2\log\left(1+ie^{-\text{sech}^{-1}(ax)}\right)\right) + 2\,\text{PolyLog}\left(2,-\cdots\right.}{a}$$

input
```
Integrate[ArcSech[a*x]^2,x]
```

output
```
(I*(ArcSech[a*x]*((-I)*a*x*ArcSech[a*x] + 2*Log[1 - I/E^ArcSech[a*x]] - 2*
Log[1 + I/E^ArcSech[a*x]]) + 2*PolyLog[2, (-I)/E^ArcSech[a*x]] - 2*PolyLog
[2, I/E^ArcSech[a*x]]))/a
```

3.5.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6833, 5941, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^{-1}(ax)^2\,dx$$

$$\Big\downarrow \text{ 6833}$$

$$-\frac{\int ax\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2d\text{sech}^{-1}(ax)}{a}$$

$$\Big\downarrow \text{ 5941}$$

$$-\frac{2\int ax\text{sech}^{-1}(ax)d\text{sech}^{-1}(ax)-ax\text{sech}^{-1}(ax)^2}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-ax\text{sech}^{-1}(ax)^2+2\int \text{sech}^{-1}(ax)\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)}{a}$$

$$\Big\downarrow \text{ 4668}$$

$$-\frac{-ax\text{sech}^{-1}(ax)^2+2\left(-i\int \log\left(1-ie^{\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)+i\int \log\left(1+ie^{\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)+2\text{sech}^{-1}\right.}{a}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{-ax\text{sech}^{-1}(ax)^2+2\left(-i\int e^{-\text{sech}^{-1}(ax)}\log\left(1-ie^{\text{sech}^{-1}(ax)}\right)de^{\text{sech}^{-1}(ax)}+i\int e^{-\text{sech}^{-1}(ax)}\log\left(1+ie^{\text{sech}^{-1}(ax)}\right)\right.}{a}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{-ax\text{sech}^{-1}(ax)^2+2\left(2\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)-i\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)+i\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(a\right.}\right.}{a}$$

input
```
Int[ArcSech[a*x]^2,x]
```

output
```
-((-(a*x*ArcSech[a*x]^2) + 2*(2*ArcSech[a*x]*ArcTan[E^ArcSech[a*x]] - I*Po
lyLog[2, (-I)*E^ArcSech[a*x]] + I*PolyLog[2, I*E^ArcSech[a*x]]))/a)
```

3.5.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 6833
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-c^(-1)   S
ubst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
 b, c, n}, x] && IGtQ[n, 0]
```

3.5.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.90

method	result
derivativedivides	$\frac{\operatorname{arcsech}(ax)^2 ax+2i\operatorname{arcsech}(ax)\ln\left(1+i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)\right)-2i\operatorname{arcsech}(ax)\ln\left(1-i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)\right)+2i}{a}$
default	$\frac{\operatorname{arcsech}(ax)^2 ax+2i\operatorname{arcsech}(ax)\ln\left(1+i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)\right)-2i\operatorname{arcsech}(ax)\ln\left(1-i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)\right)+2i}{a}$

input
```
int(arcsech(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(arcsech(a*x)^2*a*x+2*I*arcsech(a*x)*ln(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+
1/a/x)^(1/2)))-2*I*arcsech(a*x)*ln(1-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1
/2)))+2*I*dilog(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))-2*I*dilog(1-I
*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))))
```

3.5.5 Fricas [F]

$$\int \operatorname{sech}^{-1}(ax)^2\,dx = \int \operatorname{arsech}(ax)^2\,dx$$

input
```
integrate(arcsech(a*x)^2,x, algorithm="fricas")
```

output
```
integral(arcsech(a*x)^2, x)
```

3.5.6 Sympy [F]

$$\int \operatorname{sech}^{-1}(ax)^2\,dx = \int \operatorname{asech}^2(ax)\,dx$$

input
```
integrate(asech(a*x)**2,x)
```

output
```
Integral(asech(a*x)**2, x)
```

3.5.7 Maxima [F]

$$\int \operatorname{sech}^{-1}(ax)^2\,dx = \int \operatorname{arsech}(ax)^2\,dx$$

input
```
integrate(arcsech(a*x)^2,x, algorithm="maxima")
```

output
```
x*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1)^2 - integrate(-(a^2*x^2*log(a)^2 +
 (a^2*x^2 - 1)*log(x)^2 + (a^2*x^2*log(a)^2 + (a^2*x^2 - 1)*log(x)^2 - log
(a)^2 + 2*(a^2*x^2*log(a) - log(a))*log(x))*sqrt(a*x + 1)*sqrt(-a*x + 1) -
 2*(a^2*x^2*log(a) + (a^2*x^2*(log(a) + 1) + (a^2*x^2 - 1)*log(x) - log(a)
)*sqrt(a*x + 1)*sqrt(-a*x + 1) + (a^2*x^2 - 1)*log(x) - log(a))*log(sqrt(a
*x + 1)*sqrt(-a*x + 1) + 1) - log(a)^2 + 2*(a^2*x^2*log(a) - log(a))*log(x
))/(a^2*x^2 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1) - 1), x)
```

3.5.8 Giac [F]

$$\int \operatorname{sech}^{-1}(ax)^2\,dx = \int \operatorname{arsech}(ax)^2\,dx$$

input
```
integrate(arcsech(a*x)^2,x, algorithm="giac")
```

output
```
integrate(arcsech(a*x)^2, x)
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^{-1}(ax)^2\,dx = \int \operatorname{acosh}\left(\frac{1}{a\,x}\right)^2\,dx$$

input
```
int(acosh(1/(a*x))^2,x)
```

output
```
int(acosh(1/(a*x))^2, x)
```

3.6 $\int \frac{\operatorname{sech}^{-1}(ax)^2}{x} \, dx$

3.6.1 Optimal result

Integrand size = 10, antiderivative size = 64

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x} \, dx = \frac{1}{3}\operatorname{sech}^{-1}(ax)^3 - \operatorname{sech}^{-1}(ax)^2 \log\left(1 + e^{2\operatorname{sech}^{-1}(ax)}\right) - \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{1}{2} \operatorname{PolyLog}\left(3, -e^{2\operatorname{sech}^{-1}(ax)}\right)$$

output
```
1/3*arcsech(a*x)^3-arcsech(a*x)^2*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1
/2))^2)-arcsech(a*x)*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
+1/2*polylog(3,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
```

3.6.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x} \, dx = -\frac{1}{3}\operatorname{sech}^{-1}(ax)^3 - \operatorname{sech}^{-1}(ax)^2 \log\left(1 + e^{-2\operatorname{sech}^{-1}(ax)}\right) + \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, -e^{-2\operatorname{sech}^{-1}(ax)}\right) + \frac{1}{2} \operatorname{PolyLog}\left(3, -e^{-2\operatorname{sech}^{-1}(ax)}\right)$$

input
```
Integrate[ArcSech[a*x]^2/x,x]
```

output
```
-1/3*ArcSech[a*x]^3 - ArcSech[a*x]^2*Log[1 + E^(-2*ArcSech[a*x])] + ArcSec
h[a*x]*PolyLog[2, -E^(-2*ArcSech[a*x])] + PolyLog[3, -E^(-2*ArcSech[a*x])]
/2
```

3.6.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6839, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(ax)^2}{x} \, dx$$

$$\Big\downarrow \text{ 6839}$$

$$-\int \sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2 d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{ 3042}$$

$$-\int -i\text{sech}^{-1}(ax)^2 \tan\left(i\text{sech}^{-1}(ax)\right) d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{ 26}$$

$$i\int \text{sech}^{-1}(ax)^2 \tan\left(i\text{sech}^{-1}(ax)\right) d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{ 4201}$$

$$i\left(2i\int \frac{e^{2\text{sech}^{-1}(ax)}\text{sech}^{-1}(ax)^2}{1+e^{2\text{sech}^{-1}(ax)}} d\text{sech}^{-1}(ax) - \frac{1}{3}i\text{sech}^{-1}(ax)^3\right)$$

$$\Big\downarrow \text{ 2620}$$

$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(ax)^2 \log\left(e^{2\text{sech}^{-1}(ax)}+1\right) - \int \text{sech}^{-1}(ax) \log\left(1+e^{2\text{sech}^{-1}(ax)}\right) d\text{sech}^{-1}(ax)\right) - \frac{1}{3}i\text{sech}^{-1}(ax)^3\right)$$

$$\Big\downarrow \text{ 3011}$$

$$i\left(2i\left(-\frac{1}{2}\int \text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right) d\text{sech}^{-1}(ax) + \frac{1}{2}\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right) + \frac{1}{2}\text{sech}^{-1}(ax)^2 \text{lo}\right.\right.$$

$$\Big\downarrow \ 2720$$

$$i\left(2i\left(-\frac{1}{4}\int e^{-2\text{sech}^{-1}(ax)}\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)de^{2\text{sech}^{-1}(ax)}+\frac{1}{2}\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)+\frac{1}{2}\text{se}\right.\right.$$

$$\Big\downarrow \ 7143$$

$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)-\frac{1}{4}\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(ax)}\right)+\frac{1}{2}\text{sech}^{-1}(ax)^2\log\left(e^{2\text{sech}^{-1}(ax)}+\right.\right.\right.$$

input
```
Int[ArcSech[a*x]^2/x,x]
```

output
```
I*((-1/3*I)*ArcSech[a*x]^3 + (2*I)*((ArcSech[a*x]^2*Log[1 + E^(2*ArcSech[a
*x])])/2 + (ArcSech[a*x]*PolyLog[2, -E^(2*ArcSech[a*x])])/2 - PolyLog[3, -
E^(2*ArcSech[a*x])]/4))
```

3.6.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.6.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.12

method	result
derivativedivides	$\frac{\operatorname{arcsech}(ax)^3}{3} - \operatorname{arcsech}(ax)^2 \ln\left(1 + \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1}\,\sqrt{1 + \frac{1}{ax}}\right)^2\right) - \operatorname{arcsech}(ax) \operatorname{polylog}$
default	$\frac{\operatorname{arcsech}(ax)^3}{3} - \operatorname{arcsech}(ax)^2 \ln\left(1 + \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1}\,\sqrt{1 + \frac{1}{ax}}\right)^2\right) - \operatorname{arcsech}(ax) \operatorname{polylog}$

input
```
int(arcsech(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*arcsech(a*x)^3-arcsech(a*x)^2*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1
/2))^2)-arcsech(a*x)*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
+1/2*polylog(3,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
```

3.6.5 Fricas [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arsech}\left(ax\right)^2}{x}\,dx$$

input
```
integrate(arcsech(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral(arcsech(a*x)^2/x, x)
```

3.6.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{asech}^2\left(ax\right)}{x}\,dx$$

input
```
integrate(asech(a*x)**2/x,x)
```

output
```
Integral(asech(a*x)**2/x, x)
```

3.6.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arsech}\left(ax\right)^2}{x}\,dx$$

input
```
integrate(arcsech(a*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(arcsech(a*x)^2/x, x)
```

3.6.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arsech}(ax)^2}{x}\,dx$$

input
```
integrate(arcsech(a*x)^2/x,x, algorithm="giac")
```

output
```
integrate(arcsech(a*x)^2/x, x)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^2}{x}\,dx$$

input
```
int(acosh(1/(a*x))^2/x,x)
```

output
```
int(acosh(1/(a*x))^2/x, x)
```

3.7 $\int \frac{\text{sech}^{-1}(ax)^2}{x^2} dx$

3.7.1 Optimal result

Integrand size = 10, antiderivative size = 49

$$\int \frac{\text{sech}^{-1}(ax)^2}{x^2} dx = -\frac{2}{x} + \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{x} - \frac{\text{sech}^{-1}(ax)^2}{x}$$

output
```
-2/x-arcsech(a*x)^2/x+2*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)/x
```

3.7.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{\text{sech}^{-1}(ax)^2}{x^2} dx = -\frac{2-2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)+\text{sech}^{-1}(ax)^2}{x}$$

input
```
Integrate[ArcSech[a*x]^2/x^2,x]
```

output
```
-((2 - 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] + ArcSech[a*x]^2
)/x)
```

3.7.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.41, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6839, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{ 6839}$$

$$-a\int \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{ax}d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{ 3042}$$

$$-a\int -i\text{sech}^{-1}(ax)^2\sin\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{ 26}$$

$$ia\int \text{sech}^{-1}(ax)^2\sin\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{ 3777}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^2}{ax}-2i\int \frac{\text{sech}^{-1}(ax)}{ax}d\text{sech}^{-1}(ax)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^2}{ax}-2i\int \text{sech}^{-1}(ax)\sin\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)\right)$$

$$\Big\downarrow \text{ 3777}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^2}{ax}-2i\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{ax}-i\int -\frac{i\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax}d\text{sech}^{-1}(ax)\right)\right)$$

$$\Big\downarrow \text{ 26}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^2}{ax}-2i\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{ax}-\int \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax}d\text{sech}^{-1}(ax)\right)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^2}{ax}-2i\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{ax}-\int -i\sin\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)\right)\right)$$

$$\Big\downarrow \textbf{26}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^2}{ax}-2i\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{ax}+i\int \sin\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)\right)\right)$$

$$\Big\downarrow \textbf{3118}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^2}{ax}-2i\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{ax}-\frac{1}{ax}\right)\right)$$

input
```
Int[ArcSech[a*x]^2/x^2,x]
```

output
```
I*a*((I*ArcSech[a*x]^2)/(a*x) - (2*I)*(-(1/(a*x)) + (Sqrt[(1 - a*x)/(1 + a
*x)]*(1 + a*x)*ArcSech[a*x])/(a*x)))
```

3.7.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.7.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$a\left(-\frac{\operatorname{arcsech}(ax)^2}{ax}+2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)-\frac{2}{ax}\right)$	61
default	$a\left(-\frac{\operatorname{arcsech}(ax)^2}{ax}+2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)-\frac{2}{ax}\right)$	61

input
```
int(arcsech(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/a/x*arcsech(a*x)^2+2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*arcsec
h(a*x)-2/a/x)
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(47) = 94$.

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.98

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2}\,dx = \frac{2\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)-\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2-2}{x}$$

input
```
integrate(arcsech(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
(2*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^
2)) + 1)/(a*x)) - log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 -
2)/x
```

3.7.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{asech}^2(ax)}{x^2}\,dx$$

input
```
integrate(asech(a*x)**2/x**2,x)
```

output
```
Integral(asech(a*x)**2/x**2, x)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2}\,dx = 2\,a\sqrt{\frac{1}{a^2x^2}-1}\,\operatorname{arsech}(ax) - \frac{\operatorname{arsech}(ax)^2}{x} - \frac{2}{x}$$

input
```
integrate(arcsech(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
2*a*sqrt(1/(a^2*x^2) - 1)*arcsech(a*x) - arcsech(a*x)^2/x - 2/x
```

3.7.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arsech}(ax)^2}{x^2}\,dx$$

input
```
integrate(arcsech(a*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(arcsech(a*x)^2/x^2, x)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^2}{x^2}\,dx$$

input
```
int(acosh(1/(a*x))^2/x^2,x)
```

output
```
int(acosh(1/(a*x))^2/x^2, x)
```

3.8 $\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3} \, dx$

3.8.1 Optimal result

Integrand size = 10, antiderivative size = 90

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3} \, dx = -\frac{(1-ax)(1+ax)}{4x^2} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{2x^2} - \frac{1}{4}a^2\operatorname{sech}^{-1}(ax)^2 - \frac{(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)^2}{2x^2}$$

output
```
-1/4*(-a*x+1)*(a*x+1)/x^2-1/4*a^2*arcsech(a*x)^2-1/2*(-a*x+1)*(a*x+1)*arcs
ech(a*x)^2/x^2+1/2*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)/x^2
```

3.8.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3} \, dx = \frac{-1 + 2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax) + (-2+a^2x^2)\operatorname{sech}^{-1}(ax)^2}{4x^2}$$

input
```
Integrate[ArcSech[a*x]^2/x^3,x]
```

output
```
(-1 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] + (-2 + a^2*x^2)*
ArcSech[a*x]^2)/(4*x^2)
```

3.8.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6839, 5895, 3042, 25, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(ax)^2}{x^3}dx$$

$$\Big\downarrow \text{6839}$$

$$-a^2\int \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{a^2x^2}d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{5895}$$

$$-a^2\left(\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^2}{2a^2x^2}-\int \frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)}{a^2x^2}d\text{sech}^{-1}(ax)\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^2\left(\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^2}{2a^2x^2}-\int -\text{sech}^{-1}(ax)\sin\left(i\text{sech}^{-1}(ax)\right)^2d\text{sech}^{-1}(ax)\right)$$

$$\Big\downarrow \text{25}$$

$$-a^2\left(\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^2}{2a^2x^2}+\int \text{sech}^{-1}(ax)\sin\left(i\text{sech}^{-1}(ax)\right)^2d\text{sech}^{-1}(ax)\right)$$

$$\Big\downarrow \text{3791}$$

$$-a^2\left(\frac{1}{2}\int \text{sech}^{-1}(ax)d\text{sech}^{-1}(ax)+\frac{(1-ax)(ax+1)}{4a^2x^2}+\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^2}{2a^2x^2}-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(}{2a^2x^2}\right.$$

$$\Big\downarrow \text{15}$$

$$-a^2\left(\frac{(1-ax)(ax+1)}{4a^2x^2}+\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^2}{2a^2x^2}-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{2a^2x^2}+\frac{1}{4}\text{sech}^{-1}(ax)^2\right)$$

input
```
Int[ArcSech[a*x]^2/x^3,x]
```

output
```
-(a^2*(((1 - a*x)*(1 + a*x))/(4*a^2*x^2) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 +
 a*x)*ArcSech[a*x])/(2*a^2*x^2) + ArcSech[a*x]^2/4 + ((1 - a*x)*(1 + a*x)*
ArcSech[a*x]^2)/(2*a^2*x^2)))
```

3.8.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.8.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$a^{2}\left(-\frac{\operatorname{arcsech}(ax)^{2}}{2a^{2}x^{2}}+\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)}{2ax}+\frac{\operatorname{arcsech}(ax)^{2}}{4}-\frac{1}{4a^{2}x^{2}}\right)$	77
default	$a^{2}\left(-\frac{\operatorname{arcsech}(ax)^{2}}{2a^{2}x^{2}}+\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)}{2ax}+\frac{\operatorname{arcsech}(ax)^{2}}{4}-\frac{1}{4a^{2}x^{2}}\right)$	77

input
```
int(arcsech(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/a^2/x^2*arcsech(a*x)^2+1/2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1
/2)/a/x*arcsech(a*x)+1/4*arcsech(a*x)^2-1/4/a^2/x^2)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.18

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3}\,dx = \frac{2\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)+(a^2x^2-2)\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2-1}{4\,x^2}$$

input
```
integrate(arcsech(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
1/4*(2*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^
2*x^2)) + 1)/(a*x)) + (a^2*x^2 - 2)*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)
) + 1)/(a*x))^2 - 1)/x^2
```

3.8.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3}\,dx = \int \frac{\operatorname{asech}^2(ax)}{x^3}\,dx$$

input
```
integrate(asech(a*x)**2/x**3,x)
```

output
```
Integral(asech(a*x)**2/x**3, x)
```

3.8.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3}\,dx = \int \frac{\operatorname{arsech}(ax)^2}{x^3}\,dx$$

input
```
integrate(arcsech(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate(arcsech(a*x)^2/x^3, x)
```

3.8.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3}\,dx = \int \frac{\operatorname{arsech}(ax)^2}{x^3}\,dx$$

input
```
integrate(arcsech(a*x)^2/x^3,x, algorithm="giac")
```

output
```
integrate(arcsech(a*x)^2/x^3, x)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{sech}^{-1}(ax)^2}{x^3}\,dx = \int \frac{\text{acosh}\left(\frac{1}{a\,x}\right)^2}{x^3}\,dx$$

input
```
int(acosh(1/(a*x))^2/x^3,x)
```

output
```
int(acosh(1/(a*x))^2/x^3, x)
```

3.9 $\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4} dx$

3.9.1 Optimal result

Integrand size = 10, antiderivative size = 102

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4} dx = -\frac{2}{27x^3} - \frac{4a^2}{9x} + \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{9x^3} + \frac{4a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{9x} - \frac{\operatorname{sech}^{-1}(ax)^2}{3x^3}$$

output
```
-2/27/x^3-4/9*a^2/x-1/3*arcsech(a*x)^2/x^3+2/9*(a*x+1)*arcsech(a*x)*((-a*x
+1)/(a*x+1))^(1/2)/x^3+4/9*a^2*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/
2)/x
```

3.9.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4} dx = \frac{-2(1+6a^2x^2) + 6\sqrt{\frac{1-ax}{1+ax}}(1+ax+2a^2x^2+2a^3x^3)\operatorname{sech}^{-1}(ax) - 9\operatorname{sech}^{-1}(ax)^2}{27x^3}$$

input
```
Integrate[ArcSech[a*x]^2/x^4,x]
```

output
```
(-2*(1 + 6*a^2*x^2) + 6*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x + 2*a^2*x^2 + 2
*a^3*x^3)*ArcSech[a*x] - 9*ArcSech[a*x]^2)/(27*x^3)
```

3.9.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6839, 5896, 3042, 3791, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(ax)^2}{x^4}dx$$

$$\Big\downarrow \text{6839}$$

$$-a^3\int \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{a^3x^3}d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{5896}$$

$$-a^3\left(\frac{\text{sech}^{-1}(ax)^2}{3a^3x^3}-\frac{2}{3}\int \frac{\text{sech}^{-1}(ax)}{a^3x^3}d\text{sech}^{-1}(ax)\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3\left(\frac{\text{sech}^{-1}(ax)^2}{3a^3x^3}-\frac{2}{3}\int \text{sech}^{-1}(ax)\sin\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^3d\text{sech}^{-1}(ax)\right)$$

$$\Big\downarrow \text{3791}$$

$$-a^3\left(\frac{\text{sech}^{-1}(ax)^2}{3a^3x^3}-\frac{2}{3}\left(\frac{2}{3}\int \frac{\text{sech}^{-1}(ax)}{ax}d\text{sech}^{-1}(ax)-\frac{1}{9a^3x^3}+\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{3a^3x^3}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3\left(\frac{\text{sech}^{-1}(ax)^2}{3a^3x^3}-\frac{2}{3}\left(\frac{2}{3}\int \text{sech}^{-1}(ax)\sin\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)-\frac{1}{9a^3x^3}+\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(a\right.\right.$$

$$\Big\downarrow \text{3777}$$

$$-a^3\left(\frac{\operatorname{sech}^{-1}(ax)^2}{3a^3x^3}-\frac{2}{3}\left(\frac{2}{3}\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{ax}-i\int -\frac{i\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax}d\operatorname{sech}^{-1}(ax)\right)-\frac{1}{9a^3x^3}+\right.\right.$$

$$\Big\downarrow \; 26$$

$$-a^3\left(\frac{\operatorname{sech}^{-1}(ax)^2}{3a^3x^3}-\frac{2}{3}\left(\frac{2}{3}\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{ax}-\int \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax}d\operatorname{sech}^{-1}(ax)\right)-\frac{1}{9a^3x^3}+\right.\right.$$

$$\Big\downarrow \; 3042$$

$$-a^3\left(\frac{\operatorname{sech}^{-1}(ax)^2}{3a^3x^3}-\frac{2}{3}\left(\frac{2}{3}\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{ax}-\int -i\sin\left(i\operatorname{sech}^{-1}(ax)\right)d\operatorname{sech}^{-1}(ax)\right)-\frac{1}{9a^3x^3}+\right.\right.$$

$$\Big\downarrow \; 26$$

$$-a^3\left(\frac{\operatorname{sech}^{-1}(ax)^2}{3a^3x^3}-\frac{2}{3}\left(\frac{2}{3}\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{ax}+i\int \sin\left(i\operatorname{sech}^{-1}(ax)\right)d\operatorname{sech}^{-1}(ax)\right)-\frac{1}{9a^3x^3}+\right.\right.$$

$$\Big\downarrow \; 3118$$

$$-a^3\left(\frac{\operatorname{sech}^{-1}(ax)^2}{3a^3x^3}-\frac{2}{3}\left(-\frac{1}{9a^3x^3}+\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{3a^3x^3}+\frac{2}{3}\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{ax}-\frac{1}{ax}\right)\right)\right)$$

input
```
Int[ArcSech[a*x]^2/x^4,x]
```

output
```
-(a^3*(ArcSech[a*x]^2/(3*a^3*x^3) - (2*(-1/9*1/(a^3*x^3) + (Sqrt[(1 - a*x)
/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/(3*a^3*x^3) + (2*(-(1/(a*x)) + (Sqrt[(
1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/(a*x)))/3))/3))
```

3.9. $\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4}\,dx$

3.9.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.9.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10

method	result
derivativedivides	$a^{3}\left(-\frac{\operatorname{arcsech}(ax)^{2}}{3a^{3}x^{3}}+\frac{4\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)}{9}+\frac{2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)}{9a^{2}x^{2}}-\frac{4}{9ax}-\frac{2}{27x^{3}a^{3}}\right)$
default	$a^{3}\left(-\frac{\operatorname{arcsech}(ax)^{2}}{3a^{3}x^{3}}+\frac{4\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)}{9}+\frac{2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)}{9a^{2}x^{2}}-\frac{4}{9ax}-\frac{2}{27x^{3}a^{3}}\right)$

input
```
int(arcsech(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3/a^3/x^3*arcsech(a*x)^2+4/9*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1
/2)*arcsech(a*x)+2/9/a^2/x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*arcs
ech(a*x)-4/9/a/x-2/27/x^3/a^3)
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4}\,dx = -\frac{12\,a^2x^2-6\,(2\,a^3x^3+ax)\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)+9\,\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2+2}{27\,x^3}$$

input
```
integrate(arcsech(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
-1/27*(12*a^2*x^2 - 6*(2*a^3*x^3 + a*x)*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log
((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)) + 9*log((a*x*sqrt(-(a^2*x
^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 + 2)/x^3
```

3.9.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{asech}^2(ax)}{x^4}\,dx$$

input
```
integrate(asech(a*x)**2/x***4,x)
```

output
```
Integral(asech(a*x)**2/x***4, x)
```

3.9.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arsech}(ax)^2}{x^4}\,dx$$

input
```
integrate(arcsech(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
integrate(arcsech(a*x)^2/x^4, x)
```

3.9.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arsech}(ax)^2}{x^4}\,dx$$

input
```
integrate(arcsech(a*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate(arcsech(a*x)^2/x^4, x)
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4} \, dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^2}{x^4} \, dx$$

input
```
int(acosh(1/(a*x))^2/x^4,x)
```

output
```
int(acosh(1/(a*x))^2/x^4, x)
```

3.10 $\int x^4 \mathrm{sech}^{-1}(ax)^3 \, dx$

3.10.1 Optimal result

Integrand size = 10, antiderivative size = 297

$$
\begin{aligned}
\int x^4 \mathrm{sech}^{-1}(ax)^3 \, dx = {} & \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{20a^4} - \frac{9x\mathrm{sech}^{-1}(ax)}{20a^4} - \frac{x^3\mathrm{sech}^{-1}(ax)}{10a^2} \\
& - \frac{9x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\mathrm{sech}^{-1}(ax)^2}{40a^4} - \frac{3x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\mathrm{sech}^{-1}(ax)^2}{20a^2} \\
& + \frac{1}{5}x^5\mathrm{sech}^{-1}(ax)^3 - \frac{9\mathrm{sech}^{-1}(ax)^2\arctan\left(e^{\mathrm{sech}^{-1}(ax)}\right)}{20a^5} \\
& + \frac{\arctan\left(\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax}\right)}{2a^5} + \frac{9i\mathrm{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,-ie^{\mathrm{sech}^{-1}(ax)}\right)}{20a^5} \\
& - \frac{9i\mathrm{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,ie^{\mathrm{sech}^{-1}(ax)}\right)}{20a^5} \\
& - \frac{9i\operatorname{PolyLog}\left(3,-ie^{\mathrm{sech}^{-1}(ax)}\right)}{20a^5} + \frac{9i\operatorname{PolyLog}\left(3,ie^{\mathrm{sech}^{-1}(ax)}\right)}{20a^5}
\end{aligned}
$$

output
```
-9/20*x*arcsech(a*x)/a^4-1/10*x^3*arcsech(a*x)/a^2+1/5*x^5*arcsech(a*x)^3-
9/20*arcsech(a*x)^2*arctan(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/a^5+1/2*
arctan((a*x+1)*((-a*x+1)/(a*x+1))^(1/2)/a/x)/a^5+9/20*I*arcsech(a*x)*polyl
og(2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^5-9/20*I*arcsech(a*x)*p
olylog(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^5-9/20*I*polylog(3,-
I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^5+9/20*I*polylog(3,I*(1/a/x+(
1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^5+1/20*x*(a*x+1)*((-a*x+1)/(a*x+1))^(1/
2)/a^4-9/40*x*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/a^4-3/20*x^3
*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/a^2
```

3.10.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.95

$$\int x^4\text{sech}^{-1}(ax)^3\,dx$$
$$= \frac{2ax\sqrt{\frac{1-ax}{1+ax}}(1+ax) - 18ax\text{sech}^{-1}(ax) - 4a^3x^3\text{sech}^{-1}(ax) - 9ax\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2 - 6a^3x^3\sqrt{\frac{1-}{1+}}$$

input
```
Integrate[x^4*ArcSech[a*x]^3,x]
```

output
```
(2*a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - 18*a*x*ArcSech[a*x] - 4*a^3*x
^3*ArcSech[a*x] - 9*a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2
 - 6*a^3*x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 + 8*a^5*x^
5*ArcSech[a*x]^3 + 40*ArcTan[Tanh[ArcSech[a*x]/2]] + (9*I)*ArcSech[a*x]^2*
Log[1 - I/E^ArcSech[a*x]] - (9*I)*ArcSech[a*x]^2*Log[1 + I/E^ArcSech[a*x]]
 + (18*I)*ArcSech[a*x]*PolyLog[2, (-I)/E^ArcSech[a*x]] - (18*I)*ArcSech[a*
x]*PolyLog[2, I/E^ArcSech[a*x]] + (18*I)*PolyLog[3, (-I)/E^ArcSech[a*x]] -
 (18*I)*PolyLog[3, I/E^ArcSech[a*x]])/(40*a^5)
```

3.10.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {6839, 5941, 3042, 4674, 3042, 4255, 3042, 4257, 4674, 3042, 4257, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\text{sech}^{-1}(ax)^3\,dx$$

$$\Big\downarrow \text{ 6839}$$

$$-\frac{\int a^5x^5\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^3d\text{sech}^{-1}(ax)}{a^5}$$

$$\Big\downarrow \text{ 5941}$$

$$-\frac{\frac{3}{5}\int a^5x^5\text{sech}^{-1}(ax)^2d\text{sech}^{-1}(ax)-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^3}{a^5}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^3+\frac{3}{5}\int \text{sech}^{-1}(ax)^2\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^5d\text{sech}^{-1}(ax)}{a^5}$$

$$\Big\downarrow \text{ 4674}$$

$$-\frac{\frac{3}{5}\left(-\frac{1}{6}\int a^3x^3d\text{sech}^{-1}(ax)+\frac{3}{4}\int a^3x^3\text{sech}^{-1}(ax)^2d\text{sech}^{-1}(ax)+\frac{1}{4}a^3x^3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+\frac{1}{6}a^3x^3\text{sech}^{-1}\right.}{a^5}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^3+\frac{3}{5}\left(-\frac{1}{6}\int\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^3d\text{sech}^{-1}(ax)+\frac{3}{4}\int\text{sech}^{-1}(ax)^2\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^3ds\right.}{a^5}$$

$$\Big\downarrow \text{ 4255}$$

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^3+\frac{3}{5}\left(\frac{1}{6}\left(-\frac{1}{2}\int axd\text{sech}^{-1}(ax)-\frac{1}{2}ax\sqrt{\frac{1-ax}{ax+1}}(ax+1)\right)+\frac{3}{4}\int\text{sech}^{-1}(ax)^2\csc\left(i\text{sech}^{-1}(ax)+\right.\right.}{a^5}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^3+\frac{3}{5}\left(\frac{1}{6}\left(-\frac{1}{2}ax\sqrt{\frac{1-ax}{ax+1}}(ax+1)-\frac{1}{2}\int\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)\right)+\frac{3}{4}\int\text{sech}^{-1}(ax)^2\right.}{a^5}$$

$$\Big\downarrow \text{ 4257}$$

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^3+\frac{3}{5}\left(\frac{3}{4}\int\text{sech}^{-1}(ax)^2\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^3d\text{sech}^{-1}(ax)+\frac{1}{4}a^3x^3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(a\right.}{a^5}$$

$$\Big\downarrow \text{ 4674}$$

$$-\frac{\frac{3}{5}\left(\frac{3}{4}\left(-\int ax d\text{sech}^{-1}(ax)+\frac{1}{2}\int ax\text{sech}^{-1}(ax)^2 d\text{sech}^{-1}(ax)+\frac{1}{2}ax\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+ax\text{sech}^{-1}(ax)\right)\right.}{}$$

$\downarrow$ 3042

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^3+\frac{3}{5}\left(\frac{3}{4}\left(-\int \csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)+\frac{1}{2}\int \text{sech}^{-1}(ax)^2\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)ds\right.\right.}{}$$

$\downarrow$ 4257

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^3+\frac{3}{5}\left(\frac{3}{4}\left(\frac{1}{2}\int \text{sech}^{-1}(ax)^2\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)-\arctan\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax}\right)+\frac{1}{2}a\right.\right.}{}$$

$\downarrow$ 4668

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^3+\frac{3}{5}\left(\frac{3}{4}\left(\frac{1}{2}\left(-2i\int \text{sech}^{-1}(ax)\log\left(1-ie^{\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)+2i\int \text{sech}^{-1}(ax)\log\left(1+\right.\right.\right.\right.}{}$$

$\downarrow$ 3011

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^3+\frac{3}{5}\left(\frac{3}{4}\left(\frac{1}{2}\left(2i\left(\int \text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)-\text{sech}^{-1}(ax)\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}}\right.\right.\right.\right.\right.}{}$$

$\downarrow$ 2720

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^3+\frac{3}{5}\left(\frac{3}{4}\left(\frac{1}{2}\left(2i\left(\int e^{-\text{sech}^{-1}(ax)}\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)de^{\text{sech}^{-1}(ax)}-\text{sech}^{-1}(ax)\text{PolyLog}\right.\right.\right.\right.}{}$$

$\downarrow$ 7143

$$-\frac{-\frac{1}{5}a^5x^5\text{sech}^{-1}(ax)^3+\frac{3}{5}\left(\frac{1}{4}a^3x^3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+\frac{1}{6}a^3x^3\text{sech}^{-1}(ax)+\frac{3}{4}\left(\frac{1}{2}\left(2\text{sech}^{-1}(ax)^2\arctan\left(\right.\right.\right.\right.}{}$$

input
```
Int[x^4*ArcSech[a*x]^3,x]
```

3.10. $\int x^4\text{sech}^{-1}(ax)^3\,dx$

output
```
-((-1/5*(a^5*x^5*ArcSech[a*x]^3) + (3*((a^3*x^3*ArcSech[a*x])/6 + (a^3*x^3
*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/4 + (-1/2*(a*x*Sqrt[(
1 - a*x)/(1 + a*x)]*(1 + a*x)) - ArcTan[(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*
x))/(a*x)]/2)/6 + (3*(a*x*ArcSech[a*x] + (a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1
 + a*x)*ArcSech[a*x]^2)/2 - ArcTan[(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(
a*x)] + (2*ArcSech[a*x]^2*ArcTan[E^ArcSech[a*x]] + (2*I)*(-(ArcSech[a*x]*P
olyLog[2, (-I)*E^ArcSech[a*x]]) + PolyLog[3, (-I)*E^ArcSech[a*x]]) - (2*I)
*(-(ArcSech[a*x]*PolyLog[2, I*E^ArcSech[a*x]]) + PolyLog[3, I*E^ArcSech[a*
x]]))/2))/4))/5)/a^5)
```

3.10.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.10. $\int x^4 \text{sech}^{-1}(ax)^3 \, dx$

3.10.4 Maple [F]

$$\int x^4 \operatorname{arcsech}(ax)^3 \, dx$$

input
```
int(x^4*arcsech(a*x)^3,x)
```

output
```
int(x^4*arcsech(a*x)^3,x)
```

3.10.5 Fricas [F]

$$\int x^4 \operatorname{sech}^{-1}(ax)^3 \, dx = \int x^4 \operatorname{arsech}(ax)^3 \, dx$$

input
```
integrate(x^4*arcsech(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^4*arcsech(a*x)^3, x)
```

3.10.6 Sympy [F]

$$\int x^4 \operatorname{sech}^{-1}(ax)^3 \, dx = \int x^4 \operatorname{asech}^3(ax) \, dx$$

input
```
integrate(x**4*asech(a*x)**3,x)
```

output
```
Integral(x**4*asech(a*x)**3, x)
```

3.10.7 Maxima [F]

$$\int x^4 \text{sech}^{-1}(ax)^3 \, dx = \int x^4 \text{arsech}\,(ax)^3 \, dx$$

input
```
integrate(x^4*arcsech(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^4*arcsech(a*x)^3, x)
```

3.10.8 Giac [F]

$$\int x^4 \text{sech}^{-1}(ax)^3 \, dx = \int x^4 \text{arsech}\,(ax)^3 \, dx$$

input
```
integrate(x^4*arcsech(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^4*arcsech(a*x)^3, x)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \text{sech}^{-1}(ax)^3 \, dx = \int x^4 \,\text{acosh}\left(\frac{1}{a\,x}\right)^3 \, dx$$

input
```
int(x^4*acosh(1/(a*x))^3,x)
```

output
```
int(x^4*acosh(1/(a*x))^3, x)
```

3.11 $\int x^3 \text{sech}^{-1}(ax)^3 \, dx$

3.11.1 Optimal result

Integrand size = 10, antiderivative size = 184

$$\int x^3 \text{sech}^{-1}(ax)^3 \, dx = \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{4a^4} - \frac{x^2 \text{sech}^{-1}(ax)}{4a^2} - \frac{\text{sech}^{-1}(ax)^2}{2a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{4a^2} + \frac{1}{4}x^4 \text{sech}^{-1}(ax)^3 + \frac{\text{sech}^{-1}(ax)\log\left(1+e^{2\text{sech}^{-1}(ax)}\right)}{a^4} + \frac{\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)}{2a^4}$$

output
```
-1/4*x^2*arcsech(a*x)/a^2-1/2*arcsech(a*x)^2/a^4+1/4*x^4*arcsech(a*x)^3+ar
csech(a*x)*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)/a^4+1/2*polylog
(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)/a^4+1/4*(a*x+1)*((-a*x+1)/(
a*x+1))^(1/2)/a^4-1/2*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/a^4-
1/4*x^2*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/a^2
```

3.11.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.02

$$\int x^3 \text{sech}^{-1}(ax)^3 \, dx$$
$$= \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax) - \left(-2+2\sqrt{\frac{1-ax}{1+ax}}+2ax\sqrt{\frac{1-ax}{1+ax}}+a^2x^2\sqrt{\frac{1-ax}{1+ax}}+a^3x^3\sqrt{\frac{1-ax}{1+ax}}\right)\text{sech}^{-1}(ax)^2+a^4x^4\text{sech}^{-1}(}{4a^4}$$

input
```
Integrate[x^3*ArcSech[a*x]^3,x]
```

output
```
(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - (-2 + 2*Sqrt[(1 - a*x)/(1 + a*x)] +
 2*a*x*Sqrt[(1 - a*x)/(1 + a*x)] + a^2*x^2*Sqrt[(1 - a*x)/(1 + a*x)] + a^3
*x^3*Sqrt[(1 - a*x)/(1 + a*x)])*ArcSech[a*x]^2 + a^4*x^4*ArcSech[a*x]^3 +
ArcSech[a*x]*(-(a^2*x^2) + 4*Log[1 + E^(-2*ArcSech[a*x])]) - 2*PolyLog[2,
-E^(-2*ArcSech[a*x])])/(4*a^4)
```

3.11.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {6839, 5941, 3042, 4674, 3042, 4254, 24, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{sech}^{-1}(ax)^3 \, dx$$
$$\Big\downarrow \text{ 6839}$$
$$-\frac{\int a^4x^4\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^3 d\text{sech}^{-1}(ax)}{a^4}$$
$$\Big\downarrow \text{ 5941}$$
$$-\frac{\frac{3}{4}\int a^4x^4\text{sech}^{-1}(ax)^2 d\text{sech}^{-1}(ax)-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^3}{a^4}$$
$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^3+\frac{3}{4}\int \text{sech}^{-1}(ax)^2\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^4d\text{sech}^{-1}(ax)}{a^4}$$

$$\Big\downarrow \text{ 4674}$$

$$-\frac{\frac{3}{4}\left(-\frac{1}{3}\int a^2x^2d\text{sech}^{-1}(ax)+\frac{2}{3}\int a^2x^2\text{sech}^{-1}(ax)^2d\text{sech}^{-1}(ax)+\frac{1}{3}a^2x^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+\frac{1}{3}a^2x^2\text{sech}^{-1}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^3+\frac{3}{4}\left(-\frac{1}{3}\int\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^2d\text{sech}^{-1}(ax)+\frac{2}{3}\int\text{sech}^{-1}(ax)^2\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^2ds$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^3+\frac{3}{4}\left(-\frac{1}{3}i\int 1d\left(-i\sqrt{\frac{1-ax}{ax+1}}(ax+1)\right)+\frac{2}{3}\int\text{sech}^{-1}(ax)^2\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^2d\text{sech}^{-1}(ax)\right.$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^3+\frac{3}{4}\left(\frac{2}{3}\int\text{sech}^{-1}(ax)^2\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^2d\text{sech}^{-1}(ax)+\frac{1}{3}a^2x^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(a$$

$$\Big\downarrow \text{ 4672}$$

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^3+\frac{3}{4}\left(\frac{2}{3}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2-2i\int -i\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)d\text{sech}^{-1}(ax)\right)+\frac{1}{3}a$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{\frac{3}{4}\left(\frac{2}{3}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2-2\int\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)d\text{sech}^{-1}(ax)\right)+\frac{1}{3}a^2x^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^3+\frac{3}{4}\left(\frac{2}{3}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2-2\int -i\text{sech}^{-1}(ax)\tan\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)\right)+$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^3+\frac{3}{4}\left(\frac{2}{3}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+2i\int\text{sech}^{-1}(ax)\tan\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)\right)+\frac{1}{3}$$

3.11. $\int x^3\text{sech}^{-1}(ax)^3\,dx$

$\big\downarrow$ 4201

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^3+\frac{3}{4}\left(\frac{2}{3}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+2i\left(2i\int\frac{e^{2\text{sech}^{-1}(ax)}\text{sech}^{-1}(ax)}{1+e^{2\text{sech}^{-1}(ax)}}d\text{sech}^{-1}(ax)-\frac{1}{2}i\text{sech}^{-}\right.\right.\right.}{a^4}$$

$\big\downarrow$ 2620

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^3+\frac{3}{4}\left(\frac{2}{3}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+2i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(ax)\log\left(e^{2\text{sech}^{-1}(ax)}+1\right)-\frac{1}{2}\int\log\left(\right.\right.\right.\right.\right.}{}$$

$\big\downarrow$ 2715

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^3+\frac{3}{4}\left(\frac{2}{3}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+2i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(ax)\log\left(e^{2\text{sech}^{-1}(ax)}+1\right)-\frac{1}{4}\int e^{-2\text{se}}\right.\right.\right.\right.}{}$$

$\big\downarrow$ 2838

$$-\frac{-\frac{1}{4}a^4x^4\text{sech}^{-1}(ax)^3+\frac{3}{4}\left(\frac{1}{3}a^2x^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+\frac{1}{3}a^2x^2\text{sech}^{-1}(ax)+\frac{2}{3}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax\right.\right.}{}$$

input
```
Int[x^3*ArcSech[a*x]^3,x]
```

output
```
-((-1/4*(a^4*x^4*ArcSech[a*x]^3) + (3*(-1/3*(Sqrt[(1 - a*x)/(1 + a*x)]*(1
+ a*x)) + (a^2*x^2*ArcSech[a*x])/3 + (a^2*x^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1
 + a*x)*ArcSech[a*x]^2)/3 + (2*(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSec
h[a*x]^2 + (2*I)*((-1/2*I)*ArcSech[a*x]^2 + (2*I)*((ArcSech[a*x]*Log[1 + E
^(2*ArcSech[a*x])])/2 + PolyLog[2, -E^(2*ArcSech[a*x])]/4))))/3))/4)/a^4)
```

3.11.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.11. $\int x^3 \text{sech}^{-1}(ax)^3 \, dx$

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.11.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{\frac{a^4x^4\,\mathrm{arcsech}(ax)^3}{4}-\frac{\mathrm{arcsech}(ax)^2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,a^3x^3}{4}-\frac{\mathrm{arcsech}(ax)^2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax}{2}-\frac{\mathrm{arcsech}(ax)a^2x^2}{4}+\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\cdots}}{4}}{a^4}$
default	$\frac{\frac{a^4x^4\,\mathrm{arcsech}(ax)^3}{4}-\frac{\mathrm{arcsech}(ax)^2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,a^3x^3}{4}-\frac{\mathrm{arcsech}(ax)^2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax}{2}-\frac{\mathrm{arcsech}(ax)a^2x^2}{4}+\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\cdots}}{4}}{a^4}$

input
```
int(x^3*arcsech(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*a^4*x^4*arcsech(a*x)^3-1/4*arcsech(a*x)^2*(-(a*x-1)/a/x)^(1/2)*
((a*x+1)/a/x)^(1/2)*a^3*x^3-1/2*arcsech(a*x)^2*(-(a*x-1)/a/x)^(1/2)*((a*x+
1)/a/x)^(1/2)*a*x-1/4*arcsech(a*x)*a^2*x^2+1/4*(-(a*x-1)/a/x)^(1/2)*((a*x+
1)/a/x)^(1/2)*a*x-1/2*arcsech(a*x)^2-1/4+arcsech(a*x)*ln(1+(1/a/x+(1/a/x-1
)^(1/2)*(1+1/a/x)^(1/2))^2)+1/2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x
)^(1/2))^2))
```

3.11.5 Fricas [F]

$$\int x^3 \text{sech}^{-1}(ax)^3 \, dx = \int x^3 \operatorname{arsech}(ax)^3 \, dx$$

input
```
integrate(x^3*arcsech(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^3*arcsech(a*x)^3, x)
```

3.11.6 Sympy [F]

$$\int x^3 \text{sech}^{-1}(ax)^3 \, dx = \int x^3 \operatorname{asech}^3(ax) \, dx$$

input
```
integrate(x**3*asech(a*x)**3,x)
```

output
```
Integral(x**3*asech(a*x)**3, x)
```

3.11.7 Maxima [F]

$$\int x^3 \text{sech}^{-1}(ax)^3 \, dx = \int x^3 \operatorname{arsech}(ax)^3 \, dx$$

input
```
integrate(x^3*arcsech(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^3*arcsech(a*x)^3, x)
```

3.11.8 Giac [F]

$$\int x^3 \text{sech}^{-1}(ax)^3 \, dx = \int x^3 \, \text{arsech}\,(ax)^3 \, dx$$

input
```
integrate(x^3*arcsech(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^3*arcsech(a*x)^3, x)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \text{sech}^{-1}(ax)^3 \, dx = \int x^3 \, \text{acosh}\left(\frac{1}{a\,x}\right)^3 \, dx$$

input
```
int(x^3*acosh(1/(a*x))^3,x)
```

output
```
int(x^3*acosh(1/(a*x))^3, x)
```

3.12 $\int x^2\text{sech}^{-1}(ax)^3\,dx$

3.12.1 Optimal result

Integrand size = 10, antiderivative size = 198

$$\begin{aligned}\int x^2\text{sech}^{-1}(ax)^3\,dx =& -\frac{x\text{sech}^{-1}(ax)}{a^2}-\frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} \\ &+\frac{1}{3}x^3\text{sech}^{-1}(ax)^3-\frac{\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{a^3} \\ &+\frac{\arctan\left(\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax}\right)}{a^3}+\frac{i\text{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{a^3} \\ &-\frac{i\text{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{a^3} \\ &-\frac{i\operatorname{PolyLog}\left(3,-ie^{\text{sech}^{-1}(ax)}\right)}{a^3}+\frac{i\operatorname{PolyLog}\left(3,ie^{\text{sech}^{-1}(ax)}\right)}{a^3}\end{aligned}$$

output
```
-x*arcsech(a*x)/a^2+1/3*x^3*arcsech(a*x)^3-arcsech(a*x)^2*arctan(1/a/x+(1/
a/x-1)^(1/2)*(1+1/a/x)^(1/2))/a^3+arctan((a*x+1)*((-a*x+1)/(a*x+1))^(1/2)/
a/x)/a^3+I*arcsech(a*x)*polylog(2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2
)))/a^3-I*arcsech(a*x)*polylog(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))
)/a^3-I*polylog(3,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^3+I*polylo
g(3,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^3-1/2*x*(a*x+1)*arcsech(a
*x)^2*((-a*x+1)/(a*x+1))^(1/2)/a^2
```

3.12.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.01

$$\int x^2\text{sech}^{-1}(ax)^3\,dx$$
$$= \frac{-6ax\text{sech}^{-1}(ax) - 3ax\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2 + 2a^3x^3\text{sech}^{-1}(ax)^3 + 3i\Big(-4i\arctan\left(\tanh\left(\tfrac{1}{2}\text{sech}^{-1}(\right.\right.}{}$$

input
```
Integrate[x^2*ArcSech[a*x]^3,x]
```

output
```
(-6*a*x*ArcSech[a*x] - 3*a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a
*x]^2 + 2*a^3*x^3*ArcSech[a*x]^3 + (3*I)*((-4*I)*ArcTan[Tanh[ArcSech[a*x]/
2]] + ArcSech[a*x]^2*Log[1 - I/E^ArcSech[a*x]] - ArcSech[a*x]^2*Log[1 + I/
E^ArcSech[a*x]] + 2*ArcSech[a*x]*PolyLog[2, (-I)/E^ArcSech[a*x]] - 2*ArcSe
ch[a*x]*PolyLog[2, I/E^ArcSech[a*x]] + 2*PolyLog[3, (-I)/E^ArcSech[a*x]] -
 2*PolyLog[3, I/E^ArcSech[a*x]]))/(6*a^3)
```

3.12.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6839, 5941, 3042, 4674, 3042, 4257, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{sech}^{-1}(ax)^3\,dx$$
$$\Big\downarrow \text{6839}$$
$$-\frac{\int a^3x^3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^3d\text{sech}^{-1}(ax)}{a^3}$$
$$\Big\downarrow \text{5941}$$
$$-\frac{\int a^3x^3\text{sech}^{-1}(ax)^2d\text{sech}^{-1}(ax) - \frac{1}{3}a^3x^3\text{sech}^{-1}(ax)^3}{a^3}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{-\frac{1}{3}a^3x^3\text{sech}^{-1}(ax)^3 + \int \text{sech}^{-1}(ax)^2\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^3d\text{sech}^{-1}(ax)}{a^3}$$

$\big\downarrow$ 4674

$$-\frac{-\int axd\text{sech}^{-1}(ax)+\frac{1}{2}\int ax\text{sech}^{-1}(ax)^2d\text{sech}^{-1}(ax)-\frac{1}{3}a^3x^3\text{sech}^{-1}(ax)^3+\frac{1}{2}ax\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+}{a^3}$$

$\big\downarrow$ 3042

$$-\frac{-\int \csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)+\frac{1}{2}\int \text{sech}^{-1}(ax)^2\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)-\frac{1}{3}a^3x^3\text{sech}^{-1}(ax}{a^3}$$

$\big\downarrow$ 4257

$$-\frac{\frac{1}{2}\int \text{sech}^{-1}(ax)^2\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)-\frac{1}{3}a^3x^3\text{sech}^{-1}(ax)^3-\arctan\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax}\right)+\frac{1}{2}ax\sqrt{\frac{1-ax}{ax+1}}}{a^3}$$

$\big\downarrow$ 4668

$$-\frac{\frac{1}{2}\left(-2i\int \text{sech}^{-1}(ax)\log\left(1-ie^{\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)+2i\int \text{sech}^{-1}(ax)\log\left(1+ie^{\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)+2s\right.}{}$$

$\big\downarrow$ 3011

$$-\frac{\frac{1}{2}\left(2i\left(\int \text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)-\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)\right)-2i\left(\int \text{PolyLog}\left(2,i\right.\right.\right.}{}$$

$\big\downarrow$ 2720

$$-\frac{\frac{1}{2}\left(2i\left(\int e^{-\text{sech}^{-1}(ax)}\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)de^{\text{sech}^{-1}(ax)}-\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)\right)-2i\left(\int e^{-}\right.\right.}{}$$

$\big\downarrow$ 7143

$$-\frac{-\frac{1}{3}a^3x^3\text{sech}^{-1}(ax)^3+\frac{1}{2}\left(2\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{sech}^{-1}(ax)}\right)-\text{sech}^{-1}(ax)\,\text{PolyL}\right.\right.}{}$$

input
```
Int[x^2*ArcSech[a*x]^3,x]
```

3.12. $\int x^2\text{sech}^{-1}(ax)^3\,dx$

output
```
-((a*x*ArcSech[a*x] + (a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x
]^2)/2 - (a^3*x^3*ArcSech[a*x]^3)/3 - ArcTan[(Sqrt[(1 - a*x)/(1 + a*x)]*(1
 + a*x))/(a*x)] + (2*ArcSech[a*x]^2*ArcTan[E^ArcSech[a*x]] + (2*I)*(-(ArcS
ech[a*x]*PolyLog[2, (-I)*E^ArcSech[a*x]]) + PolyLog[3, (-I)*E^ArcSech[a*x]
]) - (2*I)*(-(ArcSech[a*x]*PolyLog[2, I*E^ArcSech[a*x]]) + PolyLog[3, I*E^
ArcSech[a*x]]))/2)/a^3)
```

3.12.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.12.4 Maple [F]

$$\int x^2 \operatorname{arcsech}(ax)^3\,dx$$

input
```
int(x^2*arcsech(a*x)^3,x)
```

output
```
int(x^2*arcsech(a*x)^3,x)
```

3.12.5 Fricas [F]

$$\int x^2 \text{sech}^{-1}(ax)^3 \, dx = \int x^2 \,\text{arsech}\,(ax)^3 \, dx$$

input
```
integrate(x^2*arcsech(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^2*arcsech(a*x)^3, x)
```

3.12.6 Sympy [F]

$$\int x^2 \text{sech}^{-1}(ax)^3 \, dx = \int x^2 \,\text{asech}^3\,(ax) \, dx$$

input
```
integrate(x**2*asech(a*x)**3,x)
```

output
```
Integral(x**2*asech(a*x)**3, x)
```

3.12.7 Maxima [F]

$$\int x^2 \text{sech}^{-1}(ax)^3 \, dx = \int x^2 \,\text{arsech}\,(ax)^3 \, dx$$

input
```
integrate(x^2*arcsech(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^2*arcsech(a*x)^3, x)
```

3.12.8 Giac [F]

$$\int x^2 \text{sech}^{-1}(ax)^3 \, dx = \int x^2 \operatorname{arsech}(ax)^3 \, dx$$

input
```
integrate(x^2*arcsech(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^2*arcsech(a*x)^3, x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{sech}^{-1}(ax)^3 \, dx = \int x^2 \operatorname{acosh}\left(\frac{1}{a\,x}\right)^3 \, dx$$

input
```
int(x^2*acosh(1/(a*x))^3,x)
```

output
```
int(x^2*acosh(1/(a*x))^3, x)
```

3.13 $\int x\text{sech}^{-1}(ax)^3\,dx$

3.13.1 Optimal result

Integrand size = 8, antiderivative size = 102

$$\int x\text{sech}^{-1}(ax)^3\,dx=-\frac{3\text{sech}^{-1}(ax)^2}{2a^2}-\frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2}+\frac{1}{2}x^2\text{sech}^{-1}(ax)^3+\frac{3\text{sech}^{-1}(ax)\log\left(1+e^{2\text{sech}^{-1}(ax)}\right)}{a^2}+\frac{3\operatorname{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)}{2a^2}$$

output
```
-3/2*arcsech(a*x)^2/a^2+1/2*x^2*arcsech(a*x)^3+3*arcsech(a*x)*ln(1+(1/a/x+
(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)/a^2+3/2*polylog(2,-(1/a/x+(1/a/x-1)^(1
/2)*(1+1/a/x)^(1/2))^2)/a^2-3/2*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^
(1/2)/a^2
```

3.13.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

$$\int x\text{sech}^{-1}(ax)^3\,dx$$
$$=\frac{\text{sech}^{-1}(ax)\left(-3\left(-1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)\text{sech}^{-1}(ax)+a^2x^2\text{sech}^{-1}(ax)^2+6\log\left(1+e^{-2\text{sech}^{-1}(ax)}\right)\right)-}{2a^2}$$

input
```
Integrate[x*ArcSech[a*x]^3,x]
```

output
```
(ArcSech[a*x]*(-3*(-1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1
+ a*x)])*ArcSech[a*x] + a^2*x^2*ArcSech[a*x]^2 + 6*Log[1 + E^(-2*ArcSech[a
*x])]) - 3*PolyLog[2, -E^(-2*ArcSech[a*x])])/(2*a^2)
```

3.13.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.375, Rules used = {6839, 5941, 3042, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{sech}^{-1}(ax)^3\,dx$$

$$\Big\downarrow \text{6839}$$

$$-\frac{\int a^2x^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^3d\text{sech}^{-1}(ax)}{a^2}$$

$$\Big\downarrow \text{5941}$$

$$-\frac{\frac{3}{2}\int a^2x^2\text{sech}^{-1}(ax)^2d\text{sech}^{-1}(ax)-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^3}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^3+\frac{3}{2}\int \text{sech}^{-1}(ax)^2\csc\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^2d\text{sech}^{-1}(ax)}{a^2}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^3+\frac{3}{2}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2-2i\int -i\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)d\text{sech}^{-1}(ax)\right)}{a^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{3}{2}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2-2\int \sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)d\text{sech}^{-1}(ax)\right)-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^3}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^3+\frac{3}{2}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2-2\int -i\text{sech}^{-1}(ax)\tan\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)\right)}{a^2}$$

$\Big\downarrow$ 26

$$-\frac{-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^3+\frac{3}{2}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+2i\int \text{sech}^{-1}(ax)\tan\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)\right)}{a^2}$$

$\Big\downarrow$ 4201

$$-\frac{-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^3+\frac{3}{2}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+2i\left(2i\int \frac{e^{2\text{sech}^{-1}(ax)}\text{sech}^{-1}(ax)}{1+e^{2\text{sech}^{-1}(ax)}}d\text{sech}^{-1}(ax)-\frac{1}{2}i\text{sech}^{-1}(a\right.\right.}{a^2}$$

$\Big\downarrow$ 2620

$$-\frac{-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^3+\frac{3}{2}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+2i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(ax)\log\left(e^{2\text{sech}^{-1}(ax)}+1\right)-\frac{1}{2}\int\log\left(1+\right.\right.\right.\right.}{a^2}$$

$\Big\downarrow$ 2715

$$-\frac{-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^3+\frac{3}{2}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+2i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(ax)\log\left(e^{2\text{sech}^{-1}(ax)}+1\right)-\frac{1}{4}\int e^{-2\text{sech}^{-}}\right.\right.\right.}{a^2}$$

$\Big\downarrow$ 2838

$$-\frac{-\frac{1}{2}a^2x^2\text{sech}^{-1}(ax)^3+\frac{3}{2}\left(\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2+2i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)+\frac{1}{2}\text{sech}^{-1}(ax)\log\right.\right.\right.}{a^2}$$

input
```
Int[x*ArcSech[a*x]^3,x]
```

output
```
-((-1/2*(a^2*x^2*ArcSech[a*x]^3) + (3*(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)
*ArcSech[a*x]^2 + (2*I)*((-1/2*I)*ArcSech[a*x]^2 + (2*I)*((ArcSech[a*x]*Lo
g[1 + E^(2*ArcSech[a*x])])/2 + PolyLog[2, -E^(2*ArcSech[a*x])]/4))))/2)/a^
2)
```

3.13. $\int x\text{sech}^{-1}(ax)^3\,dx$

3.13.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.13.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsech}(ax)^2\left(-3\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,ax+\operatorname{arcsech}(ax)a^2x^2+3\right)}{2}-3\operatorname{arcsech}(ax)^2+3\operatorname{arcsech}(ax)\ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\sqrt{1+\frac{1}{ax}}\right.\right.}{a^2}$
default	$\frac{\frac{\operatorname{arcsech}(ax)^2\left(-3\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,ax+\operatorname{arcsech}(ax)a^2x^2+3\right)}{2}-3\operatorname{arcsech}(ax)^2+3\operatorname{arcsech}(ax)\ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\sqrt{1+\frac{1}{ax}}\right.\right.}{a^2}$

input
```
int(x*arcsech(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2*arcsech(a*x)^2*(-3*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*a*x
+arcsech(a*x)*a^2*x^2+3)-3*arcsech(a*x)^2+3*arcsech(a*x)*ln(1+(1/a/x+(1/a/
x-1)^(1/2)*(1+1/a/x)^(1/2))^2)+3/2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/
a/x)^(1/2))^2))
```

3.13.5 Fricas [F]

$$\int x\operatorname{sech}^{-1}(ax)^3\,dx=\int x\operatorname{arsech}(ax)^3\,dx$$

input
```
integrate(x*arcsech(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x*arcsech(a*x)^3, x)
```

3.13.6 Sympy [F]

$$\int x \operatorname{sech}^{-1}(ax)^3 \, dx = \int x \operatorname{asech}^3(ax) \, dx$$

input
```
integrate(x*asech(a*x)**3,x)
```

output
```
Integral(x*asech(a*x)**3, x)
```

3.13.7 Maxima [F]

$$\int x \operatorname{sech}^{-1}(ax)^3 \, dx = \int x \operatorname{arsech}(ax)^3 \, dx$$

input
```
integrate(x*arcsech(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x*arcsech(a*x)^3, x)
```

3.13.8 Giac [F]

$$\int x \operatorname{sech}^{-1}(ax)^3 \, dx = \int x \operatorname{arsech}(ax)^3 \, dx$$

input
```
integrate(x*arcsech(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x*arcsech(a*x)^3, x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int x\text{sech}^{-1}(ax)^3 \, dx = \int x \,\text{acosh}\left(\frac{1}{a\,x}\right)^3 \,dx$$

input
```
int(x*acosh(1/(a*x))^3,x)
```

output
```
int(x*acosh(1/(a*x))^3, x)
```

3.14 $\int \operatorname{sech}^{-1}(ax)^3 \, dx$

3.14.1 Optimal result

Integrand size = 6, antiderivative size = 111

$$\int \operatorname{sech}^{-1}(ax)^3 \, dx = x\operatorname{sech}^{-1}(ax)^3 - \frac{6\operatorname{sech}^{-1}(ax)^2 \arctan\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i\operatorname{PolyLog}\left(3,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i\operatorname{PolyLog}\left(3,ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}$$

output
```
x*arcsech(a*x)^3-6*arcsech(a*x)^2*arctan(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(
1/2))/a+6*I*arcsech(a*x)*polylog(2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/
2)))/a-6*I*arcsech(a*x)*polylog(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)
))/a-6*I*polylog(3,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a+6*I*polyl
og(3,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a
```

3.14.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int \operatorname{sech}^{-1}(ax)^3\,dx = x\operatorname{sech}^{-1}(ax)^3$$
$$-\frac{3i\left(-\operatorname{sech}^{-1}(ax)^2\left(\log\left(1-ie^{-\operatorname{sech}^{-1}(ax)}\right)-\log\left(1+ie^{-\operatorname{sech}^{-1}(ax)}\right)\right)-2\operatorname{sech}^{-1}(ax)\left(\operatorname{PolyLog}\left(2,-ie^{-\operatorname{s}}\right.\right.\right.}{}$$

input

```
Integrate[ArcSech[a*x]^3,x]
```

output

```
x*ArcSech[a*x]^3 - ((3*I)*(-(ArcSech[a*x]^2*(Log[1 - I/E^ArcSech[a*x]] - L
og[1 + I/E^ArcSech[a*x]])) - 2*ArcSech[a*x]*(PolyLog[2, (-I)/E^ArcSech[a*x
]] - PolyLog[2, I/E^ArcSech[a*x]]) - 2*(PolyLog[3, (-I)/E^ArcSech[a*x]] -
PolyLog[3, I/E^ArcSech[a*x]])))/a
```

3.14.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {6833, 5941, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^{-1}(ax)^3\,dx$$
$$\Big\downarrow \text{6833}$$
$$-\frac{\int ax\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^3 d\operatorname{sech}^{-1}(ax)}{a}$$
$$\Big\downarrow \text{5941}$$
$$-\frac{3\int ax\operatorname{sech}^{-1}(ax)^2 d\operatorname{sech}^{-1}(ax)-ax\operatorname{sech}^{-1}(ax)^3}{a}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{-ax\operatorname{sech}^{-1}(ax)^3+3\int \operatorname{sech}^{-1}(ax)^2\csc\left(i\operatorname{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\operatorname{sech}^{-1}(ax)}{a}$$
$$\Big\downarrow \text{4668}$$

$$-\frac{-ax\text{sech}^{-1}(ax)^3+3\left(-2i\int \text{sech}^{-1}(ax)\log\left(1-ie^{\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)+2i\int \text{sech}^{-1}(ax)\log\left(1+ie^{\text{sech}^{-1}(ax)}\right)\right.}{a}$$

$\downarrow$ 3011

$$-\frac{-ax\text{sech}^{-1}(ax)^3+3\left(2i\left(\int \text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)-\text{sech}^{-1}(ax)\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)\right)-\right.}{}$$

$\downarrow$ 2720

$$-\frac{-ax\text{sech}^{-1}(ax)^3+3\left(2i\left(\int e^{-\text{sech}^{-1}(ax)}\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)de^{\text{sech}^{-1}(ax)}-\text{sech}^{-1}(ax)\text{PolyLog}\left(2,-ie^{\text{sech}}\right.\right.\right.}{}$$

$\downarrow$ 7143

$$-\frac{-ax\text{sech}^{-1}(ax)^3+3\left(2\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{sech}^{-1}(ax)}\right)-\text{sech}^{-1}(ax)\text{PolyLog}\right.\right.}{a}$$

input
```
Int[ArcSech[a*x]^3,x]
```

output
```
-((-(a*x*ArcSech[a*x]^3) + 3*(2*ArcSech[a*x]^2*ArcTan[E^ArcSech[a*x]] + (2
*I)*(-(ArcSech[a*x]*PolyLog[2, (-I)*E^ArcSech[a*x]]) + PolyLog[3, (-I)*E^A
rcSech[a*x]]) - (2*I)*(-(ArcSech[a*x]*PolyLog[2, I*E^ArcSech[a*x]]) + Poly
Log[3, I*E^ArcSech[a*x]])))/a)
```

3.14.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 6833
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-c^(-1)   S
ubst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
 b, c, n}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.14.4 Maple [F]

$$\int \operatorname{arcsech}(ax)^3 \, dx$$

input
```
int(arcsech(a*x)^3,x)
```

output
```
int(arcsech(a*x)^3,x)
```

3.14.5 Fricas [F]

$$\int \operatorname{sech}^{-1}(ax)^3\,dx = \int \operatorname{arsech}(ax)^3\,dx$$

input
```
integrate(arcsech(a*x)^3,x, algorithm="fricas")
```

output
```
integral(arcsech(a*x)^3, x)
```

3.14.6 Sympy [F]

$$\int \operatorname{sech}^{-1}(ax)^3\,dx = \int \operatorname{asech}^3(ax)\,dx$$

input
```
integrate(asech(a*x)**3,x)
```

output
```
Integral(asech(a*x)**3, x)
```

3.14.7 Maxima [F]

$$\int \operatorname{sech}^{-1}(ax)^3\,dx = \int \operatorname{arsech}(ax)^3\,dx$$

input
```
integrate(arcsech(a*x)^3,x, algorithm="maxima")
```

output
```
x*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1)^3 - integrate((a^2*x^2*log(a)^3 +
(a^2*x^2 - 1)*log(x)^3 + 3*(a^2*x^2*log(a) + (a^2*x^2*(log(a) + 1) + (a^2*
x^2 - 1)*log(x) - log(a))*sqrt(a*x + 1)*sqrt(-a*x + 1) + (a^2*x^2 - 1)*log
(x) - log(a))*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1)^2 - log(a)^3 + 3*(a^2*
x^2*log(a) - log(a))*log(x)^2 + (a^2*x^2*log(a)^3 + (a^2*x^2 - 1)*log(x)^3
 - log(a)^3 + 3*(a^2*x^2*log(a) - log(a))*log(x)^2 + 3*(a^2*x^2*log(a)^2 -
 log(a)^2)*log(x))*sqrt(a*x + 1)*sqrt(-a*x + 1) - 3*(a^2*x^2*log(a)^2 + (a
^2*x^2 - 1)*log(x)^2 + (a^2*x^2*log(a)^2 + (a^2*x^2 - 1)*log(x)^2 - log(a)
^2 + 2*(a^2*x^2*log(a) - log(a))*log(x))*sqrt(a*x + 1)*sqrt(-a*x + 1) - lo
g(a)^2 + 2*(a^2*x^2*log(a) - log(a))*log(x))*log(sqrt(a*x + 1)*sqrt(-a*x +
 1) + 1) + 3*(a^2*x^2*log(a)^2 - log(a)^2)*log(x))/(a^2*x^2 + (a^2*x^2 - 1
)*sqrt(a*x + 1)*sqrt(-a*x + 1) - 1), x)
```

3.14.8 Giac [F]

$$\int \operatorname{sech}^{-1}(ax)^3\,dx = \int \operatorname{arsech}\left(ax\right)^3\,dx$$

input
```
integrate(arcsech(a*x)^3,x, algorithm="giac")
```

output
```
integrate(arcsech(a*x)^3, x)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^{-1}(ax)^3\,dx = \int \operatorname{acosh}\left(\frac{1}{a\,x}\right)^3\,dx$$

input
```
int(acosh(1/(a*x))^3,x)
```

output
```
int(acosh(1/(a*x))^3, x)
```

3.15 $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x} dx$

3.15.1 Optimal result

Integrand size = 10, antiderivative size = 88

$$\begin{aligned}\int \frac{\operatorname{sech}^{-1}(ax)^3}{x} dx = {} & \frac{1}{4}\operatorname{sech}^{-1}(ax)^4 - \operatorname{sech}^{-1}(ax)^3 \log\left(1 + e^{2\operatorname{sech}^{-1}(ax)}\right) \\ & - \frac{3}{2}\operatorname{sech}^{-1}(ax)^2 \operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ax)}\right) \\ & + \frac{3}{2}\operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(3, -e^{2\operatorname{sech}^{-1}(ax)}\right) \\ & - \frac{3}{4} \operatorname{PolyLog}\left(4, -e^{2\operatorname{sech}^{-1}(ax)}\right)\end{aligned}$$

output
```
1/4*arcsech(a*x)^4-arcsech(a*x)^3*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1
/2))^2)-3/2*arcsech(a*x)^2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/
2))^2)+3/2*arcsech(a*x)*polylog(3,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))
^2)-3/4*polylog(4,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \frac{\operatorname{sech}^{-1}(ax)^3}{x} dx = {} & \frac{1}{4}\Big(-\operatorname{sech}^{-1}(ax)^4 - 4\operatorname{sech}^{-1}(ax)^3 \log\left(1 + e^{-2\operatorname{sech}^{-1}(ax)}\right) \\ & + 6\operatorname{sech}^{-1}(ax)^2 \operatorname{PolyLog}\left(2, -e^{-2\operatorname{sech}^{-1}(ax)}\right) \\ & + 6\operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(3, -e^{-2\operatorname{sech}^{-1}(ax)}\right) \\ & + 3 \operatorname{PolyLog}\left(4, -e^{-2\operatorname{sech}^{-1}(ax)}\right)\Big)\end{aligned}$$

input
```
Integrate[ArcSech[a*x]^3/x,x]
```

output
```
(-ArcSech[a*x]^4 - 4*ArcSech[a*x]^3*Log[1 + E^(-2*ArcSech[a*x])] + 6*ArcSe
ch[a*x]^2*PolyLog[2, -E^(-2*ArcSech[a*x])] + 6*ArcSech[a*x]*PolyLog[3, -E^
(-2*ArcSech[a*x])] + 3*PolyLog[4, -E^(-2*ArcSech[a*x])])/4
```

3.15.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6839, 3042, 26, 4201, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(ax)^3}{x} \, dx$$

$$\Big\downarrow \text{6839}$$

$$-\int \sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^3d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{3042}$$

$$-\int -i\text{sech}^{-1}(ax)^3\tan\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{26}$$

$$i\int \text{sech}^{-1}(ax)^3\tan\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{4201}$$

$$i\left(2i\int \frac{e^{2\text{sech}^{-1}(ax)}\text{sech}^{-1}(ax)^3}{1+e^{2\text{sech}^{-1}(ax)}}d\text{sech}^{-1}(ax)-\frac{1}{4}i\text{sech}^{-1}(ax)^4\right)$$

$$\Big\downarrow \text{2620}$$

$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(ax)^3\log\left(e^{2\text{sech}^{-1}(ax)}+1\right)-\frac{3}{2}\int \text{sech}^{-1}(ax)^2\log\left(1+e^{2\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)\right)-\frac{1}{4}i\text{sech}^{-1}(ax\right.$$

$$\Big\downarrow \text{3011}$$

$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(ax)^3\log\left(e^{2\text{sech}^{-1}(ax)}+1\right)-\frac{3}{2}\left(\int\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)-\frac{1}{2}\text{sech}^{-1}(\right.\right.\right.$$

$\downarrow$ 7163

$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(ax)^3\log\left(e^{2\text{sech}^{-1}(ax)}+1\right)-\frac{3}{2}\left(-\frac{1}{2}\int\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(ax)}\right)d\text{sech}^{-1}(ax)-\frac{1}{2}\text{sech}^{-1}(ax)^2\,\text{Po}\right.\right.\right.$$

$\downarrow$ 2720

$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(ax)^3\log\left(e^{2\text{sech}^{-1}(ax)}+1\right)-\frac{3}{2}\left(-\frac{1}{4}\int e^{-2\text{sech}^{-1}(ax)}\,\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(ax)}\right)de^{2\text{sech}^{-1}(ax)}-\frac{1}{2}\text{sec}\right.\right.\right.$$

$\downarrow$ 7143

$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(ax)^3\log\left(e^{2\text{sech}^{-1}(ax)}+1\right)-\frac{3}{2}\left(-\frac{1}{2}\text{sech}^{-1}(ax)^2\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)+\frac{1}{2}\text{sech}^{-1}(ax)\,\text{PolyLo}\right.\right.\right.$$

input
```
Int[ArcSech[a*x]^3/x,x]
```

output
```
I*((-1/4*I)*ArcSech[a*x]^4 + (2*I)*((ArcSech[a*x]^3*Log[1 + E^(2*ArcSech[a
*x])])/2 - (3*(-1/2*(ArcSech[a*x]^2*PolyLog[2, -E^(2*ArcSech[a*x])]) + (Ar
cSech[a*x]*PolyLog[3, -E^(2*ArcSech[a*x])])/2 - PolyLog[4, -E^(2*ArcSech[a
*x])]/4))/2))
```

3.15.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.15. $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x} \, dx$

3.15.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.06

method	result
derivativedivides	$\frac{\operatorname{arcsech}(ax)^4}{4} - \operatorname{arcsech}(ax)^3 \ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\sqrt{1+\frac{1}{ax}}\right)^2\right) - \frac{3\operatorname{arcsech}(ax)^2 \operatorname{polylog}\left(2,-\right.}{}$
default	$\frac{\operatorname{arcsech}(ax)^4}{4} - \operatorname{arcsech}(ax)^3 \ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\sqrt{1+\frac{1}{ax}}\right)^2\right) - \frac{3\operatorname{arcsech}(ax)^2 \operatorname{polylog}\left(2,-\right.}{}$

input
```
int(arcsech(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*arcsech(a*x)^4-arcsech(a*x)^3*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1
/2))^2)-3/2*arcsech(a*x)^2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/
2))^2)+3/2*arcsech(a*x)*polylog(3,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))
^2)-3/4*polylog(4,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
```

3.15.5 Fricas [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arsech}(ax)^3}{x}\,dx$$

input
```
integrate(arcsech(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral(arcsech(a*x)^3/x, x)
```

3.15.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{asech}^3(ax)}{x}\,dx$$

input
```
integrate(asech(a*x)**3/x,x)
```

output
```
Integral(asech(a*x)**3/x, x)
```

3.15.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arsech}\left(ax\right)^3}{x}\,dx$$

input
```
integrate(arcsech(a*x)^3/x,x, algorithm="maxima")
```

output
```
integrate(arcsech(a*x)^3/x, x)
```

3.15.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arsech}\left(ax\right)^3}{x}\,dx$$

input
```
integrate(arcsech(a*x)^3/x,x, algorithm="giac")
```

output
```
integrate(arcsech(a*x)^3/x, x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^3}{x}\,dx$$

input
```
int(acosh(1/(a*x))^3/x,x)
```

output
```
int(acosh(1/(a*x))^3/x, x)
```

3.16 $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2} dx$

3.16.1 Optimal result

Integrand size = 10, antiderivative size = 83

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2} dx = \frac{6\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{x} - \frac{6\operatorname{sech}^{-1}(ax)}{x} + \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{x} - \frac{\operatorname{sech}^{-1}(ax)^3}{x}$$

output
```
-6*arcsech(a*x)/x-arcsech(a*x)^3/x+6*(a*x+1)*((-a*x+1)/(a*x+1))^(1/2)/x+3*
(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/x
```

3.16.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2} dx = \frac{6\sqrt{\frac{1-ax}{1+ax}}(1+ax) - 6\operatorname{sech}^{-1}(ax) + 3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2 - \operatorname{sech}^{-1}(ax)^3}{x}$$

input
```
Integrate[ArcSech[a*x]^3/x^2,x]
```

output
```
(6*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - 6*ArcSech[a*x] + 3*Sqrt[(1 - a*x)
/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 - ArcSech[a*x]^3)/x
```

3.16.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.39, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {6839, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{6839}$$

$$-a\int \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^3}{ax}d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{3042}$$

$$-a\int -i\text{sech}^{-1}(ax)^3\sin\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{26}$$

$$ia\int \text{sech}^{-1}(ax)^3\sin\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{3777}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^3}{ax}-3i\int \frac{\text{sech}^{-1}(ax)^2}{ax}d\text{sech}^{-1}(ax)\right)$$

$$\Big\downarrow \text{3042}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^3}{ax}-3i\int \text{sech}^{-1}(ax)^2\sin\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)\right)$$

$$\Big\downarrow \text{3777}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^3}{ax}-3i\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{ax}-2i\int -\frac{i\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{ax}d\text{sech}^{-1}(ax)\right)\right)$$

$$\Big\downarrow \text{26}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^3}{ax}-3i\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{ax}-2\int \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{ax}d\text{sech}^{-1}(ax)\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^3}{ax}-3i\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{ax}-2\int -i\text{sech}^{-1}(ax)\sin\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)\right)\right)$$

$$\Big\downarrow \text{26}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^3}{ax}-3i\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{ax}+2i\int \text{sech}^{-1}(ax)\sin\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)\right)\right)$$

$$\Big\downarrow \text{3777}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^3}{ax}-3i\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{ax}+2i\left(\frac{i\text{sech}^{-1}(ax)}{ax}-i\int\frac{1}{ax}d\text{sech}^{-1}(ax)\right)\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^3}{ax}-3i\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{ax}+2i\left(\frac{i\text{sech}^{-1}(ax)}{ax}-i\int\sin\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax\right.\right.\right.$$

$$\Big\downarrow \text{3117}$$

$$ia\left(\frac{i\text{sech}^{-1}(ax)^3}{ax}-3i\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{ax}+2i\left(\frac{i\text{sech}^{-1}(ax)}{ax}-\frac{i\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax}\right)\right)\right)$$

input
```
Int[ArcSech[a*x]^3/x^2,x]
```

output
```
I*a*((I*ArcSech[a*x]^3)/(a*x) - (3*I)*((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x
)*ArcSech[a*x]^2)/(a*x) + (2*I)*(((-I)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)
)/(a*x) + (I*ArcSech[a*x])/(a*x))))
```

3.16. $\int \frac{\text{sech}^{-1}(ax)^3}{x^2}\,dx$

3.16.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.16.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

method	result
derivativedivides	$a\left(-\frac{\operatorname{arcsech}(ax)^3}{ax}+3\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)^2-\frac{6\operatorname{arcsech}(ax)}{ax}+6\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\right)$
default	$a\left(-\frac{\operatorname{arcsech}(ax)^3}{ax}+3\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)^2-\frac{6\operatorname{arcsech}(ax)}{ax}+6\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\right)$

input
```
int(arcsech(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/a/x*arcsech(a*x)^3+3*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*arcsec
h(a*x)^2-6/a/x*arcsech(a*x)+6*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.87

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2}\,dx = \frac{3\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2 - \log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^3 + 6\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}} - 6\,\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)}{x}$$

input
```
integrate(arcsech(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
(3*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^
2)) + 1)/(a*x))^2 - log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^3
+ 6*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) - 6*log((a*x*sqrt(-(a^2*x^2 - 1)/(a
^2*x^2)) + 1)/(a*x)))/x
```

3.16.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{asech}^3(ax)}{x^2}\,dx$$

input
```
integrate(asech(a*x)**3/x**2,x)
```

output
```
Integral(asech(a*x)**3/x**2, x)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2}\,dx = 3\,a\sqrt{\frac{1}{a^2x^2}-1}\,\operatorname{arsech}(ax)^2 - \frac{\operatorname{arsech}(ax)^3}{x} + 6\,a\sqrt{\frac{1}{a^2x^2}-1} - \frac{6\,\operatorname{arsech}(ax)}{x}$$

input
```
integrate(arcsech(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
3*a*sqrt(1/(a^2*x^2) - 1)*arcsech(a*x)^2 - arcsech(a*x)^3/x + 6*a*sqrt(1/(
a^2*x^2) - 1) - 6*arcsech(a*x)/x
```

3.16.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arsech}\left(ax\right)^3}{x^2}\,dx$$

input
```
integrate(arcsech(a*x)^3/x^2,x, algorithm="giac")
```

output
```
integrate(arcsech(a*x)^3/x^2, x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^3}{x^2}\,dx$$

input
```
int(acosh(1/(a*x))^3/x^2,x)
```

output
```
int(acosh(1/(a*x))^3/x^2, x)
```

3.17 $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx$

3.17.1 Optimal result

Integrand size = 10, antiderivative size = 137

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx = \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{8x^2} - \frac{3}{8}a^2\operatorname{sech}^{-1}(ax) - \frac{3(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)}{4x^2} + \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{4x^2} - \frac{1}{4}a^2\operatorname{sech}^{-1}(ax)^3 - \frac{(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)^3}{2x^2}$$

output
```
-3/8*a^2*arcsech(a*x)-3/4*(-a*x+1)*(a*x+1)*arcsech(a*x)/x^2-1/4*a^2*arcsec
h(a*x)^3-1/2*(-a*x+1)*(a*x+1)*arcsech(a*x)^3/x^2+3/8*(a*x+1)*((-a*x+1)/(a*
x+1))^(1/2)/x^2+3/4*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/x^2
```

3.17.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx = \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax) - 6\operatorname{sech}^{-1}(ax) + 6\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2 + 2(-2+a^2x^2)\operatorname{sech}^{-1}(ax)^3 - 3a^2x^2\log(}{8x^2}$$

input
```
Integrate[ArcSech[a*x]^3/x^3,x]
```

output
```
(3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - 6*ArcSech[a*x] + 6*Sqrt[(1 - a*x)
/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 + 2*(-2 + a^2*x^2)*ArcSech[a*x]^3 - 3
*a^2*x^2*Log[x] + 3*a^2*x^2*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(
1 - a*x)/(1 + a*x)]])/(8*x^2)
```

3.17.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6839, 5895, 3042, 25, 3792, 15, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(ax)^3}{x^3}dx$$

$$\Big\downarrow \text{6839}$$

$$-a^2\int \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^3}{a^2x^2}d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{5895}$$

$$-a^2\left(\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^3}{2a^2x^2}-\frac{3}{2}\int \frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^2}{a^2x^2}d\text{sech}^{-1}(ax)\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^2\left(\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^3}{2a^2x^2}-\frac{3}{2}\int -\text{sech}^{-1}(ax)^2\sin\left(i\text{sech}^{-1}(ax)\right)^2d\text{sech}^{-1}(ax)\right)$$

$$\Big\downarrow \text{25}$$

$$-a^2\left(\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^3}{2a^2x^2}+\frac{3}{2}\int \text{sech}^{-1}(ax)^2\sin\left(i\text{sech}^{-1}(ax)\right)^2d\text{sech}^{-1}(ax)\right)$$

$$\Big\downarrow \text{3792}$$

$$-a^2\left(\frac{3}{2}\left(\frac{1}{2}\int -\frac{(1-ax)(ax+1)}{a^2x^2}d\text{sech}^{-1}(ax)+\frac{1}{2}\int \text{sech}^{-1}(ax)^2d\text{sech}^{-1}(ax)-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{2a^2x^2}+\right.\right.$$

$\downarrow$ 15

$$-a^2\left(\frac{3}{2}\left(\frac{1}{2}\int -\frac{(1-ax)(ax+1)}{a^2x^2}d\text{sech}^{-1}(ax)-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{2a^2x^2}+\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)}{2a^2x^2}+\frac{1}{6}\right.\right.$$

$\downarrow$ 25

$$-a^2\left(\frac{3}{2}\left(-\frac{1}{2}\int \frac{(1-ax)(ax+1)}{a^2x^2}d\text{sech}^{-1}(ax)-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{2a^2x^2}+\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)}{2a^2x^2}+\frac{1}{6}\right.\right.$$

$\downarrow$ 3042

$$-a^2\left(\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^3}{2a^2x^2}+\frac{3}{2}\left(-\frac{1}{2}\int -\sin\left(i\text{sech}^{-1}(ax)\right)^2d\text{sech}^{-1}(ax)-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{2a^2x^2}\right.\right.$$

$\downarrow$ 25

$$-a^2\left(\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^3}{2a^2x^2}+\frac{3}{2}\left(\frac{1}{2}\int \sin\left(i\text{sech}^{-1}(ax)\right)^2d\text{sech}^{-1}(ax)-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{2a^2x^2}+\right.\right.$$

$\downarrow$ 3115

$$-a^2\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int 1d\text{sech}^{-1}(ax)-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{2a^2x^2}\right)-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{2a^2x^2}+\frac{(1-ax)(ax+1)\text{sech}^{-1}(a}{2a^2x^2}\right.\right.$$

$\downarrow$ 24

$$-a^2\left(\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^3}{2a^2x^2}+\frac{3}{2}\left(-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{2a^2x^2}+\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)}{2a^2x^2}+\frac{1}{2}\left(\frac{1}{2}\text{se}\right.\right.\right.$$

input
```
Int[ArcSech[a*x]^3/x^3,x]
```

output
```
-(a^2*(((1 - a*x)*(1 + a*x)*ArcSech[a*x]^3)/(2*a^2*x^2) + (3*((-1/2*(Sqrt[
(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a^2*x^2) + ArcSech[a*x]/2)/2 + ((1 - a*x)
*(1 + a*x)*ArcSech[a*x])/(2*a^2*x^2) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x
)*ArcSech[a*x]^2)/(2*a^2*x^2) + ArcSech[a*x]^3/6))/2))
```

3.17. $\int \frac{\text{sech}^{-1}(ax)^3}{x^3}\,dx$

3.17.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.17. $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx$

3.17.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arcsech}(ax)^3}{2a^2x^2}+\frac{3\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)^2}{4ax}+\frac{\operatorname{arcsech}(ax)^3}{4}-\frac{3\operatorname{arcsech}(ax)}{4a^2x^2}+\frac{3\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}}{8ax}\right.$
default	$a^2\left(-\frac{\operatorname{arcsech}(ax)^3}{2a^2x^2}+\frac{3\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)^2}{4ax}+\frac{\operatorname{arcsech}(ax)^3}{4}-\frac{3\operatorname{arcsech}(ax)}{4a^2x^2}+\frac{3\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}}{8ax}\right.$

input
```
int(arcsech(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/a^2/x^2*arcsech(a*x)^3+3/4*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1
/2)/a/x*arcsech(a*x)^2+1/4*arcsech(a*x)^3-3/4/a^2/x^2*arcsech(a*x)+3/8*(-(
a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)/a/x+3/8*arcsech(a*x))
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.27

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx$$
$$= \frac{6\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2+2\left(a^2x^2-2\right)\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^3+3\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+3\left(a^2x^2-2\right)}{8\,x^2}$$

input
```
integrate(arcsech(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
1/8*(6*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^
2*x^2)) + 1)/(a*x))^2 + 2*(a^2*x^2 - 2)*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*
x^2)) + 1)/(a*x))^3 + 3*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 3*(a^2*x^2 -
2)*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)))/x^2
```

3.17.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{asech}^3(ax)}{x^3}\,dx$$

input
```
integrate(asech(a*x)**3/x**3,x)
```

output
```
Integral(asech(a*x)**3/x**3, x)
```

3.17.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{arsech}(ax)^3}{x^3}\,dx$$

input
```
integrate(arcsech(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
integrate(arcsech(a*x)^3/x^3, x)
```

3.17.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{arsech}(ax)^3}{x^3}\,dx$$

input
```
integrate(arcsech(a*x)^3/x^3,x, algorithm="giac")
```

output
```
integrate(arcsech(a*x)^3/x^3, x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3} \, dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^3}{x^3} \, dx$$

input
```
int(acosh(1/(a*x))^3/x^3,x)
```

output
```
int(acosh(1/(a*x))^3/x^3, x)
```

3.18 $\int \frac{\text{sech}^{-1}(ax)^3}{x^4}\,dx$

3.18.1 Optimal result

Integrand size = 10, antiderivative size = 179

$$\begin{aligned}\int \frac{\text{sech}^{-1}(ax)^3}{x^4}\,dx &= \frac{14a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{9x} + \frac{2\left(\frac{1-ax}{1+ax}\right)^{3/2}(1+ax)^3}{27x^3} - \frac{2\text{sech}^{-1}(ax)}{9x^3} \\ &\quad - \frac{4a^2\text{sech}^{-1}(ax)}{3x} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{3x^3} \\ &\quad + \frac{2a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{3x} - \frac{\text{sech}^{-1}(ax)^3}{3x^3}\end{aligned}$$

output
```
2/27*((-a*x+1)/(a*x+1))^(3/2)*(a*x+1)^3/x^3-2/9*arcsech(a*x)/x^3-4/3*a^2*a
rcsech(a*x)/x-1/3*arcsech(a*x)^3/x^3+14/9*a^2*(a*x+1)*((-a*x+1)/(a*x+1))^(
1/2)/x+1/3*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/x^3+2/3*a^2*(a*
x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/x
```

3.18.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.67

$$\begin{aligned}&\int \frac{\text{sech}^{-1}(ax)^3}{x^4}\,dx \\ &= \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax+20a^2x^2+20a^3x^3) - 6(1+6a^2x^2)\,\text{sech}^{-1}(ax) + 9\sqrt{\frac{1-ax}{1+ax}}(1+ax+2a^2x^2+2a^3x^3)\,\text{sech}}{27x^3}\end{aligned}$$

input
```
Integrate[ArcSech[a*x]^3/x^4,x]
```

output
```
(2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x + 20*a^2*x^2 + 20*a^3*x^3) - 6*(1 +
6*a^2*x^2)*ArcSech[a*x] + 9*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x + 2*a^2*x^2
 + 2*a^3*x^3)*ArcSech[a*x]^2 - 9*ArcSech[a*x]^3)/(27*x^3)
```

3.18.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.37, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {6839, 5896, 3042, 3792, 3042, 3113, 2009, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(ax)^3}{x^4}dx$$

$$\Big\downarrow \text{6839}$$

$$-a^3\int \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^3}{a^3x^3}d\text{sech}^{-1}(ax)$$

$$\Big\downarrow \text{5896}$$

$$-a^3\left(\frac{\text{sech}^{-1}(ax)^3}{3a^3x^3}-\int \frac{\text{sech}^{-1}(ax)^2}{a^3x^3}d\text{sech}^{-1}(ax)\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3\left(\frac{\text{sech}^{-1}(ax)^3}{3a^3x^3}-\int \text{sech}^{-1}(ax)^2\sin\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^3d\text{sech}^{-1}(ax)\right)$$

$$\Big\downarrow \text{3792}$$

$$-a^3\left(-\frac{2}{9}\int \frac{1}{a^3x^3}d\text{sech}^{-1}(ax)-\frac{2}{3}\int \frac{\text{sech}^{-1}(ax)^2}{ax}d\text{sech}^{-1}(ax)+\frac{\text{sech}^{-1}(ax)^3}{3a^3x^3}-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{3a^3x^3}+2\right.$$

$$\Big\downarrow \text{3042}$$

$$-a^3\left(-\frac{2}{3}\int \text{sech}^{-1}(ax)^2\sin\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)-\frac{2}{9}\int \sin\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)^3d\text{sech}^{-1}(ax)+\frac{\text{sech}^{-}}{3a}\right.$$

$\downarrow$ 3113

$$-a^3\left(-\frac{2}{9}i\int\left(\frac{(1-ax)(ax+1)}{a^2x^2}+1\right)d\left(-\frac{i\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax}\right)-\frac{2}{3}\int \text{sech}^{-1}(ax)^2\sin\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-}\right.$$

$\downarrow$ 2009

$$-a^3\left(-\frac{2}{3}\int \text{sech}^{-1}(ax)^2\sin\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)-\frac{2}{9}i\left(-\frac{i\left(\frac{1-ax}{ax+1}\right)^{3/2}(ax+1)^3}{3a^3x^3}-\frac{i\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax}\right)\right.$$

$\downarrow$ 3777

$$-a^3\left(-\frac{2}{3}\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{ax}-2i\int -\frac{i\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{ax}d\text{sech}^{-1}(ax)\right)-\frac{2}{9}i\left(-\frac{i\left(\frac{1-ax}{ax+1}\right)^{3/2}}{3a^3x}\right.\right.$$

$\downarrow$ 26

$$-a^3\left(-\frac{2}{3}\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{ax}-2\int \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{ax}d\text{sech}^{-1}(ax)\right)-\frac{2}{9}i\left(-\frac{i\left(\frac{1-ax}{ax+1}\right)^{3/2}(ax}{3a^3x^3}\right.\right.$$

$\downarrow$ 3042

$$-a^3\left(-\frac{2}{3}\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{ax}-2\int -i\text{sech}^{-1}(ax)\sin\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)\right)-\frac{2}{9}i\left(-\frac{i\left(\frac{1-ax}{ax+1}\right)^{3/}}{3a^3}\right.\right.$$

$\downarrow$ 26

$$-a^3\left(-\frac{2}{3}\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{ax}+2i\int \text{sech}^{-1}(ax)\sin\left(i\text{sech}^{-1}(ax)\right)d\text{sech}^{-1}(ax)\right)-\frac{2}{9}i\left(-\frac{i\left(\frac{1-ax}{ax+1}\right)^{3/2}(}{3a^3x}\right.\right.$$

$\downarrow$ 3777

3.18. $\int \frac{\text{sech}^{-1}(ax)^3}{x^4}\,dx$

$$-a^3\left(-\frac{2}{3}\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{ax}+2i\left(\frac{i\text{sech}^{-1}(ax)}{ax}-i\int\frac{1}{ax}d\text{sech}^{-1}(ax)\right)\right)-\frac{2}{9}i\left(-\frac{i\left(\frac{1-ax}{ax+1}\right)^{3/2}(ax+1\right.\right.}{3a^3x^3}$$

$$\Big\downarrow \; 3042$$

$$-a^3\left(-\frac{2}{3}\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{ax}+2i\left(\frac{i\text{sech}^{-1}(ax)}{ax}-i\int\sin\left(i\text{sech}^{-1}(ax)+\frac{\pi}{2}\right)d\text{sech}^{-1}(ax)\right)\right)-\frac{2}{9}i\left(-\right.\right.$$

$$\Big\downarrow \; 3117$$

$$-a^3\left(-\frac{2}{9}i\left(-\frac{i\left(\frac{1-ax}{ax+1}\right)^{3/2}(ax+1)^3}{3a^3x^3}-\frac{i\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax}\right)+\frac{\text{sech}^{-1}(ax)^3}{3a^3x^3}-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{3a^3x^3}+\frac{2\text{sech}^{-1}}{9a}\right.$$

input
```
Int[ArcSech[a*x]^3/x^4,x]
```

output
```
-(a^3*(((-2*I)/9)*(((-I)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a*x) - ((I/
3)*((1 - a*x)/(1 + a*x))^(3/2)*(1 + a*x)^3)/(a^3*x^3)) + (2*ArcSech[a*x])/
(9*a^3*x^3) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(3*a^3*
x^3) + ArcSech[a*x]^3/(3*a^3*x^3) - (2*((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*
x)*ArcSech[a*x]^2)/(a*x) + (2*I)*(((-I)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x
))/(a*x) + (I*ArcSech[a*x])/(a*x))))/3))
```

3.18.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.18. $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4} \, dx$

3.18.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.07

method	result
derivativedivides	$a^{3}\left(-\frac{\operatorname{arcsech}(ax)^{3}}{3a^{3}x^{3}}+\frac{2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)^{2}}{3}+\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)^{2}}{3a^{2}x^{2}}-\frac{4\operatorname{arcsech}(ax)}{3ax}+\right.$
default	$a^{3}\left(-\frac{\operatorname{arcsech}(ax)^{3}}{3a^{3}x^{3}}+\frac{2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)^{2}}{3}+\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)^{2}}{3a^{2}x^{2}}-\frac{4\operatorname{arcsech}(ax)}{3ax}+\right.$

input
```
int(arcsech(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3/a^3/x^3*arcsech(a*x)^3+2/3*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1
/2)*arcsech(a*x)^2+1/3/a^2/x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*ar
csech(a*x)^2-4/3/a/x*arcsech(a*x)+40/27*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)
^(1/2)-2/9*arcsech(a*x)/a^3/x^3+2/27*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1
/2)/a^2/x^2)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04

$$\int \frac{\operatorname{sech}^{-1}(ax)^{3}}{x^{4}}\,dx$$
$$=\frac{9\left(2\,a^{3}x^{3}+ax\right)\sqrt{-\frac{a^{2}x^{2}-1}{a^{2}x^{2}}}\log\left(\frac{ax\sqrt{-\frac{a^{2}x^{2}-1}{a^{2}x^{2}}}+1}{ax}\right)^{2}-9\,\log\left(\frac{ax\sqrt{-\frac{a^{2}x^{2}-1}{a^{2}x^{2}}}+1}{ax}\right)^{3}-6\left(6\,a^{2}x^{2}+1\right)\log\left(\frac{ax\sqrt{-\frac{a^{2}x^{2}-1}{a^{2}x^{2}}}}{ax}\right.}{27\,x^{3}}$$

input
```
integrate(arcsech(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
1/27*(9*(2*a^3*x^3 + a*x)*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a
^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - 9*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2
*x^2)) + 1)/(a*x))^3 - 6*(6*a^2*x^2 + 1)*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2
*x^2)) + 1)/(a*x)) + 2*(20*a^3*x^3 + a*x)*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)))/
x^3
```

3.18.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4} \, dx = \int \frac{\operatorname{asech}^3(ax)}{x^4} \, dx$$

input
```
integrate(asech(a*x)**3/x**4,x)
```

output
```
Integral(asech(a*x)**3/x**4, x)
```

3.18.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4} \, dx = \int \frac{\operatorname{arsech}(ax)^3}{x^4} \, dx$$

input
```
integrate(arcsech(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
integrate(arcsech(a*x)^3/x^4, x)
```

3.18.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4} \, dx = \int \frac{\operatorname{arsech}(ax)^3}{x^4} \, dx$$

input
```
integrate(arcsech(a*x)^3/x^4,x, algorithm="giac")
```

output
```
integrate(arcsech(a*x)^3/x^4, x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^3}{x^4}\,dx$$

input
```
int(acosh(1/(a*x))^3/x^4,x)
```

output
```
int(acosh(1/(a*x))^3/x^4, x)
```

3.19 $\int x^6 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

3.19.1 Optimal result

Integrand size = 12, antiderivative size = 142

$$\int x^6 \left(a+b\text{sech}^{-1}(cx)\right) \, dx=-\frac{5bx\sqrt{1-cx}}{112c^6\sqrt{\frac{1}{1+cx}}}-\frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)+\frac{5b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{112c^7}$$

output
```
1/7*x^7*(a+b*arcsech(c*x))-5/112*b*x*(-c*x+1)^(1/2)/c^6/(1/(c*x+1))^(1/2)-
5/168*b*x^3*(-c*x+1)^(1/2)/c^4/(1/(c*x+1))^(1/2)-1/42*b*x^5*(-c*x+1)^(1/2)
/c^2/(1/(c*x+1))^(1/2)+5/112*b*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)
/c^7
```

3.19.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01

$$\int x^6 \left(a+b\text{sech}^{-1}(cx)\right) \, dx=\frac{ax^7}{7}+b\sqrt{\frac{1-cx}{1+cx}}\left(-\frac{5x}{112c^6}-\frac{5x^2}{112c^5}-\frac{5x^3}{168c^4}-\frac{5x^4}{168c^3}-\frac{x^5}{42c^2}-\frac{x^6}{42c}\right)+\frac{1}{7}bx^7\text{sech}^{-1}(cx)+\frac{5ib\log\left(-2icx+2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)}{112c^7}$$

input
```
Integrate[x^6*(a + b*ArcSech[c*x]),x]
```

output
```
(a*x^7)/7 + b*Sqrt[(1 - c*x)/(1 + c*x)]*((-5*x)/(112*c^6) - (5*x^2)/(112*c
^5) - (5*x^3)/(168*c^4) - (5*x^4)/(168*c^3) - x^5/(42*c^2) - x^6/(42*c)) +
 (b*x^7*ArcSech[c*x])/7 + (((5*I)/112)*b*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)
/(1 + c*x)]*(1 + c*x)])/c^7
```

3.19.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6837, 111, 27, 111, 27, 101, 25, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{6837}$$

$$\frac{1}{7} b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \int \frac{x^6}{\sqrt{1-cx}\sqrt{cx+1}}dx+\frac{1}{7} x^7 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{111}$$

$$\frac{1}{7} b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \left(-\frac{\int -\frac{5x^4}{\sqrt{1-cx}\sqrt{cx+1}}dx}{6c^2}-\frac{x^5\sqrt{1-cx}\sqrt{cx+1}}{6c^2}\right)+\frac{1}{7} x^7 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7} b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \left(\frac{5\int \frac{x^4}{\sqrt{1-cx}\sqrt{cx+1}}dx}{6c^2}-\frac{x^5\sqrt{1-cx}\sqrt{cx+1}}{6c^2}\right)+\frac{1}{7} x^7 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{111}$$

$$\frac{1}{7} b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \left(\frac{5\left(-\frac{\int -\frac{3x^2}{\sqrt{1-cx}\sqrt{cx+1}}dx}{4c^2}-\frac{x^3\sqrt{1-cx}\sqrt{cx+1}}{4c^2}\right)}{6c^2}-\frac{x^5\sqrt{1-cx}\sqrt{cx+1}}{6c^2}\right)+\frac{1}{7} x^7 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{5\left(\frac{3\int\frac{x^2}{\sqrt{1-cx}\sqrt{cx+1}}dx}{4c^2}-\frac{x^3\sqrt{1-cx}\sqrt{cx+1}}{4c^2}\right)}{6c^2}-\frac{x^5\sqrt{1-cx}\sqrt{cx+1}}{6c^2}\right)+\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \; 101$$

$$\frac{1}{7}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{5\left(\frac{3\left(-\frac{\int-\frac{1}{\sqrt{1-cx}\sqrt{cx+1}}dx}{2c^2}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-cx}\sqrt{cx+1}}{4c^2}\right)}{6c^2}-\frac{x^5\sqrt{1-cx}\sqrt{cx+1}}{6c^2}\right)+\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{7}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{5\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{1-cx}\sqrt{cx+1}}dx}{2c^2}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-cx}\sqrt{cx+1}}{4c^2}\right)}{6c^2}-\frac{x^5\sqrt{1-cx}\sqrt{cx+1}}{6c^2}\right)+\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \; 39$$

$$\frac{1}{7}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{5\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-cx}\sqrt{cx+1}}{4c^2}\right)}{6c^2}-\frac{x^5\sqrt{1-cx}\sqrt{cx+1}}{6c^2}\right)+\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \; 223$$

3.19. $\int x^6\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

$$\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)+$$
$$\frac{1}{7}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{5\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-cx}\sqrt{cx+1}}{4c^2}\right)}{6c^2}-\frac{x^5\sqrt{1-cx}\sqrt{cx+1}}{6c^2}\right)$$

input
```
Int[x^6*(a + b*ArcSech[c*x]),x]
```

output
```
(x^7*(a + b*ArcSech[c*x]))/7 + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-1/6
*(x^5*Sqrt[1 - c*x]*Sqrt[1 + c*x])/c^2 + (5*(-1/4*(x^3*Sqrt[1 - c*x]*Sqrt[
1 + c*x])/c^2 + (3*(-1/2*(x*Sqrt[1 - c*x]*Sqrt[1 + c*x])/c^2 + ArcSin[c*x]
/(2*c^3)))/(4*c^2)))/(6*c^2)))/7
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6837
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Simp[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)]   Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x])
, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

3.19.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

method	result
parts	$\frac{a\,x^{7}}{7}+\frac{b\left(\frac{c^{7} x^{7} \operatorname{arcsech}\left(c x \right)}{7}-\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(8 \sqrt{-c^{2} x^{2}+1}\, c^{5} x^{5}+10 c^{3} x^{3} \sqrt{-c^{2} x^{2}+1}+15 c x \sqrt{-c^{2} x^{2}+1}-15 \arcsin \left(c x \right)\right)}{336 \sqrt{-c^{2} x^{2}+1}}\right)}{c^{7}}$
derivativedivides	$\frac{\frac{a\,c^{7} x^{7}}{7}+b\left(\frac{c^{7} x^{7} \operatorname{arcsech}\left(c x \right)}{7}-\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(8 \sqrt{-c^{2} x^{2}+1}\, c^{5} x^{5}+10 c^{3} x^{3} \sqrt{-c^{2} x^{2}+1}+15 c x \sqrt{-c^{2} x^{2}+1}-15 \arcsin \left(c x \right)\right)}{336 \sqrt{-c^{2} x^{2}+1}}\right)}{c^{7}}$
default	$\frac{\frac{a\,c^{7} x^{7}}{7}+b\left(\frac{c^{7} x^{7} \operatorname{arcsech}\left(c x \right)}{7}-\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(8 \sqrt{-c^{2} x^{2}+1}\, c^{5} x^{5}+10 c^{3} x^{3} \sqrt{-c^{2} x^{2}+1}+15 c x \sqrt{-c^{2} x^{2}+1}-15 \arcsin \left(c x \right)\right)}{336 \sqrt{-c^{2} x^{2}+1}}\right)}{c^{7}}$

input
```
int(x^6*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/7*a*x^7+b/c^7*(1/7*c^7*x^7*arcsech(c*x)-1/336*(-(c*x-1)/c/x)^(1/2)*c*x*(
(c*x+1)/c/x)^(1/2)*(8*(-c^2*x^2+1)^(1/2)*c^5*x^5+10*c^3*x^3*(-c^2*x^2+1)^(
1/2)+15*c*x*(-c^2*x^2+1)^(1/2)-15*arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.29

$$\int x^6 \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$= \frac{48\,ac^7x^7 - 48\,bc^7\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 30\,b\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + 48\,(bc^7x^7 - bc^7)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right.}{336\,c^7}$$

input
```
integrate(x^6*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/336*(48*a*c^7*x^7 - 48*b*c^7*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1
)/x) - 30*b*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + 48*(b
*c^7*x^7 - b*c^7)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (8
*b*c^6*x^6 + 10*b*c^4*x^4 + 15*b*c^2*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/
c^7
```

3.19.6 Sympy [F]

$$\int x^6 \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \int x^6 (a + b\operatorname{asech}(cx))\,dx$$

input
```
integrate(x**6*(a+b*asech(c*x)),x)
```

output
```
Integral(x**6*(a + b*asech(c*x)), x)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

$$\int x^6 \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{1}{7}\,ax^7$$
$$+ \frac{1}{336}\left(48\,x^7\operatorname{arsech}(cx) - \frac{\frac{15\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+40\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^2x^2}-1}}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6} + \frac{15\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^6}}{c}\right)b$$

input
```
integrate(x^6*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*x^7 + 1/336*(48*x^7*arcsech(c*x) - ((15*(1/(c^2*x^2) - 1)^(5/2) + 40
*(1/(c^2*x^2) - 1)^(3/2) + 33*sqrt(1/(c^2*x^2) - 1))/(c^6*(1/(c^2*x^2) - 1
)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*arct
an(sqrt(1/(c^2*x^2) - 1))/c^6)/c)*b
```

3.19.8 Giac [F]

$$\int x^6 \left(a + b\text{sech}^{-1}(cx)\right) dx = \int (b \operatorname{arsech}(cx) + a)x^6 \, dx$$

input
```
integrate(x^6*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^6, x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int x^6 \left(a + b\text{sech}^{-1}(cx)\right) dx = \int x^6 \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right) dx$$

input
```
int(x^6*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^6*(a + b*acosh(1/(c*x))), x)
```

3.20 $\int x^5 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

3.20.1 Optimal result

Integrand size = 12, antiderivative size = 109

$$\int x^5 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = -\frac{4b\sqrt{1-cx}}{45c^6\sqrt{\frac{1}{1+cx}}}-\frac{2bx^2\sqrt{1-cx}}{45c^4\sqrt{\frac{1}{1+cx}}} -\frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{6}x^6\left(a+b\text{sech}^{-1}(cx)\right)$$

output
```
1/6*x^6*(a+b*arcsech(c*x))-4/45*b*(-c*x+1)^(1/2)/c^6/(1/(c*x+1))^(1/2)-2/4
5*b*x^2*(-c*x+1)^(1/2)/c^4/(1/(c*x+1))^(1/2)-1/30*b*x^4*(-c*x+1)^(1/2)/c^2
/(1/(c*x+1))^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int x^5 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \frac{ax^6}{6}+b\sqrt{\frac{1-cx}{1+cx}}\left(-\frac{4}{45c^6}-\frac{4x}{45c^5}-\frac{2x^2}{45c^4}-\frac{2x^3}{45c^3}-\frac{x^4}{30c^2}-\frac{x^5}{30c}\right) +\frac{1}{6}bx^6\text{sech}^{-1}(cx)$$

input
```
Integrate[x^5*(a + b*ArcSech[c*x]),x]
```

output
```
(a*x^6)/6 + b*Sqrt[(1 - c*x)/(1 + c*x)]*(-4/(45*c^6) - (4*x)/(45*c^5) - (2
*x^2)/(45*c^4) - (2*x^3)/(45*c^3) - x^4/(30*c^2) - x^5/(30*c)) + (b*x^6*Ar
cSech[c*x])/6
```

3.20.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6837, 111, 27, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{6837}$$

$$\frac{1}{6} b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \int \frac{x^5}{\sqrt{1-cx}\sqrt{cx+1}}dx+\frac{1}{6} x^6 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{111}$$

$$\frac{1}{6} b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \left(-\frac{\int -\frac{4x^3}{\sqrt{1-cx}\sqrt{cx+1}}dx}{5c^2}-\frac{x^4\sqrt{1-cx}\sqrt{cx+1}}{5c^2}\right)+\frac{1}{6} x^6 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6} b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \left(\frac{4\int \frac{x^3}{\sqrt{1-cx}\sqrt{cx+1}}dx}{5c^2}-\frac{x^4\sqrt{1-cx}\sqrt{cx+1}}{5c^2}\right)+\frac{1}{6} x^6 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{111}$$

$$\frac{1}{6} b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \left(\frac{4\left(-\frac{\int -\frac{2x}{\sqrt{1-cx}\sqrt{cx+1}}dx}{3c^2}-\frac{x^2\sqrt{1-cx}\sqrt{cx+1}}{3c^2}\right)}{5c^2}-\frac{x^4\sqrt{1-cx}\sqrt{cx+1}}{5c^2}\right)+$$
$$\frac{1}{6} x^6 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{4\left(\frac{2\int\frac{x}{\sqrt{1-cx}\sqrt{cx+1}}dx}{3c^2}-\frac{x^2\sqrt{1-cx}\sqrt{cx+1}}{3c^2}\right)}{5c^2}-\frac{x^4\sqrt{1-cx}\sqrt{cx+1}}{5c^2}\right)+$$
$$\frac{1}{6}x^6\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{83}$$

$$\frac{1}{6}x^6\left(a+b\text{sech}^{-1}(cx)\right)+$$
$$\frac{1}{6}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{4\left(-\frac{2\sqrt{1-cx}\sqrt{cx+1}}{3c^4}-\frac{x^2\sqrt{1-cx}\sqrt{cx+1}}{3c^2}\right)}{5c^2}-\frac{x^4\sqrt{1-cx}\sqrt{cx+1}}{5c^2}\right)$$

input
```
Int[x^5*(a + b*ArcSech[c*x]),x]
```

output
```
(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-1/5*(x^4*Sqrt[1 - c*x]*Sqrt[1 + c*
x])/c^2 + (4*((-2*Sqrt[1 - c*x]*Sqrt[1 + c*x])/(3*c^4) - (x^2*Sqrt[1 - c*x
]*Sqrt[1 + c*x])/(3*c^2)))/(5*c^2)))/6 + (x^6*(a + b*ArcSech[c*x]))/6
```

3.20.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 6837
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Simp[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)]   Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x])
, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

3.20.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

method	result	size
parts	$\frac{a\,x^{6}}{6}+\frac{b\left(\frac{c^{6} x^{6} \operatorname{arcsech}\left(c x\right)}{6}-\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(3 c^{4} x^{4}+4 c^{2} x^{2}+8\right)}{90}\right)}{c^{6}}$	77
derivativedivides	$\frac{\frac{a\,c^{6} x^{6}}{6}+b\left(\frac{c^{6} x^{6} \operatorname{arcsech}\left(c x\right)}{6}-\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(3 c^{4} x^{4}+4 c^{2} x^{2}+8\right)}{90}\right)}{c^{6}}$	81
default	$\frac{\frac{a\,c^{6} x^{6}}{6}+b\left(\frac{c^{6} x^{6} \operatorname{arcsech}\left(c x\right)}{6}-\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(3 c^{4} x^{4}+4 c^{2} x^{2}+8\right)}{90}\right)}{c^{6}}$	81

input
```
int(x^5*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x^6+b/c^6*(1/6*c^6*x^6*arcsech(c*x)-1/90*(-(c*x-1)/c/x)^(1/2)*c*x*((
c*x+1)/c/x)^(1/2)*(3*c^4*x^4+4*c^2*x^2+8))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{15\,bc^5x^6\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+15\,ac^5x^6-\left(3\,bc^4x^5+4\,bc^2x^3+8\,bx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{90\,c^5}$$

input
```
integrate(x^5*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/90*(15*b*c^5*x^6*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 1
5*a*c^5*x^6 - (3*b*c^4*x^5 + 4*b*c^2*x^3 + 8*b*x)*sqrt(-(c^2*x^2 - 1)/(c^2
*x^2)))/c^5
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int x^5\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \begin{cases} \frac{ax^6}{6}+\frac{bx^6\operatorname{asech}(cx)}{6}-\frac{bx^4\sqrt{-c^2x^2+1}}{30c^2}-\frac{2bx^2\sqrt{-c^2x^2+1}}{45c^4}-\frac{4b\sqrt{-c^2x^2+1}}{45c^6} & \text{for } c\neq 0\\ \frac{x^6(a+\infty b)}{6} & \text{otherwise}\end{cases}$$

input
```
integrate(x**5*(a+b*asech(c*x)),x)
```

output
```
Piecewise((a*x**6/6 + b*x**6*asech(c*x)/6 - b*x**4*sqrt(-c**2*x**2 + 1)/(3
0*c**2) - 2*b*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 4*b*sqrt(-c**2*x**2 +
1)/(45*c**6), Ne(c, 0)), (x**6*(a + oo*b)/6, True))
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int x^5\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \frac{1}{6}\,ax^6 + \frac{1}{90}\left(15\,x^6\operatorname{arsech}(cx)-\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^5}\right)b$$

input
```
integrate(x^5*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/90*(15*x^6*arcsech(c*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2)
 - 10*c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) - 1))/c^5)*b
```

3.20.8 Giac [F]

$$\int x^5\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int(b\,\text{arsech}\,(cx)+a)x^5\,dx$$

input
```
integrate(x^5*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^5, x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int x^5\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^5\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^5*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^5*(a + b*acosh(1/(c*x))), x)
```

3.21 $\int x^4 \left(a + b\text{sech}^{-1}(cx)\right) \, dx$

3.21.1 Optimal result

Integrand size = 12, antiderivative size = 110

$$\int x^4 \left(a + b\text{sech}^{-1}(cx)\right) \, dx = -\frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{1+cx}}} - \frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{5}x^5\left(a + b\text{sech}^{-1}(cx)\right) + \frac{3b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{40c^5}$$

output
```
1/5*x^5*(a+b*arcsech(c*x))-3/40*b*x*(-c*x+1)^(1/2)/c^4/(1/(c*x+1))^(1/2)-1
/20*b*x^3*(-c*x+1)^(1/2)/c^2/(1/(c*x+1))^(1/2)+3/40*b*arcsin(c*x)*(1/(c*x+
1))^(1/2)*(c*x+1)^(1/2)/c^5
```

3.21.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12

$$\int x^4 \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \frac{ax^5}{5} + b\sqrt{\frac{1-cx}{1+cx}}\left(-\frac{3x}{40c^4} - \frac{3x^2}{40c^3} - \frac{x^3}{20c^2} - \frac{x^4}{20c}\right) + \frac{1}{5}bx^5\text{sech}^{-1}(cx) + \frac{3ib\log\left(-2icx + 2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)}{40c^5}$$

input
```
Integrate[x^4*(a + b*ArcSech[c*x]),x]
```

output
```
(a*x^5)/5 + b*Sqrt[(1 - c*x)/(1 + c*x)]*((-3*x)/(40*c^4) - (3*x^2)/(40*c^3
) - x^3/(20*c^2) - x^4/(20*c)) + (b*x^5*ArcSech[c*x])/5 + (((3*I)/40)*b*Lo
g[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/c^5
```

3.21.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6837, 111, 27, 101, 25, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{ 6837}$$

$$\frac{1}{5} b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \int \frac{x^4}{\sqrt{1-cx}\sqrt{cx+1}}dx+\frac{1}{5} x^5 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 111}$$

$$\frac{1}{5} b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \left(-\frac{\int -\frac{3x^2}{\sqrt{1-cx}\sqrt{cx+1}}dx}{4c^2}-\frac{x^3\sqrt{1-cx}\sqrt{cx+1}}{4c^2}\right)+\frac{1}{5} x^5 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5} b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \left(\frac{3\int \frac{x^2}{\sqrt{1-cx}\sqrt{cx+1}}dx}{4c^2}-\frac{x^3\sqrt{1-cx}\sqrt{cx+1}}{4c^2}\right)+\frac{1}{5} x^5 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 101}$$

$$\frac{1}{5} b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \left(\frac{3\left(-\frac{\int -\frac{1}{\sqrt{1-cx}\sqrt{cx+1}}dx}{2c^2}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-cx}\sqrt{cx+1}}{4c^2}\right)+\frac{1}{5} x^5 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{5}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-cx}\sqrt{cx+1}}dx}{2c^2}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-cx}\sqrt{cx+1}}{4c^2}\right)+$$
$$\frac{1}{5}x^5\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \; 39$$

$$\frac{1}{5}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-cx}\sqrt{cx+1}}{4c^2}\right)+$$
$$\frac{1}{5}x^5\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \; 223$$

$$\frac{1}{5}x^5\left(a+b\text{sech}^{-1}(cx)\right)+$$
$$\frac{1}{5}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-cx}\sqrt{cx+1}}{4c^2}\right)$$

input
```
Int[x^4*(a + b*ArcSech[c*x]),x]
```

output
```
(x^5*(a + b*ArcSech[c*x]))/5 + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-1/4
*(x^3*Sqrt[1 - c*x]*Sqrt[1 + c*x])/c^2 + (3*(-1/2*(x*Sqrt[1 - c*x]*Sqrt[1
+ c*x])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/5
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6837
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Simp[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)]   Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x])
, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

3.21.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04

method	result	size
parts	$\frac{a\,x^{5}}{5}+\frac{b\left(\frac{c^{5} x^{5} \operatorname{arcsech}\left(c x \right)}{5}+\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(-2 c^{3} x^{3} \sqrt{-c^{2} x^{2}+1}-3 c x \sqrt{-c^{2} x^{2}+1}+3 \arcsin \left(c x \right)\right)}{40 \sqrt{-c^{2} x^{2}+1}}\right)}{c^{5}}$	114
derivativedivides	$\frac{\frac{a\,c^{5} x^{5}}{5}+b\left(\frac{c^{5} x^{5} \operatorname{arcsech}\left(c x \right)}{5}+\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(-2 c^{3} x^{3} \sqrt{-c^{2} x^{2}+1}-3 c x \sqrt{-c^{2} x^{2}+1}+3 \arcsin \left(c x \right)\right)}{40 \sqrt{-c^{2} x^{2}+1}}\right)}{c^{5}}$	118
default	$\frac{\frac{a\,c^{5} x^{5}}{5}+b\left(\frac{c^{5} x^{5} \operatorname{arcsech}\left(c x \right)}{5}+\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(-2 c^{3} x^{3} \sqrt{-c^{2} x^{2}+1}-3 c x \sqrt{-c^{2} x^{2}+1}+3 \arcsin \left(c x \right)\right)}{40 \sqrt{-c^{2} x^{2}+1}}\right)}{c^{5}}$	118

input
```
int(x^4*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+b/c^5*(1/5*c^5*x^5*arcsech(c*x)+1/40*(-(c*x-1)/c/x)^(1/2)*c*x*((
c*x+1)/c/x)^(1/2)*(-2*c^3*x^3*(-c^2*x^2+1)^(1/2)-3*c*x*(-c^2*x^2+1)^(1/2)+
3*arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(70) = 140.

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.58

$$\int x^4\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{8\,ac^5x^5-8\,bc^5\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)-6\,b\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)+8\left(bc^5x^5-bc^5\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-}{40\,c^5}$$

input
```
integrate(x^4*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/40*(8*a*c^5*x^5 - 8*b*c^5*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x
) - 6*b*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + 8*(b*c^5*
x^5 - b*c^5)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (2*b*c^
4*x^4 + 3*b*c^2*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^5
```

3.21.6 Sympy [F]

$$\int x^4\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^4\left(a+b\,\text{asech}\,(cx)\right)\,dx$$

input
```
integrate(x**4*(a+b*asech(c*x)),x)
```

output
```
Integral(x**4*(a + b*asech(c*x)), x)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

$$\int x^4 \left(a + b\text{sech}^{-1}(cx)\right) \, dx$$
$$= \frac{1}{5}ax^5 + \frac{1}{40}\left(8\,x^5\,\text{arsech}\,(cx) - \frac{\frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+5\sqrt{\frac{1}{c^2x^2}-1}}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4} + \frac{3\,\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4}}{c}\right)b$$

input
```
integrate(x^4*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/40*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1)^(3/2) + 5*sqr
t(1/(c^2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c
^4) + 3*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b
```

3.21.8 Giac [F]

$$\int x^4 \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int (b\,\text{arsech}\,(cx) + a)x^4 \, dx$$

input
```
integrate(x^4*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^4, x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int x^4 \left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int(x^4*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^4*(a + b*acosh(1/(c*x))), x)
```

3.22 $\int x^3 \left(a + b\text{sech}^{-1}(cx)\right) dx$

3.22.1 Optimal result

Integrand size = 12, antiderivative size = 77

$$\int x^3 \left(a + b\text{sech}^{-1}(cx)\right) dx = -\frac{b\sqrt{1-cx}}{6c^4\sqrt{\frac{1}{1+cx}}} - \frac{bx^2\sqrt{1-cx}}{12c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{4}x^4\left(a + b\text{sech}^{-1}(cx)\right)$$

output
```
1/4*x^4*(a+b*arcsech(c*x))-1/6*b*(-c*x+1)^(1/2)/c^4/(1/(c*x+1))^(1/2)-1/12
*b*x^2*(-c*x+1)^(1/2)/c^2/(1/(c*x+1))^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int x^3 \left(a + b\text{sech}^{-1}(cx)\right) dx = \frac{ax^4}{4} + b\sqrt{\frac{1-cx}{1+cx}}\left(-\frac{1}{6c^4} - \frac{x}{6c^3} - \frac{x^2}{12c^2} - \frac{x^3}{12c}\right) + \frac{1}{4}bx^4\text{sech}^{-1}(cx)$$

input
```
Integrate[x^3*(a + b*ArcSech[c*x]),x]
```

output
```
(a*x^4)/4 + b*Sqrt[(1 - c*x)/(1 + c*x)]*(-1/6*1/c^4 - x/(6*c^3) - x^2/(12*
c^2) - x^3/(12*c)) + (b*x^4*ArcSech[c*x])/4
```

3.22.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6837, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{ 6837}$$

$$\frac{1}{4}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x^3}{\sqrt{1-cx}\sqrt{cx+1}}dx+\frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 111}$$

$$\frac{1}{4}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{2x}{\sqrt{1-cx}\sqrt{cx+1}}dx}{3c^2}-\frac{x^2\sqrt{1-cx}\sqrt{cx+1}}{3c^2}\right)+\frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2\int \frac{x}{\sqrt{1-cx}\sqrt{cx+1}}dx}{3c^2}-\frac{x^2\sqrt{1-cx}\sqrt{cx+1}}{3c^2}\right)+\frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 83}$$

$$\frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{4}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{2\sqrt{1-cx}\sqrt{cx+1}}{3c^4}-\frac{x^2\sqrt{1-cx}\sqrt{cx+1}}{3c^2}\right)$$

input
```
Int[x^3*(a + b*ArcSech[c*x]),x]
```

output
```
(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-2*Sqrt[1 - c*x]*Sqrt[1 + c*x])/(3
*c^4) - (x^2*Sqrt[1 - c*x]*Sqrt[1 + c*x])/(3*c^2)))/4 + (x^4*(a + b*ArcSec
h[c*x]))/4
```

3.22.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 6837
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Simp[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)]   Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x])
, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

3.22.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

method	result	size
parts	$\frac{a\,x^{4}}{4}+\frac{b\left(\frac{c^{4} x^{4} \operatorname{arcsech}\left(c x \right)}{4}-\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(c^{2} x^{2}+2\right)}{12}\right)}{c^{4}}$	68
derivativedivides	$\frac{\frac{a\,c^{4} x^{4}}{4}+b \left(\frac{c^{4} x^{4} \operatorname{arcsech}\left(c x \right)}{4}-\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(c^{2} x^{2}+2\right)}{12}\right)}{c^{4}}$	72
default	$\frac{\frac{a\,c^{4} x^{4}}{4}+b \left(\frac{c^{4} x^{4} \operatorname{arcsech}\left(c x \right)}{4}-\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(c^{2} x^{2}+2\right)}{12}\right)}{c^{4}}$	72

input
```
int(x^3*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+b/c^4*(1/4*c^4*x^4*arcsech(c*x)-1/12*(-(c*x-1)/c/x)^(1/2)*c*x*((
c*x+1)/c/x)^(1/2)*(c^2*x^2+2))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int x^3\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\frac{3\,bc^3x^4\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+3\,ac^3x^4-\left(bc^2x^3+2\,bx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{12\,c^3}$$

input
```
integrate(x^3*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*b*c^3*x^4*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 3*
a*c^3*x^4 - (b*c^2*x^3 + 2*b*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^3
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int x^3\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\begin{cases}\frac{ax^4}{4}+\frac{bx^4\,\text{asech}\,(cx)}{4}-\frac{bx^2\sqrt{-c^2x^2+1}}{12c^2}-\frac{b\sqrt{-c^2x^2+1}}{6c^4} & \text{for } c\neq 0\\ \frac{x^4(a+\infty b)}{4} & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*(a+b*asech(c*x)),x)
```

output
```
Piecewise((a*x**4/4 + b*x**4*asech(c*x)/4 - b*x**2*sqrt(-c**2*x**2 + 1)/(1
2*c**2) - b*sqrt(-c**2*x**2 + 1)/(6*c**4), Ne(c, 0)), (x**4*(a + oo*b)/4,
True))
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\frac{1}{4}\,ax^4+\frac{1}{12}\left(3\,x^4\operatorname{arsech}\left(cx\right)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)b$$

input
```
integrate(x^3*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) -
3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b
```

3.22.8 Giac [F]

$$\int x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int(b\operatorname{arsech}\left(cx\right)+a)x^3\,dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^3, x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int x^3\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^3*(a + b*acosh(1/(c*x))), x)
```

3.23 $\int x^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

3.23.1 Optimal result

Integrand size = 12, antiderivative size = 78

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = -\frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right) + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{6c^3}$$

output
```
1/3*x^3*(a+b*arcsech(c*x))-1/6*b*x*(-c*x+1)^(1/2)/c^2/(1/(c*x+1))^(1/2)+1/
6*b*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^3
```

3.23.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.32

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \frac{ax^3}{3}+b\sqrt{\frac{1-cx}{1+cx}}\left(-\frac{x}{6c^2}-\frac{x^2}{6c}\right)+\frac{1}{3}bx^3\text{sech}^{-1}(cx) + \frac{ib\log\left(-2icx+2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)}{6c^3}$$

input
```
Integrate[x^2*(a + b*ArcSech[c*x]),x]
```

output
```
(a*x^3)/3 + b*Sqrt[(1 - c*x)/(1 + c*x)]*(-1/6*x/c^2 - x^2/(6*c)) + (b*x^3*
ArcSech[c*x])/3 + ((I/6)*b*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1
 + c*x)])/c^3
```

3.23.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6837, 101, 25, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{6837}$$

$$\frac{1}{3} b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x^2}{\sqrt{1-cx}\sqrt{cx+1}}dx+\frac{1}{3} x^3 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{101}$$

$$\frac{1}{3} b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{1}{\sqrt{1-cx}\sqrt{cx+1}}dx}{2c^2}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)+\frac{1}{3} x^3 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3} b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{1}{\sqrt{1-cx}\sqrt{cx+1}}dx}{2c^2}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)+\frac{1}{3} x^3 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{39}$$

$$\frac{1}{3} b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)+\frac{1}{3} x^3 \left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{3} x^3 \left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3} b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)$$

input
```
Int[x^2*(a + b*ArcSech[c*x]),x]
```

output
```
(x^3*(a + b*ArcSech[c*x]))/3 + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-1/2
*(x*Sqrt[1 - c*x]*Sqrt[1 + c*x])/c^2 + ArcSin[c*x]/(2*c^3)))/3
```

3.23.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6837
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Simp[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)]   Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x])
, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

3.23.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

method	result	size
parts	$\frac{a\,x^3}{3}+\frac{b\left(\frac{c^3x^3\,\operatorname{arcsech}(cx)}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(-cx\sqrt{-c^2x^2+1}+\arcsin(cx)\right)}{6\sqrt{-c^2x^2+1}}\right)}{c^3}$	92
derivativedivides	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\,\operatorname{arcsech}(cx)}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(-cx\sqrt{-c^2x^2+1}+\arcsin(cx)\right)}{6\sqrt{-c^2x^2+1}}\right)}{c^3}$	96
default	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\,\operatorname{arcsech}(cx)}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(-cx\sqrt{-c^2x^2+1}+\arcsin(cx)\right)}{6\sqrt{-c^2x^2+1}}\right)}{c^3}$	96

input
```
int(x^2*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+b/c^3*(1/3*c^3*x^3*arcsech(c*x)+1/6*(-(c*x-1)/c/x)^(1/2)*c*x*((c
*x+1)/c/x)^(1/2)*(-c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(46) = 92.

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.08

$$\int x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$= \frac{2\,ac^3x^3 - bc^2x^2\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 2\,bc^3\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 2\,b\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + 2\left(bc^3x^3-bc^3\right)\log\left(\right.}{6\,c^3}$$

input
```
integrate(x^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*x^3 - b*c^2*x^2*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*b*c^3*log(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2*b*arctan((c*x*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + 2*(b*c^3*x^3 - b*c^3)*log((c*x*sqrt(-(c^2
*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c^3
```

3.23.6 Sympy [F]

$$\int x^2 \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int x^2 (a + b \operatorname{asech}(cx)) \, dx$$

input
```
integrate(x**2*(a+b*asech(c*x)),x)
```

output
```
Integral(x**2*(a + b*asech(c*x)), x)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \frac{1}{3} a x^3 + \frac{1}{6} \left(2 x^3 \operatorname{arsech}(cx) - \frac{\frac{\sqrt{\frac{1}{c^2 x^2} - 1}}{c^2 \left(\frac{1}{c^2 x^2} - 1\right) + c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2 x^2} - 1}\right)}{c^2}}{c} \right) b$$

input
```
integrate(x^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*
x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b
```

3.23.8 Giac [F]

$$\int x^2 \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int (b \operatorname{arsech}(cx) + a) x^2 \, dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^2, x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (a + b\text{sech}^{-1}(cx)) \, dx = \int x^2 \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int(x^2*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^2*(a + b*acosh(1/(c*x))), x)
```

3.24 $\int x\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.24.1 Optimal result

Integrand size = 10, antiderivative size = 45

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)\,dx=-\frac{b\sqrt{1-cx}}{2c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)$$

output
```
1/2*x^2*(a+b*arcsech(c*x))-1/2*b*(-c*x+1)^(1/2)/c^2/(1/(c*x+1))^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\frac{ax^2}{2}+b\left(-\frac{1}{2c^2}-\frac{x}{2c}\right)\sqrt{\frac{1-cx}{1+cx}}+\frac{1}{2}bx^2\text{sech}^{-1}(cx)$$

input
```
Integrate[x*(a + b*ArcSech[c*x]),x]
```

output
```
(a*x^2)/2 + b*(-1/2*1/c^2 - x/(2*c))*Sqrt[(1 - c*x)/(1 + c*x)] + (b*x^2*Ar
cSech[c*x])/2
```

3.24.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6837, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\text{sech}^{-1}(cx))\,dx$$

$$\Big\downarrow \text{6837}$$

$$\frac{1}{2}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x}{\sqrt{1-cx}\sqrt{cx+1}}dx+\frac{1}{2}x^2(a+b\text{sech}^{-1}(cx))$$

$$\Big\downarrow \text{83}$$

$$\frac{1}{2}x^2(a+b\text{sech}^{-1}(cx))-\frac{b\sqrt{1-cx}}{2c^2\sqrt{\frac{1}{cx+1}}}$$

input
```
Int[x*(a + b*ArcSech[c*x]),x]
```

output
```
-1/2*(b*Sqrt[1 - c*x])/(c^2*Sqrt[(1 + c*x)^(-1)]) + (x^2*(a + b*ArcSech[c*
x]))/2
```

3.24.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 6837
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Simp[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)]   Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x])
, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

3.24.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

method	result	size
parts	$\frac{a\,x^{2}}{2}+\frac{b\left(\frac{c^{2}x^{2}\,\mathrm{arcsech}(cx)}{2}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}}{2}\right)}{c^{2}}$	59
derivativedivides	$\frac{\frac{a\,c^{2}x^{2}}{2}+b\left(\frac{c^{2}x^{2}\,\mathrm{arcsech}(cx)}{2}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}}{2}\right)}{c^{2}}$	63
default	$\frac{\frac{a\,c^{2}x^{2}}{2}+b\left(\frac{c^{2}x^{2}\,\mathrm{arcsech}(cx)}{2}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}}{2}\right)}{c^{2}}$	63

input
```
int(x*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+b/c^2*(1/2*c^2*x^2*arcsech(c*x)-1/2*(-(c*x-1)/c/x)^(1/2)*c*x*((c
*x+1)/c/x)^(1/2))
```

3.24.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int x\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\frac{bcx^{2}\log\left(\frac{cx\sqrt{-\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}+1}{cx}\right)+acx^{2}-bx\sqrt{-\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}{2\,c}$$

input
```
integrate(x*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/2*(b*c*x^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + a*c*x^2
 - b*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int x\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\begin{cases}\frac{ax^2}{2}+\frac{bx^2\operatorname{asech}(cx)}{2}-\frac{b\sqrt{-c^2x^2+1}}{2c^2} & \text{for } c\neq 0\\ \frac{x^2(a+\infty b)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(a+b*asech(c*x)),x)
```

output
```
Piecewise((a*x**2/2 + b*x**2*asech(c*x)/2 - b*sqrt(-c**2*x**2 + 1)/(2*c**2
), Ne(c, 0)), (x**2*(a + oo*b)/2, True))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int x\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\operatorname{arsech}(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)b$$

input
```
integrate(x*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/2*(x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)/c)*b
```

3.24.8 Giac [F]

$$\int x\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int(b\operatorname{arsech}(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x, x)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int x\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\frac{a\,x^2}{2}+\frac{b\,x^2\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{2}-\frac{b\,x\,\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}{2\,c}$$

input
```
int(x*(a + b*acosh(1/(c*x))),x)
```

output
```
(a*x^2)/2 + (b*x^2*acosh(1/(c*x)))/2 - (b*x*(1/(c*x) - 1)^(1/2)*(1/(c*x) +
 1)^(1/2))/(2*c)
```

3.25 $\int (a + b\text{sech}^{-1}(cx))\,dx$

3.25.1 Optimal result

Integrand size = 8, antiderivative size = 40

$$\int (a + b\text{sech}^{-1}(cx))\,dx = ax + bx\text{sech}^{-1}(cx) + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c}$$

output
```
a*x+b*x*arcsech(c*x)+b*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c
```

3.25.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

$$\int (a + b\text{sech}^{-1}(cx))\,dx = ax + bx\text{sech}^{-1}(cx) + \frac{2b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\arctan\left(\frac{\sqrt{1-c^2x^2}}{1-cx}\right)}{c-c^2x}$$

input
```
Integrate[a + b*ArcSech[c*x],x]
```

output
```
a*x + b*x*ArcSech[c*x] + (2*b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*
ArcTan[Sqrt[1 - c^2*x^2]/(1 - c*x)])/(c - c^2*x)
```

3.25.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)}{c}+bx\text{sech}^{-1}(cx)$$

input
```
Int[a + b*ArcSech[c*x],x]
```

output
```
a*x + b*x*ArcSech[c*x] + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x]
)/c
```

3.25.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.25.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

method	result	size
default	$ax+bx\,\operatorname{arcsech}(cx)-\frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)}{c}$	42
parts	$ax+bx\,\operatorname{arcsech}(cx)-\frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)}{c}$	42
derivativedivides	$\frac{acx+b\left(cx\,\operatorname{arcsech}(cx)-\arctan\left(\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\right)}{c}$	46

input
```
int(a+b*arcsech(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arcsech(c*x)-b/c*arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
```

3.25.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.98

$$\int \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \frac{acx - bc\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 2\,b\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + (bcx - bc)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{c}$$

input
```
integrate(a+b*arcsech(c*x),x, algorithm="fricas")
```

output
```
(a*c*x - b*c*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2*b*arctan(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + (b*c*x - b*c)*log((c*x*s
qrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c
```

3.25.6 Sympy [F]

$$\int \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int \left(a + b\,\text{asech}\,(cx)\right) \, dx$$

input
```
integrate(a+b*asech(c*x),x)
```

output
```
Integral(a + b*asech(c*x), x)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = ax + \frac{\left(cx\operatorname{arsech}\left(cx\right) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)b}{c}$$

input
```
integrate(a+b*arcsech(c*x),x, algorithm="maxima")
```

output
```
a*x + (c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b/c
```

3.25.8 Giac [F]

$$\int \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \int b\operatorname{arsech}\left(cx\right)+a\,dx$$

input
```
integrate(a+b*arcsech(c*x),x, algorithm="giac")
```

output
```
integrate(b*arcsech(c*x) + a, x)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = a\,x + b\,x\operatorname{acosh}\left(\frac{1}{c\,x}\right) + \frac{b\operatorname{atan}\left(\frac{1}{\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}\right)}{c}$$

input
```
int(a + b*acosh(1/(c*x)),x)
```

output
```
a*x + b*x*acosh(1/(c*x)) + (b*atan(1/((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1
/2))))/c
```

3.26 $\int \frac{a+b\text{sech}^{-1}(cx)}{x}\,dx$

3.26.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x}\,dx=-\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2b}-\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)+\frac{1}{2}b\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)$$

output
```
-1/2*(a+b*arcsech(c*x))^2/b-(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))^2)+1/2*b*polylog(2,-1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))^2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x}\,dx=a\log(x)+\frac{1}{2}b\Big(-\text{sech}^{-1}(cx)\left(\text{sech}^{-1}(cx)+2\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)\Big)$$

input
```
Integrate[(a + b*ArcSech[c*x])/x,x]
```

output
```
a*Log[x] + (b*(-(ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x]
)])) + PolyLog[2, -E^(-2*ArcSech[c*x])]))/2
```

3.26.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.39, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6835, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x} \, dx$$

$$\Big\downarrow \text{ 6835}$$

$$-\int x\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 6297}$$

$$-\frac{\int -\left(\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{b}\right)\right)d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{b}\right)d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -i\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\tan\left(\frac{ia}{b}-\frac{i\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{b}\right)d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\int \left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\tan\left(\frac{ia}{b}-\frac{i\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{b}\right)d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{b}$$

$$\Big\downarrow \text{ 4201}$$

$$-\frac{i\left(2i\int \frac{e^{-2\text{arccosh}\left(\frac{1}{cx}\right)}\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{1+e^{-2\text{arccosh}\left(\frac{1}{cx}\right)}}d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)-\frac{i}{2x^2}\right)}{b}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{i\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{-2\text{arccosh}\left(\frac{1}{cx}\right)}\right)d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)-\frac{1}{2}b\log\left(e^{-2\text{arccosh}\left(\frac{1}{cx}\right)}+1\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\right)-\frac{i}{2x^2}\right)}{b}$$

$\downarrow$ 2715

$$-\frac{i\left(2i\left(-\frac{1}{4}b^2\int x\log\left(1+e^{-2\text{arccosh}\left(\frac{1}{cx}\right)}\right)de^{-2\text{arccosh}\left(\frac{1}{cx}\right)}-\frac{1}{2}b\log\left(e^{-2\text{arccosh}\left(\frac{1}{cx}\right)}+1\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\right)-\frac{i}{2x^2}\right)}{b}$$

$\downarrow$ 2838

$$-\frac{i\left(2i\left(\frac{1}{4}b^2\text{PolyLog}\left(2,-a-b\text{arccosh}\left(\frac{1}{cx}\right)\right)-\frac{1}{2}b\log\left(e^{-2\text{arccosh}\left(\frac{1}{cx}\right)}+1\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\right)-\frac{i}{2x^2}\right)}{b}$$

input
```
Int[(a + b*ArcSech[c*x])/x,x]
```

output
```
((-I)*((-1/2*I)/x^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[1/(c*x)])*Log[1 + E^(-
2*ArcCosh[1/(c*x)])]) + (b^2*PolyLog[2, -a - b*ArcCosh[1/(c*x)]])/4)))/b
```

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6835
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

3.26.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.75

method	result
parts	$a\ln(x)+b\left(\frac{\operatorname{arcsech}(cx)^2}{2}-\operatorname{arcsech}(cx)\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2\right)-\frac{\operatorname{polylog}(}{}$
derivativedivides	$a\ln(cx)+b\left(\frac{\operatorname{arcsech}(cx)^2}{2}-\operatorname{arcsech}(cx)\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2\right)-\frac{\operatorname{polylog}}{}$
default	$a\ln(cx)+b\left(\frac{\operatorname{arcsech}(cx)^2}{2}-\operatorname{arcsech}(cx)\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2\right)-\frac{\operatorname{polylog}}{}$

input
```
int((a+b*arcsech(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*(1/2*arcsech(c*x)^2-arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2))^2)-1/2*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^
2))
```

3.26.5 Fricas [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x,x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)/x, x)
```

3.26.6 Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x}\,dx=\int \frac{a+b\,\text{asech}\,(cx)}{x}\,dx$$

input
```
integrate((a+b*asech(c*x))/x,x)
```

output
```
Integral((a + b*asech(c*x))/x, x)
```

3.26.7 Maxima [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x,x, algorithm="maxima")
```

output
```
b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x) + a*l
og(x)
```

3.26.8 Giac [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/x, x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x}\,dx=\int \frac{a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/x,x)
```

output
```
int((a + b*acosh(1/(c*x)))/x, x)
```

3.27 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^2}\,dx$

3.27.1 Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2}\,dx=\frac{b\sqrt{1-cx}}{x\sqrt{\frac{1}{1+cx}}}-\frac{a+b\text{sech}^{-1}(cx)}{x}$$

output
```
(-a-b*arcsech(c*x))/x+b*(-c*x+1)^(1/2)/x/(1/(c*x+1))^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2}\,dx=-\frac{a}{x}+b\left(c+\frac{1}{x}\right)\sqrt{\frac{1-cx}{1+cx}}-\frac{b\text{sech}^{-1}(cx)}{x}$$

input
```
Integrate[(a + b*ArcSech[c*x])/x^2,x]
```

output
```
-(a/x) + b*(c + x^(-1))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech[c*x])/x
```

3.27.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6837, 106}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2} \, dx$$

$$\Big\downarrow \text{ 6837}$$

$$-b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x^2\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{a+b\text{sech}^{-1}(cx)}{x}$$

$$\Big\downarrow \text{ 106}$$

$$\frac{b\sqrt{1-cx}}{x\sqrt{\frac{1}{cx+1}}}-\frac{a+b\text{sech}^{-1}(cx)}{x}$$

input
```
Int[(a + b*ArcSech[c*x])/x^2,x]
```

output
```
(b*Sqrt[1 - c*x])/(x*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/x
```

3.27.3.1 Defintions of rubi rules used

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 6837
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Simp[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)]   Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x])
, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

3.27.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

method	result	size
parts	$-\frac{a}{x}+bc\left(-\frac{\operatorname{arcsech}(cx)}{cx}+\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\right)$	54
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arcsech}(cx)}{cx}+\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\right)\right)$	58
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arcsech}(cx)}{cx}+\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\right)\right)$	58

input
```
int((a+b*arcsech(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+b*c*(-1/c/x*arcsech(c*x)+(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2}\,dx=\frac{bcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-b\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-a}{x}$$

input
```
integrate((a+b*arcsech(c*x))/x^2,x, algorithm="fricas")
```

output
```
(b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - b*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)) + 1)/(c*x)) - a)/x
```

3.27.6 Sympy [F]

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2}\,dx=\int \frac{a+b\operatorname{asech}(cx)}{x^2}\,dx$$

input
```
integrate((a+b*asech(c*x))/x**2,x)
```

output
```
Integral((a + b*asech(c*x))/x**2, x)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2}\,dx = \left(c\sqrt{\frac{1}{c^2x^2}-1}-\frac{\text{arsech}\,(cx)}{x}\right)b-\frac{a}{x}$$

input
```
integrate((a+b*arcsech(c*x))/x^2,x, algorithm="maxima")
```

output
```
(c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b - a/x
```

3.27.8 Giac [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2}\,dx = \int \frac{b\,\text{arsech}\,(cx)+a}{x^2}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/x^2, x)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2}\,dx = b\,c\,\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}-\frac{b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x}-\frac{a}{x}$$

input
```
int((a + b*acosh(1/(c*x)))/x^2,x)
```

output
```
b*c*(1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) - (b*acosh(1/(c*x)))/x - a/x
```

3.28 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^3} dx$

3.28.1 Optimal result

Integrand size = 12, antiderivative size = 94

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3} dx = \frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{2x^2} + \frac{1}{4}bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-cx}\sqrt{1+cx}\right)$$

output
```
1/2*(-a-b*arcsech(c*x))/x^2+1/4*b*(-c*x+1)^(1/2)/x^2/(1/(c*x+1))^(1/2)+1/4
*b*c^2*arctanh((-c*x+1)^(1/2)*(c*x+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/
2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3} dx = -\frac{a}{2x^2} + b\left(\frac{1}{4x^2} + \frac{c}{4x}\right)\sqrt{\frac{1-cx}{1+cx}} - \frac{b\text{sech}^{-1}(cx)}{2x^2} - \frac{1}{4}bc^2\log(x) + \frac{1}{4}bc^2\log\left(1+\sqrt{\frac{1-cx}{1+cx}} + cx\sqrt{\frac{1-cx}{1+cx}}\right)$$

input
```
Integrate[(a + b*ArcSech[c*x])/x^3,x]
```

output
```
-1/2*a/x^2 + b*(1/(4*x^2) + c/(4*x))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSec
h[c*x])/(2*x^2) - (b*c^2*Log[x])/4 + (b*c^2*Log[1 + Sqrt[(1 - c*x)/(1 + c*
x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/4
```

3.28.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6837, 114, 25, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3} \, dx$$

$$\Big\downarrow \text{6837}$$

$$-\frac{1}{2}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x^3\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{a+b\text{sech}^{-1}(cx)}{2x^2}$$

$$\Big\downarrow \text{114}$$

$$-\frac{1}{2}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{1}{2}\int -\frac{c^2}{x\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)-\frac{a+b\text{sech}^{-1}(cx)}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{2}\int \frac{c^2}{x\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)-\frac{a+b\text{sech}^{-1}(cx)}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{2}c^2\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)-\frac{a+b\text{sech}^{-1}(cx)}{2x^2}$$

$$\Big\downarrow \text{103}$$

$$-\frac{1}{2}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{1}{2}c^3\int \frac{1}{c-c(1-cx)(cx+1)}d\left(\sqrt{1-cx}\sqrt{cx+1}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)-\frac{a+b\text{sech}^{-1}(cx)}{2x^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a+b\text{sech}^{-1}(cx)}{2x^2}-\frac{1}{2}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{1}{2}c^2\text{arctanh}\left(\sqrt{1-cx}\sqrt{cx+1}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)$$

input
```
Int[(a + b*ArcSech[c*x])/x^3,x]
```

output
```
-1/2*(a + b*ArcSech[c*x])/x^2 - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-1/
2*(Sqrt[1 - c*x]*Sqrt[1 + c*x])/x^2 - (c^2*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 +
c*x]])/2))/2
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6837
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Simp[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)]   Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x])
, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

3.28.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

method	result	size
parts	$-\frac{a}{2x^2}+b\,c^2\left(-\frac{\operatorname{arcsech}(cx)}{2c^2x^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^2x^2+\sqrt{-c^2x^2+1}\right)}{4cx\sqrt{-c^2x^2+1}}\right)$	108
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\operatorname{arcsech}(cx)}{2c^2x^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^2x^2+\sqrt{-c^2x^2+1}\right)}{4cx\sqrt{-c^2x^2+1}}\right)\right)$	112
default	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\operatorname{arcsech}(cx)}{2c^2x^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^2x^2+\sqrt{-c^2x^2+1}\right)}{4cx\sqrt{-c^2x^2+1}}\right)\right)$	112

input
```
int((a+b*arcsech(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*c^2*(-1/2/c^2/x^2*arcsech(c*x)+1/4*(-(c*x-1)/c/x)^(1/2)/c/x*(
(c*x+1)/c/x)^(1/2)*(arctanh(1/(-c^2*x^2+1)^(1/2))*c^2*x^2+(-c^2*x^2+1)^(1/
2))/(-c^2*x^2+1)^(1/2))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3}\,dx=\frac{bcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+(bc^2x^2-2\,b)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-2\,a}{4\,x^2}$$

input
```
integrate((a+b*arcsech(c*x))/x^3,x, algorithm="fricas")
```

output
```
1/4*(b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + (b*c^2*x^2 - 2*b)*log((c*x*sqr
t(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*a)/x^2
```

3.28.6 Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3}\,dx = \int \frac{a+b\,\text{asech}\,(cx)}{x^3}\,dx$$

input
```
integrate((a+b*asech(c*x))/x**3,x)
```

output
```
Integral((a + b*asech(c*x))/x**3, x)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3}\,dx = -\frac{1}{8}\,b\left(\frac{\frac{2\,c^4x\sqrt{\frac{1}{c^2x^2}-1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1} - c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right) + c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right)}{c} + \frac{4\,\text{arsech}\,(cx)}{x^2}\right) - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*arcsech(c*x))/x^3,x, algorithm="maxima")
```

output
```
-1/8*b*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c
^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) - 1)
- 1))/c + 4*arcsech(c*x)/x^2) - 1/2*a/x^2
```

3.28.8 Giac [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3}\,dx = \int \frac{b\,\text{arsech}\,(cx)+a}{x^3}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/x^3, x)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3}\,dx = \frac{b\,\text{acosh}\left(\frac{1}{c\,x}\right)\,\left(\frac{c^2\,x}{4}-\frac{1}{2\,x}\right)}{x}-\frac{a}{2\,x^2}+\frac{b\,c\,\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}{4\,x}$$

input
```
int((a + b*acosh(1/(c*x)))/x^3,x)
```

output
```
(b*acosh(1/(c*x))*((c^2*x)/4 - 1/(2*x)))/x - a/(2*x^2) + (b*c*(1/(c*x) - 1
)^(1/2)*(1/(c*x) + 1)^(1/2))/(4*x)
```

3.29 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^4}\,dx$

3.29.1 Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^4}\,dx=\frac{b\sqrt{1-cx}}{9x^3\sqrt{\frac{1}{1+cx}}}+\frac{2bc^2\sqrt{1-cx}}{9x\sqrt{\frac{1}{1+cx}}}-\frac{a+b\text{sech}^{-1}(cx)}{3x^3}$$

output
```
1/3*(-a-b*arcsech(c*x))/x^3+1/9*b*(-c*x+1)^(1/2)/x^3/(1/(c*x+1))^(1/2)+2/9
*b*c^2*(-c*x+1)^(1/2)/x/(1/(c*x+1))^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^4}\,dx=-\frac{a}{3x^3}+b\left(\frac{2c^3}{9}+\frac{1}{9x^3}+\frac{c}{9x^2}+\frac{2c^2}{9x}\right)\sqrt{\frac{1-cx}{1+cx}}-\frac{b\text{sech}^{-1}(cx)}{3x^3}$$

input
```
Integrate[(a + b*ArcSech[c*x])/x^4,x]
```

output
```
-1/3*a/x^3 + b*((2*c^3)/9 + 1/(9*x^3) + c/(9*x^2) + (2*c^2)/(9*x))*Sqrt[(1
 - c*x)/(1 + c*x)] - (b*ArcSech[c*x])/(3*x^3)
```

3.29.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6837, 114, 27, 106}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^4}dx$$

$$\Big\downarrow \text{6837}$$

$$-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x^4\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{a+b\text{sech}^{-1}(cx)}{3x^3}$$

$$\Big\downarrow \text{114}$$

$$-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{1}{3}\int -\frac{2c^2}{x^2\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{3x^3}\right)-\frac{a+b\text{sech}^{-1}(cx)}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2}{3}c^2\int \frac{1}{x^2\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{3x^3}\right)-\frac{a+b\text{sech}^{-1}(cx)}{3x^3}$$

$$\Big\downarrow \text{106}$$

$$-\frac{a+b\text{sech}^{-1}(cx)}{3x^3}-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{2c^2\sqrt{1-cx}\sqrt{cx+1}}{3x}-\frac{\sqrt{1-cx}\sqrt{cx+1}}{3x^3}\right)$$

input
```
Int[(a + b*ArcSech[c*x])/x^4,x]
```

output
```
-1/3*(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-1/3*(Sqrt[1 - c*x]*Sqrt[1 + c
*x])/x^3 - (2*c^2*Sqrt[1 - c*x]*Sqrt[1 + c*x])/(3*x))) - (a + b*ArcSech[c*
x])/(3*x^3)
```

3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 6837
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Simp[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)]   Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x])
, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

3.29.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

method	result	size
parts	$-\frac{a}{3x^3}+b\,c^3\left(-\frac{\operatorname{arcsech}(cx)}{3c^3x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(2c^2x^2+1)}{9c^2x^2}\right)$	73
derivativedivides	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\operatorname{arcsech}(cx)}{3c^3x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(2c^2x^2+1)}{9c^2x^2}\right)\right)$	77
default	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\operatorname{arcsech}(cx)}{3c^3x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(2c^2x^2+1)}{9c^2x^2}\right)\right)$	77

input
```
int((a+b*arcsech(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arcsech(c*x)+1/9*(-(c*x-1)/c/x)^(1/2)/c^2/x
^2*((c*x+1)/c/x)^(1/2)*(2*c^2*x^2+1))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^4}\,dx = -\frac{3\,b\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(2\,bc^3x^3+bcx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+3\,a}{9\,x^3}$$

input
```
integrate((a+b*arcsech(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/9*(3*b*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (2*b*c^3*x
^3 + b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 3*a)/x^3
```

3.29.6 Sympy [F]

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^4}\,dx = \int \frac{a+b\operatorname{asech}(cx)}{x^4}\,dx$$

input
```
integrate((a+b*asech(c*x))/x**4,x)
```

output
```
Integral((a + b*asech(c*x))/x**4, x)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.73

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x^4}\,dx = \frac{1}{9}b\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} + 3\,c^4\sqrt{\frac{1}{c^2x^2}-1}}{c} - \frac{3\,\operatorname{arsech}(cx)}{x^3}\right) - \frac{a}{3\,x^3}$$

input
```
integrate((a+b*arcsech(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/9*b*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*a
rcsech(c*x)/x^3) - 1/3*a/x^3
```

3.29.8 Giac [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x^4}\,dx = \int \frac{b\operatorname{arsech}(cx) + a}{x^4}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/x^4, x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x^4}\,dx = \int \frac{a + b\operatorname{acosh}\left(\frac{1}{c\,x}\right)}{x^4}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/x^4,x)
```

output
```
int((a + b*acosh(1/(c*x)))/x^4, x)
```

3.30 $\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^5}\,dx$

3.30.1 Optimal result

Integrand size = 12, antiderivative size = 126

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^5}\,dx = \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{1+cx}}}+\frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{1+cx}}}-\frac{a+b\,\text{sech}^{-1}(cx)}{4x^4}+\frac{3}{32}bc^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-cx}\sqrt{1+cx}\right)$$

output
```
1/4*(-a-b*arcsech(c*x))/x^4+1/16*b*(-c*x+1)^(1/2)/x^4/(1/(c*x+1))^(1/2)+3/
32*b*c^2*(-c*x+1)^(1/2)/x^2/(1/(c*x+1))^(1/2)+3/32*b*c^4*arctanh((-c*x+1)^
(1/2)*(c*x+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^5}\,dx = -\frac{a}{4x^4}+b\left(\frac{1}{16x^4}+\frac{c}{16x^3}+\frac{3c^2}{32x^2}+\frac{3c^3}{32x}\right)\sqrt{\frac{1-cx}{1+cx}}-\frac{b\,\text{sech}^{-1}(cx)}{4x^4}-\frac{3}{32}bc^4\log(x)+\frac{3}{32}bc^4\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)$$

input
```
Integrate[(a + b*ArcSech[c*x])/x^5,x]
```

output
```
-1/4*a/x^4 + b*(1/(16*x^4) + c/(16*x^3) + (3*c^2)/(32*x^2) + (3*c^3)/(32*x
))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech[c*x])/(4*x^4) - (3*b*c^4*Log[x])
/32 + (3*b*c^4*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 +
 c*x)]])/32
```

3.30.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6837, 114, 27, 114, 25, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5}dx$$

$$\Big\downarrow \text{6837}$$

$$-\frac{1}{4}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x^5\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{a+b\text{sech}^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{114}$$

$$-\frac{1}{4}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{1}{4}\int -\frac{3c^2}{x^3\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{4x^4}\right)-\frac{a+b\text{sech}^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{3}{4}c^2\int \frac{1}{x^3\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{4x^4}\right)-\frac{a+b\text{sech}^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{114}$$

$$-\frac{1}{4}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{3}{4}c^2\left(-\frac{1}{2}\int -\frac{c^2}{x\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{4x^4}\right)-\frac{a+b\text{sech}^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{4}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{3}{4}c^2\left(\frac{1}{2}\int \frac{c^2}{x\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{4x^4}\right)-\frac{a+b\text{sech}^{-1}(cx)}{4x^4}$$

$\Big\downarrow$ 27

$$-\frac{1}{4}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{3}{4}c^2\left(\frac{1}{2}c^2\int\frac{1}{x\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{4x^4}\right)-\frac{a+b\text{sech}^{-1}(cx)}{4x^4}$$

$\Big\downarrow$ 103

$$-\frac{1}{4}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{3}{4}c^2\left(-\frac{1}{2}c^3\int\frac{1}{c-c(1-cx)(cx+1)}d\left(\sqrt{1-cx}\sqrt{cx+1}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{4x^4}\right)-\frac{a+b\text{sech}^{-1}(cx)}{4x^4}$$

$\Big\downarrow$ 221

$$-\frac{a+b\text{sech}^{-1}(cx)}{4x^4}-\frac{1}{4}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{3}{4}c^2\left(-\frac{1}{2}c^2\text{arctanh}\left(\sqrt{1-cx}\sqrt{cx+1}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{4x^4}\right)$$

input
```
Int[(a + b*ArcSech[c*x])/x^5,x]
```

output
```
-1/4*(a + b*ArcSech[c*x])/x^4 - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-1/
4*(Sqrt[1 - c*x]*Sqrt[1 + c*x])/x^4 + (3*c^2*(-1/2*(Sqrt[1 - c*x]*Sqrt[1 +
 c*x])/x^2 - (c^2*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/2))/4))/4
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6837
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Simp[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)]   Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x])
, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

3.30.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04

method	result
parts	$-\frac{a}{4x^4}+b\,c^4\left(-\frac{\operatorname{arcsech}(cx)}{4c^4x^4}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\left(3\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^4x^4+3\sqrt{-c^2x^2+1}\,c^2x^2+2\sqrt{-c^2x^2+1}\right)}{32c^3x^3\sqrt{-c^2x^2+1}}\right)$
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\operatorname{arcsech}(cx)}{4c^4x^4}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\left(3\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^4x^4+3\sqrt{-c^2x^2+1}\,c^2x^2+2\sqrt{-c^2x^2+1}\right)}{32c^3x^3\sqrt{-c^2x^2+1}}\right)\right)$
default	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\operatorname{arcsech}(cx)}{4c^4x^4}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\left(3\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^4x^4+3\sqrt{-c^2x^2+1}\,c^2x^2+2\sqrt{-c^2x^2+1}\right)}{32c^3x^3\sqrt{-c^2x^2+1}}\right)\right)$

input
```
int((a+b*arcsech(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/x^4+b*c^4*(-1/4/c^4/x^4*arcsech(c*x)+1/32*(-(c*x-1)/c/x)^(1/2)/c^3/
x^3*((c*x+1)/c/x)^(1/2)*(3*arctanh(1/(-c^2*x^2+1)^(1/2))*c^4*x^4+3*(-c^2*x
^2+1)^(1/2)*c^2*x^2+2*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5}\,dx = \frac{(3\,bc^4x^4-8\,b)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+(3\,bc^3x^3+2\,bcx)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-8\,a}{32\,x^4}$$

input
```
integrate((a+b*arcsech(c*x))/x^5,x, algorithm="fricas")
```

output
```
1/32*((3*b*c^4*x^4 - 8*b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*
x)) + (3*b*c^3*x^3 + 2*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 8*a)/x^4
```

3.30.6 Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5}\,dx = \int \frac{a+b\,\text{asech}\,(cx)}{x^5}\,dx$$

input
```
integrate((a+b*asech(c*x))/x**5,x)
```

output
```
Integral((a + b*asech(c*x))/x**5, x)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5}\,dx = \frac{1}{64}\,b\left(\frac{3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right)-3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right)-\frac{2\left(3\,c^8x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-5\,c^6x\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2-2\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)+1}}{c}-\frac{16\text{ a}\right. - \frac{a}{4\,x^4}$$

input
```
integrate((a+b*arcsech(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/64*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) - 3*c^5*log(c*x*sqrt(1/(
c^2*x^2) - 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) - 1)^(3/2) - 5*c^6*x*sqrt(1
/(c^2*x^2) - 1))/(c^4*x^4*(1/(c^2*x^2) - 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1
) + 1))/c - 16*arcsech(c*x)/x^4) - 1/4*a/x^4
```

3.30.8 Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^5}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{x^5}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^5,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/x^5, x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^5}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x^5}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/x^5,x)
```

output
```
int((a + b*acosh(1/(c*x)))/x^5, x)
```

3.31 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^6} dx$

3.31.1 Optimal result

Integrand size = 12, antiderivative size = 109

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^6} dx = \frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{1+cx}}} + \frac{4bc^2\sqrt{1-cx}}{75x^3\sqrt{\frac{1}{1+cx}}} + \frac{8bc^4\sqrt{1-cx}}{75x\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{5x^5}$$

output
```
1/5*(-a-b*arcsech(c*x))/x^5+1/25*b*(-c*x+1)^(1/2)/x^5/(1/(c*x+1))^(1/2)+4/
75*b*c^2*(-c*x+1)^(1/2)/x^3/(1/(c*x+1))^(1/2)+8/75*b*c^4*(-c*x+1)^(1/2)/x/
(1/(c*x+1))^(1/2)
```

3.31.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^6} dx = -\frac{a}{5x^5} + b\left(\frac{8c^5}{75} + \frac{1}{25x^5} + \frac{c}{25x^4} + \frac{4c^2}{75x^3} + \frac{4c^3}{75x^2} + \frac{8c^4}{75x}\right)\sqrt{\frac{1-cx}{1+cx}} - \frac{b\text{sech}^{-1}(cx)}{5x^5}$$

input
```
Integrate[(a + b*ArcSech[c*x])/x^6,x]
```

output
```
-1/5*a/x^5 + b*((8*c^5)/75 + 1/(25*x^5) + c/(25*x^4) + (4*c^2)/(75*x^3) +
(4*c^3)/(75*x^2) + (8*c^4)/(75*x))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech[
c*x])/(5*x^5)
```

3.31.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6837, 114, 27, 114, 27, 106}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^6}dx$$

$$\Big\downarrow \text{ 6837}$$

$$-\frac{1}{5}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x^6\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{a+b\text{sech}^{-1}(cx)}{5x^5}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{1}{5}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{1}{5}\int -\frac{4c^2}{x^4\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{5x^5}\right)-\frac{a+b\text{sech}^{-1}(cx)}{5x^5}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{5}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{4}{5}c^2\int \frac{1}{x^4\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{5x^5}\right)-\frac{a+b\text{sech}^{-1}(cx)}{5x^5}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{1}{5}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{4}{5}c^2\left(-\frac{1}{3}\int -\frac{2c^2}{x^2\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{3x^3}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{5x^5}\right)-\frac{a+b\text{sech}^{-1}(cx)}{5x^5}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{5}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{4}{5}c^2\left(\frac{2}{3}c^2\int \frac{1}{x^2\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{3x^3}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{5x^5}\right)-\frac{a+b\text{sech}^{-1}(cx)}{5x^5}$$

$$\Big\downarrow \text{ 106}$$

$$-\frac{a+b\text{sech}^{-1}(cx)}{5x^5}-\frac{1}{5}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{4}{5}c^2\left(-\frac{2c^2\sqrt{1-cx}\sqrt{cx+1}}{3x}-\frac{\sqrt{1-cx}\sqrt{cx+1}}{3x^3}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{5x^5}\right)$$

input
```
Int[(a + b*ArcSech[c*x])/x^6,x]
```

output
```
-1/5*(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-1/5*(Sqrt[1 - c*x]*Sqrt[1 + c
*x])/x^5 + (4*c^2*(-1/3*(Sqrt[1 - c*x]*Sqrt[1 + c*x])/x^3 - (2*c^2*Sqrt[1
- c*x]*Sqrt[1 + c*x])/(3*x)))/5)) - (a + b*ArcSech[c*x])/(5*x^5)
```

3.31.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 6837
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Simp[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)]   Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x])
, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

3.31.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

method	result	size
parts	$-\frac{a}{5x^5}+b\,c^5\left(-\frac{\operatorname{arcsech}(cx)}{5c^5x^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(8c^4x^4+4c^2x^2+3)}{75c^4x^4}\right)$	81
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arcsech}(cx)}{5c^5x^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(8c^4x^4+4c^2x^2+3)}{75c^4x^4}\right)\right)$	85
default	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arcsech}(cx)}{5c^5x^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(8c^4x^4+4c^2x^2+3)}{75c^4x^4}\right)\right)$	85

input
```
int((a+b*arcsech(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5+b*c^5*(-1/5/c^5/x^5*arcsech(c*x)+1/75*(-(c*x-1)/c/x)^(1/2)/c^4/
x^4*((c*x+1)/c/x)^(1/2)*(8*c^4*x^4+4*c^2*x^2+3))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^6}\,dx = -\frac{15\,b\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(8\,bc^5x^5+4\,bc^3x^3+3\,bcx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+15\,a}{75\,x^5}$$

input
```
integrate((a+b*arcsech(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/75*(15*b*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (8*b*c^5
*x^5 + 4*b*c^3*x^3 + 3*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 15*a)/x^5
```

3.31.6 Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^6} dx = \int \frac{a + b\,\text{asech}\,(cx)}{x^6} dx$$

input
```
integrate((a+b*asech(c*x))/x**6,x)
```

output
```
Integral((a + b*asech(c*x))/x**6, x)
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.67

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^6} dx \\ = \frac{1}{75} b \left(\frac{3\,c^6 \left(\frac{1}{c^2x^2} - 1\right)^{\frac{5}{2}} + 10\,c^6 \left(\frac{1}{c^2x^2} - 1\right)^{\frac{3}{2}} + 15\,c^6 \sqrt{\frac{1}{c^2x^2} - 1}}{c} - \frac{15\,\text{arsech}\,(cx)}{x^5} \right) - \frac{a}{5\,x^5}$$

input
```
integrate((a+b*arcsech(c*x))/x^6,x, algorithm="maxima")
```

output
```
1/75*b*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c^2*x^2) - 1)^(3/2) +
15*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5) - 1/5*a/x^5
```

3.31.8 Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^6} dx = \int \frac{b\,\text{arsech}\,(cx) + a}{x^6} dx$$

input
```
integrate((a+b*arcsech(c*x))/x^6,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/x^6, x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^6}\,dx=\int \frac{a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)}{x^6}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/x^6,x)
```

output
```
int((a + b*acosh(1/(c*x)))/x^6, x)
```

3.32 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^7}\,dx$

3.32.1 Optimal result

Integrand size = 12, antiderivative size = 158

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^7}\,dx = \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}}+\frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{1+cx}}}+\frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{1+cx}}}-\frac{a+b\text{sech}^{-1}(cx)}{6x^6}+\frac{5}{96}bc^6\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-cx}\sqrt{1+cx}\right)$$

output
```
1/6*(-a-b*arcsech(c*x))/x^6+1/36*b*(-c*x+1)^(1/2)/x^6/(1/(c*x+1))^(1/2)+5/
144*b*c^2*(-c*x+1)^(1/2)/x^4/(1/(c*x+1))^(1/2)+5/96*b*c^4*(-c*x+1)^(1/2)/x
^2/(1/(c*x+1))^(1/2)+5/96*b*c^6*arctanh((-c*x+1)^(1/2)*(c*x+1)^(1/2))*(1/(
c*x+1))^(1/2)*(c*x+1)^(1/2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^7}\,dx = -\frac{a}{6x^6}+b\left(\frac{1}{36x^6}+\frac{c}{36x^5}+\frac{5c^2}{144x^4}+\frac{5c^3}{144x^3}+\frac{5c^4}{96x^2}+\frac{5c^5}{96x}\right)\sqrt{\frac{1-cx}{1+cx}}-\frac{b\text{sech}^{-1}(cx)}{6x^6}-\frac{5}{96}bc^6\log(x)+\frac{5}{96}bc^6\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)$$

input
```
Integrate[(a + b*ArcSech[c*x])/x^7,x]
```

output
```
-1/6*a/x^6 + b*(1/(36*x^6) + c/(36*x^5) + (5*c^2)/(144*x^4) + (5*c^3)/(144
*x^3) + (5*c^4)/(96*x^2) + (5*c^5)/(96*x))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*
ArcSech[c*x])/(6*x^6) - (5*b*c^6*Log[x])/96 + (5*b*c^6*Log[1 + Sqrt[(1 - c
*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/96
```

3.32.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6837, 114, 27, 114, 27, 114, 25, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^7}dx$$

$$\Big\downarrow \ 6837$$

$$-\frac{1}{6}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x^7\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{a+b\text{sech}^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \ 114$$

$$-\frac{1}{6}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{1}{6}\int -\frac{5c^2}{x^5\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{6x^6}\right)-\frac{a+b\text{sech}^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{6}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{5}{6}c^2\int \frac{1}{x^5\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{6x^6}\right)-\frac{a+b\text{sech}^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \ 114$$

$$-\frac{1}{6}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{5}{6}c^2\left(-\frac{1}{4}\int -\frac{3c^2}{x^3\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{4x^4}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{6x^6}\right)-\frac{a+b\text{sech}^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{6}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\int\frac{1}{x^3\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{4x^4}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{6x^6}\right)-\frac{a+b\text{sech}^{-1}(cx)}{6x^6}$$

$\big\downarrow$ 114

$$-\frac{1}{6}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\left(-\frac{1}{2}\int-\frac{c^2}{x\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{4x^4}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{6x^6}\right)-\frac{a+b\text{sech}^{-1}(cx)}{6x^6}$$

$\big\downarrow$ 25

$$-\frac{1}{6}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\left(\frac{1}{2}\int\frac{c^2}{x\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{4x^4}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{6x^6}\right)-\frac{a+b\text{sech}^{-1}(cx)}{6x^6}$$

$\big\downarrow$ 27

$$-\frac{1}{6}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\left(\frac{1}{2}c^2\int\frac{1}{x\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{4x^4}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{6x^6}\right)-\frac{a+b\text{sech}^{-1}(cx)}{6x^6}$$

$\big\downarrow$ 103

$$-\frac{1}{6}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\left(-\frac{1}{2}c^3\int\frac{1}{c-c(1-cx)(cx+1)}d\left(\sqrt{1-cx}\sqrt{cx+1}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{4x^4}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{6x^6}\right)-\frac{a+b\text{sech}^{-1}(cx)}{6x^6}$$

$\big\downarrow$ 221

$$-\frac{a+b\text{sech}^{-1}(cx)}{6x^6}-\frac{1}{6}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\left(-\frac{1}{2}c^2\text{arctanh}\left(\sqrt{1-cx}\sqrt{cx+1}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{4x^4}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{6x^6}\right)$$

input
```
Int[(a + b*ArcSech[c*x])/x^7,x]
```

3.32. $\int \frac{a+b\text{sech}^{-1}(cx)}{x^7}\,dx$

output
```
-1/6*(a + b*ArcSech[c*x])/x^6 - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-1/
6*(Sqrt[1 - c*x]*Sqrt[1 + c*x])/x^6 + (5*c^2*(-1/4*(Sqrt[1 - c*x]*Sqrt[1 +
 c*x])/x^4 + (3*c^2*(-1/2*(Sqrt[1 - c*x]*Sqrt[1 + c*x])/x^2 - (c^2*ArcTanh
[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/2))/4))/6))/6
```

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6837
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Simp[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)]   Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x])
, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

3.32.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

method	result
parts	$-\frac{a}{6x^6}+b\,c^6\left(-\frac{\text{arcsech}(cx)}{6c^6x^6}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\left(15\,\text{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^6x^6+15\sqrt{-c^2x^2+1}\,c^4x^4+10\sqrt{-c^2x}\right.}{288c^5x^5\sqrt{-c^2x^2+1}}\right.$
derivativedivides	$c^6\left(-\frac{a}{6c^6x^6}+b\left(-\frac{\text{arcsech}(cx)}{6c^6x^6}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\left(15\,\text{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^6x^6+15\sqrt{-c^2x^2+1}\,c^4x^4+10\sqrt{-}\right.}{288c^5x^5\sqrt{-c^2x^2+1}}\right.\right.$
default	$c^6\left(-\frac{a}{6c^6x^6}+b\left(-\frac{\text{arcsech}(cx)}{6c^6x^6}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\left(15\,\text{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^6x^6+15\sqrt{-c^2x^2+1}\,c^4x^4+10\sqrt{-}\right.}{288c^5x^5\sqrt{-c^2x^2+1}}\right.\right.$

input
```
int((a+b*arcsech(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a/x^6+b*c^6*(-1/6/c^6/x^6*arcsech(c*x)+1/288*(-(c*x-1)/c/x)^(1/2)/c^5
/x^5*((c*x+1)/c/x)^(1/2)*(15*arctanh(1/(-c^2*x^2+1)^(1/2))*c^6*x^6+15*(-c^
2*x^2+1)^(1/2)*c^4*x^4+10*(-c^2*x^2+1)^(1/2)*c^2*x^2+8*(-c^2*x^2+1)^(1/2))
/(-c^2*x^2+1)^(1/2))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.63

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^7}\,dx$$
$$=\frac{3\,(5\,bc^6x^6-16\,b)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+(15\,bc^5x^5+10\,bc^3x^3+8\,bcx)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-48\,a}{288\,x^6}$$

input
```
integrate((a+b*arcsech(c*x))/x^7,x, algorithm="fricas")
```

output
```
1/288*(3*(5*b*c^6*x^6 - 16*b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)
/(c*x)) + (15*b*c^5*x^5 + 10*b*c^3*x^3 + 8*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2
*x^2)) - 48*a)/x^6
```

3.32.6 Sympy [F]

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^7}\,dx = \int \frac{a+b\,\text{asech}\,(cx)}{x^7}\,dx$$

input
```
integrate((a+b*asech(c*x))/x**7,x)
```

output
```
Integral((a + b*asech(c*x))/x**7, x)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.17

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^7}\,dx = \frac{1}{576}\,b\left(\frac{15\,c^7\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right)-15\,c^7\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right)-\frac{2\left(15\,c^{12}x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}-40\,c^{10}x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}\cdots\right.}{c^6x^6\left(\frac{1}{c^2x^2}-1\right)^3-3\,c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^2x\cdots}}{c}\right.\cdots - \frac{a}{6\,x^6}$$

input
```
integrate((a+b*arcsech(c*x))/x^7,x, algorithm="maxima")
```

output
```
1/576*b*((15*c^7*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) - 15*c^7*log(c*x*sqrt(
1/(c^2*x^2) - 1) - 1) - 2*(15*c^12*x^5*(1/(c^2*x^2) - 1)^(5/2) - 40*c^10*x
^3*(1/(c^2*x^2) - 1)^(3/2) + 33*c^8*x*sqrt(1/(c^2*x^2) - 1))/(c^6*x^6*(1/(
c^2*x^2) - 1)^3 - 3*c^4*x^4*(1/(c^2*x^2) - 1)^2 + 3*c^2*x^2*(1/(c^2*x^2) -
 1) - 1))/c - 96*arcsech(c*x)/x^6) - 1/6*a/x^6
```

3.32.8 Giac [F]

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^7}\,dx = \int \frac{b\operatorname{arsech}(cx)+a}{x^7}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^7,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/x^7, x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^7}\,dx = \int \frac{a+b\operatorname{acosh}\left(\frac{1}{cx}\right)}{x^7}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/x^7,x)
```

output
```
int((a + b*acosh(1/(c*x)))/x^7, x)
```

3.33 $\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^2 dx$

3.33.1 Optimal result

Integrand size = 14, antiderivative size = 124

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^2 dx = -\frac{b^2x^2}{12c^2} - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^4} - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{6c^2} + \frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^2 - \frac{b^2\log(x)}{3c^4}$$

output
```
-1/12*b^2*x^2/c^2+1/4*x^4*(a+b*arcsech(c*x))^2-1/3*b^2*ln(x)/c^4-1/3*b*(c*
x+1)*(a+b*arcsech(c*x))*((-c*x+1)/(c*x+1))^(1/2)/c^4-1/6*b*x^2*(c*x+1)*(a+
b*arcsech(c*x))*((-c*x+1)/(c*x+1))^(1/2)/c^2
```

3.33.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.71

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^2 dx = -\frac{b^2c^2x^2 - 3a^2c^4x^4 + 4ab\sqrt{\frac{1-cx}{1+cx}} + 4abcx\sqrt{\frac{1-cx}{1+cx}} + 2abc^2x^2\sqrt{\frac{1-cx}{1+cx}} + 2abc^3x^3\sqrt{\frac{1-cx}{1+cx}} + 2b\left(-3ac^4x^4 + b\sqrt{\frac{1}{1}}\right.}{12c^4}$$

input
```
Integrate[x^3*(a + b*ArcSech[c*x])^2,x]
```

output
```
-1/12*(b^2*c^2*x^2 - 3*a^2*c^4*x^4 + 4*a*b*Sqrt[(1 - c*x)/(1 + c*x)] + 4*a
*b*c*x*Sqrt[(1 - c*x)/(1 + c*x)] + 2*a*b*c^2*x^2*Sqrt[(1 - c*x)/(1 + c*x)]
 + 2*a*b*c^3*x^3*Sqrt[(1 - c*x)/(1 + c*x)] + 2*b*(-3*a*c^4*x^4 + b*Sqrt[(1
 - c*x)/(1 + c*x)]*(2 + 2*c*x + c^2*x^2 + c^3*x^3))*ArcSech[c*x] - 3*b^2*c
^4*x^4*ArcSech[c*x]^2 + 4*b^2*Log[x])/c^4
```

3.33.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6839, 5974, 3042, 4673, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^2 dx$$

$$\Big\downarrow \text{6839}$$

$$-\frac{\int c^4x^4\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)}{c^4}$$

$$\Big\downarrow \text{5974}$$

$$-\frac{\frac{1}{2}b\int c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^2}{c^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^2+\frac{1}{2}b\int \left(a+b\text{sech}^{-1}(cx)\right)\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^4d\text{sech}^{-1}(cx)}{c^4}$$

$$\Big\downarrow \text{4673}$$

$$-\frac{\frac{1}{2}b\left(\frac{2}{3}\int c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)+\frac{1}{3}c^2x^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{6}bc^2x^2\right)-\frac{1}{4}c^4x^4\left(a+b\right.}{c^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^2+\frac{1}{2}b\left(\frac{2}{3}\int \left(a+b\text{sech}^{-1}(cx)\right)\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^2d\text{sech}^{-1}(cx)+\frac{1}{3}c^2x^2\sqrt{\frac{1-cx}{cx+1}}(cx\right.}{c^4}$$

$$\Big\downarrow \ 4672$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^2+\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)-ib\int -i\sqrt{\frac{1-cx}{cx+1}}(cx+1)d\text{sech}^{-1}(cx)\right)+\right.}{c^4}$$

$$\Big\downarrow \ 26$$

$$-\frac{\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)-b\int\sqrt{\frac{1-cx}{cx+1}}(cx+1)d\text{sech}^{-1}(cx)\right)+\frac{1}{3}c^2x^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-}\right.\right.}{c^4}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^2+\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)-b\int -i\tan\left(i\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)\right)\right.}{c^4}$$

$$\Big\downarrow \ 26$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^2+\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)+ib\int\tan\left(i\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)\right)+\right.}{c^4}$$

$$\Big\downarrow \ 3956$$

$$-\frac{\frac{1}{2}b\left(\frac{1}{3}c^2x^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{3}\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)-b\log\left(\frac{1}{cx}\right)\right)+\frac{1}{6}bc^2x^2\right)-\frac{1}{4}}{c^4}$$

input
```
Int[x^3*(a + b*ArcSech[c*x])^2,x]
```

output
```
-((-1/4*(c^4*x^4*(a + b*ArcSech[c*x])^2) + (b*((b*c^2*x^2)/6 + (c^2*x^2*Sq
rt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/3 + (2*(Sqrt[(1 -
c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]) - b*Log[1/(c*x)]))/3))/2)/c
^4)
```

3.33. $\int x^3\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx$

3.33.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.33. $\int x^3 \left(a + b\text{sech}^{-1}(cx)\right)^2 \, dx$

3.33.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(110) = 220$.

Time = 0.72 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.81

method	result
parts	$\frac{x^4a^2}{4}+\frac{b^2\left(-\frac{\operatorname{arcsech}(cx)}{3}+\frac{\operatorname{arcsech}(cx)^2c^4x^4}{4}-\frac{\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}c^3x^3}{6}-\frac{\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}cx}{3}-\frac{c^2x^2}{12}+\cdots\right.}{c^4}$
derivativedivides	$\frac{\frac{a^2c^4x^4}{4}+b^2\left(-\frac{\operatorname{arcsech}(cx)}{3}+\frac{\operatorname{arcsech}(cx)^2c^4x^4}{4}-\frac{\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}c^3x^3}{6}-\frac{\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}cx}{3}-\frac{c^2x^2}{12}+\cdots\right.}{c^4}$
default	$\frac{\frac{a^2c^4x^4}{4}+b^2\left(-\frac{\operatorname{arcsech}(cx)}{3}+\frac{\operatorname{arcsech}(cx)^2c^4x^4}{4}-\frac{\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}c^3x^3}{6}-\frac{\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}cx}{3}-\frac{c^2x^2}{12}+\cdots\right.}{c^4}$

input
```
int(x^3*(a+b*arcsech(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*a^2+b^2/c^4*(-1/3*arcsech(c*x)+1/4*arcsech(c*x)^2*c^4*x^4-1/6*arcs
ech(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c^3*x^3-1/3*arcsech(c*x)
*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-1/12*c^2*x^2+1/3*ln(1+(1/c/x
+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))+2*a*b/c^4*(1/4*c^4*x^4*arcsech(c*x)
-1/12*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(c^2*x^2+2))
```

3.33.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(110) = 220$.

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.97

$$\int x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)^2\,dx$$
$$=\frac{3\,b^2c^4x^4\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2+3\,a^2c^4x^4-6\,abc^4\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)-b^2c^2x^2-4\,b^2\log(x)+2\left(3\,abc^4x\cdots\right.}{12\,c^4}$$

input
```
integrate(x^3*(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^2*c^4*x^4*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2
+ 3*a^2*c^4*x^4 - 6*a*b*c^4*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x
) - b^2*c^2*x^2 - 4*b^2*log(x) + 2*(3*a*b*c^4*x^4 - 3*a*b*c^4 - (b^2*c^3*x
^3 + 2*b^2*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 -
1)/(c^2*x^2)) + 1)/(c*x)) - 2*(a*b*c^3*x^3 + 2*a*b*c*x)*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)))/c^4
```

3.33.6 Sympy [F]

$$\int x^3\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int x^3(a+b\operatorname{asech}(cx))^2\,dx$$

input
```
integrate(x**3*(a+b*asech(c*x))**2,x)
```

output
```
Integral(x**3*(a + b*asech(c*x))**2, x)
```

3.33.7 Maxima [F]

$$\int x^3\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int (b\,\text{arsech}\,(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*x^4 + 1/6*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) -
 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*a*b + b^2*integrate(x^3*log(sqrt(1/(c*x)
+ 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2, x)
```

3.33.8 Giac [F]

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx=\int (b\,\text{arsech}\,(cx)+a)^2x^3 \, dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^2*x^3, x)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx=\int x^3\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2 \,d x$$

input
```
int(x^3*(a + b*acosh(1/(c*x)))^2,x)
```

output
```
int(x^3*(a + b*acosh(1/(c*x)))^2, x)
```

3.34 $\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx$

3.34.1 Optimal result

Integrand size = 14, antiderivative size = 140

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx = -\frac{b^2 x}{3c^2}-\frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^2}+\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^2-\frac{2b\left(a+b\text{sech}^{-1}(cx)\right)\arctan\left(e^{\text{sech}^{-1}(cx)}\right)}{3c^3}+\frac{ib^2\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)}{3c^3}-\frac{ib^2\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(cx)}\right)}{3c^3}$$

output
```
-1/3*b^2*x/c^2+1/3*x^3*(a+b*arcsech(c*x))^2-2/3*b*(a+b*arcsech(c*x))*arcta
n(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/c^3+1/3*I*b^2*polylog(2,-I*(1/c/
x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-1/3*I*b^2*polylog(2,I*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-1/3*b*x*(c*x+1)*(a+b*arcsech(c*x))*((-c
*x+1)/(c*x+1))^(1/2)/c^2
```

3.34.2 Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.72

$$\int x^2\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx$$
$$=\frac{1}{3}\left(a^2x^3+ab\left(2x^3\text{sech}^{-1}(cx)+\frac{\sqrt{\frac{1-cx}{1+cx}}\left(cx-c^3x^3+2\sqrt{1-c^2x^2}\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c^3(-1+cx)}\right)\right.$$
$$+\frac{b^2\left(-cx-cx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\text{sech}^{-1}(cx)+c^3x^3\text{sech}^{-1}(cx)^2+i\text{sech}^{-1}(cx)\log\left(1-ie^{-\text{sech}^{-1}(cx)}\right)-i\text{sech}}{c^3}$$

input
```
Integrate[x^2*(a + b*ArcSech[c*x])^2,x]
```

output
```
(a^2*x^3 + a*b*(2*x^3*ArcSech[c*x] + (Sqrt[(1 - c*x)/(1 + c*x)]*(c*x - c^3
*x^3 + 2*Sqrt[1 - c^2*x^2]*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))/(c^3*(-1
+ c*x))) + (b^2*(-(c*x) - c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*ArcSech[
c*x] + c^3*x^3*ArcSech[c*x]^2 + I*ArcSech[c*x]*Log[1 - I/E^ArcSech[c*x]] -
 I*ArcSech[c*x]*Log[1 + I/E^ArcSech[c*x]] + I*PolyLog[2, (-I)/E^ArcSech[c*
x]] - I*PolyLog[2, I/E^ArcSech[c*x]]))/c^3)/3
```

3.34.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6839, 5974, 3042, 4673, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx$$
$$\Big\downarrow \text{ 6839}$$
$$-\frac{\int c^3x^3\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)}{c^3}$$
$$\Big\downarrow \text{ 5974}$$
$$-\frac{\frac{2}{3}b\int c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}{c^3}$$

$$\downarrow 3042$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^2+\frac{2}{3}b\int\left(a+b\text{sech}^{-1}(cx)\right)\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^3d\text{sech}^{-1}(cx)}{c^3}$$

$$\downarrow 4673$$

$$-\frac{\frac{2}{3}b\left(\frac{1}{2}\int cx\left(a+b\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)+\frac{1}{2}cx\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)+\frac{bcx}{2}\right)-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(c\right.}{c^3}$$

$$\downarrow 3042$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^2+\frac{2}{3}b\left(\frac{1}{2}\int\left(a+b\text{sech}^{-1}(cx)\right)\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(cx)+\frac{1}{2}cx\sqrt{\frac{1-cx}{cx+1}}(cx+1\right.}{c^3}$$

$$\downarrow 4668$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^2+\frac{2}{3}b\left(\frac{1}{2}\left(-ib\int\log\left(1-ie^{\text{sech}^{-1}(cx)}\right)d\text{sech}^{-1}(cx)+ib\int\log\left(1+ie^{\text{sech}^{-1}(cx)}\right)d\text{sech}^{-}\right.\right.}{c^3}$$

$$\downarrow 2715$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^2+\frac{2}{3}b\left(\frac{1}{2}\left(-ib\int e^{-\text{sech}^{-1}(cx)}\log\left(1-ie^{\text{sech}^{-1}(cx)}\right)de^{\text{sech}^{-1}(cx)}+ib\int e^{-\text{sech}^{-1}(cx)}\log\left(1\right.\right.\right.}{}$$

$$\downarrow 2838$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^2+\frac{2}{3}b\left(\frac{1}{2}\left(2\arctan\left(e^{\text{sech}^{-1}(cx)}\right)\left(a+b\text{sech}^{-1}(cx)\right)-ib\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)+i\right.\right.}{c^3}$$

input
```
Int[x^2*(a + b*ArcSech[c*x])^2,x]
```

output
```
-((-1/3*(c^3*x^3*(a + b*ArcSech[c*x])^2) + (2*b*((b*c*x)/2 + (c*x*Sqrt[(1
- c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/2 + (2*(a + b*ArcSech[c*
x])*ArcTan[E^ArcSech[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSech[c*x]] + I*b*Pol
yLog[2, I*E^ArcSech[c*x]])/2))/3)/c^3)
```

3.34. $\int x^2\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx$

3.34.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.34.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.34

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\frac{\left(c^2x^2\operatorname{arcsech}(cx)^2-\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx-1\right)cx}{3}+\frac{i\operatorname{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\right)}{3}-\frac{i\,\mathrm{ar}\cdots}{}\right.}{}$
derivativedivides	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{\left(c^2x^2\operatorname{arcsech}(cx)^2-\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx-1\right)cx}{3}+\frac{i\operatorname{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\right)}{3}-\frac{i\,\mathrm{arc}\cdots}{}\right.}{}$
default	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{\left(c^2x^2\operatorname{arcsech}(cx)^2-\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx-1\right)cx}{3}+\frac{i\operatorname{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\right)}{3}-\frac{i\,\mathrm{arc}\cdots}{}\right.}{}$

input
```
int(x^2*(a+b*arcsech(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+b^2/c^3*(1/3*(c^2*x^2*arcsech(c*x)^2-arcsech(c*x)*(-(c*x-1)/c/
x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-1)*c*x+1/3*I*arcsech(c*x)*ln(1+I*(1/c/x+(
-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-1/3*I*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2)))+1/3*I*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2)))-1/3*I*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))))+2*a
*b/c^3*(1/3*c^3*x^3*arcsech(c*x)+1/6*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x
)^(1/2)*(-c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

3.34.5 Fricas [F]

$$\int x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)^2\,dx=\int\left(b\operatorname{arsech}(cx)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arcsech(c*x)^2 + 2*a*b*x^2*arcsech(c*x) + a^2*x^2, x)
```

3.34.6 Sympy [F]

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2 \,dx = \int x^2 (a + b \operatorname{asech}{(cx)})^2 \,dx$$

input
```
integrate(x**2*(a+b*asech(c*x))**2,x)
```

output
```
Integral(x**2*(a + b*asech(c*x))**2, x)
```

3.34.7 Maxima [F]

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2 \,dx = \int (b\,\text{arsech}\,(cx)+a)^2 x^2 \,dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/3*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^
2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*a*b + b^2*integ
rate(x^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2, x)
```

3.34.8 Giac [F]

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2 \,dx = \int (b\,\text{arsech}\,(cx)+a)^2 x^2 \,dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^2*x^2, x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2 \, dx = \int x^2 \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2 \,dx$$

input
```
int(x^2*(a + b*acosh(1/(c*x)))^2,x)
```

output
```
int(x^2*(a + b*acosh(1/(c*x)))^2, x)
```

3.35 $\int x\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx$

3.35.1 Optimal result

Integrand size = 12, antiderivative size = 65

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{c^2}+\frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)^2-\frac{b^2\log(x)}{c^2}$$

output
```
1/2*x^2*(a+b*arcsech(c*x))^2-b^2*ln(x)/c^2-b*(c*x+1)*(a+b*arcsech(c*x))*((
-c*x+1)/(c*x+1))^(1/2)/c^2
```

3.35.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.72

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\frac{a\left(ac^2x^2-2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)-2b\left(-ac^2x^2+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)\text{sech}^{-1}(cx)+b^2c^2x^2\text{sech}^{-1}(cx)^2-2b^2\log}{2c^2}$$

input
```
Integrate[x*(a + b*ArcSech[c*x])^2,x]
```

output
```
(a*(a*c^2*x^2 - 2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)) - 2*b*(-(a*c^2*x^
2) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*x] + b^2*c^2*x^2*Arc
Sech[c*x]^2 - 2*b^2*Log[c*x])/(2*c^2)
```

3.35.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6839, 5974, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^2dx$$

$$\Big\downarrow \text{ 6839}$$

$$-\frac{\int c^2x^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)}{c^2}$$

$$\Big\downarrow \text{ 5974}$$

$$-\frac{b\int c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}{c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^2+b\int\left(a+b\text{sech}^{-1}(cx)\right)\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^2d\text{sech}^{-1}(cx)}{c^2}$$

$$\Big\downarrow \text{ 4672}$$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^2+b\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)-ib\int -i\sqrt{\frac{1-cx}{cx+1}}(cx+1)d\text{sech}^{-1}(cx)\right)}{c^2}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{b\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)-b\int\sqrt{\frac{1-cx}{cx+1}}(cx+1)d\text{sech}^{-1}(cx)\right)-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}{c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^2+b\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)-b\int -i\tan\left(i\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)\right)}{c^2}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^2+b\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)+ib\int\tan\left(i\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)\right)}{c^2}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{b\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)-b\log\left(\frac{1}{cx}\right)\right)-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}{c^2}$$

input
```
Int[x*(a + b*ArcSech[c*x])^2,x]
```

output
```
-((-1/2*(c^2*x^2*(a + b*ArcSech[c*x])^2) + b*(Sqrt[(1 - c*x)/(1 + c*x)]*(1
 + c*x)*(a + b*ArcSech[c*x]) - b*Log[1/(c*x)]))/c^2)
```

3.35.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.35.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(61) = 122$.

Time = 0.64 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.54

method	result
parts	$\frac{a^2x^2}{2} + \frac{b^2\left(-2\,\mathrm{arcsech}(cx)+\frac{\mathrm{arcsech}(cx)\left(c^2x^2\,\mathrm{arcsech}(cx)-2\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+2\right)}{2}+\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)^2\right.\right.}{c^2}$
derivativedivides	$\frac{\frac{a^2c^2x^2}{2}+b^2\left(-2\,\mathrm{arcsech}(cx)+\frac{\mathrm{arcsech}(cx)\left(c^2x^2\,\mathrm{arcsech}(cx)-2\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+2\right)}{2}+\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)^2\right.\right.}{c^2}$
default	$\frac{\frac{a^2c^2x^2}{2}+b^2\left(-2\,\mathrm{arcsech}(cx)+\frac{\mathrm{arcsech}(cx)\left(c^2x^2\,\mathrm{arcsech}(cx)-2\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+2\right)}{2}+\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)^2\right.\right.}{c^2}$

input
```
int(x*(a+b*arcsech(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+b^2/c^2*(-2*arcsech(c*x)+1/2*arcsech(c*x)*(c^2*x^2*arcsech(c*x
)-2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+2)+ln(1+(1/c/x+(-1+1/c/x)
^(1/2)*(1+1/c/x)^(1/2))^2))+2*a*b/c^2*(1/2*c^2*x^2*arcsech(c*x)-1/2*(-(c*x
-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2))
```

3.35.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(61) = 122$.

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.15

$$\int x\left(a+b\mathrm{sech}^{-1}(cx)\right)^2\,dx$$
$$= \frac{b^2c^2x^2\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2 + a^2c^2x^2 - 2\,abc^2\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 2\,abcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 2\,b^2\log(x) + 2\,\left(\right.}{2\,c^2}$$

input
```
integrate(x*(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*c^2*x^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 + a
^2*c^2*x^2 - 2*a*b*c^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2
*a*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*b^2*log(x) + 2*(a*b*c^2*x^2 -
b^2*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - a*b*c^2)*log((c*x*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)) + 1)/(c*x)))/c^2
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

$$\int x\left(a+b\operatorname{sech}^{-1}(cx)\right)^2\,dx = \begin{cases} \frac{a^2x^2}{2}+abx^2\operatorname{asech}(cx)-\frac{ab\sqrt{-c^2x^2+1}}{c^2}+\frac{b^2x^2\operatorname{asech}^2(cx)}{2}-\frac{b^2\sqrt{-c^2x^2+1}\operatorname{asech}(cx)}{c^2}-\frac{b^2\log(x)}{c^2} & \text{for } c\neq 0 \\ \frac{x^2(a+\infty b)^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a+b*asech(c*x))**2,x)
```

output
```
Piecewise((a**2*x**2/2 + a*b*x**2*asech(c*x) - a*b*sqrt(-c**2*x**2 + 1)/c*
*2 + b**2*x**2*asech(c*x)**2/2 - b**2*sqrt(-c**2*x**2 + 1)*asech(c*x)/c**2
 - b**2*log(x)/c**2, Ne(c, 0)), (x**2*(a + oo*b)**2/2, True))
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int x\left(a+b\operatorname{sech}^{-1}(cx)\right)^2\,dx = \frac{1}{2}b^2x^2\operatorname{arsech}(cx)^2+\frac{1}{2}a^2x^2 + \left(x^2\operatorname{arsech}(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)ab - \left(\frac{x\sqrt{\frac{1}{c^2x^2}-1}\operatorname{arsech}(cx)}{c}+\frac{\log(x)}{c^2}\right)b^2$$

input
```
integrate(x*(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*arcsech(c*x)^2 + 1/2*a^2*x^2 + (x^2*arcsech(c*x) - x*sqrt(1/(c
^2*x^2) - 1)/c)*a*b - (x*sqrt(1/(c^2*x^2) - 1)*arcsech(c*x)/c + log(x)/c^2
)*b^2
```

3.35.8 Giac [F]

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int (b\,\text{arsech}\,(cx)+a)^2x\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^2*x, x)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int x\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

input
```
int(x*(a + b*acosh(1/(c*x)))^2,x)
```

output
```
int(x*(a + b*acosh(1/(c*x)))^2, x)
```

3.36 $\int (a+b\text{sech}^{-1}(cx))^2\,dx$

3.36.1 Optimal result

Integrand size = 10, antiderivative size = 78

$$\int (a+b\text{sech}^{-1}(cx))^2\,dx = x(a+b\text{sech}^{-1}(cx))^2-\frac{4b(a+b\text{sech}^{-1}(cx))\arctan\left(e^{\text{sech}^{-1}(cx)}\right)}{c}+\frac{2ib^2\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)}{c}-\frac{2ib^2\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(cx)}\right)}{c}$$

output
```
x*(a+b*arcsech(c*x))^2-4*b*(a+b*arcsech(c*x))*arctan(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))/c+2*I*b^2*polylog(2,-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2)))/c-2*I*b^2*polylog(2,I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/
c
```

3.36.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.62

$$\int (a+b\text{sech}^{-1}(cx))^2\,dx = a^2x+\frac{2ab\left(cx\text{sech}^{-1}(cx)-2\arctan\left(\tanh\left(\frac{1}{2}\text{sech}^{-1}(cx)\right)\right)\right)}{c}+\frac{ib^2\left(\text{sech}^{-1}(cx)\left(-icx\text{sech}^{-1}(cx)+2\log\left(1-ie^{-\text{sech}^{-1}(cx)}\right)-2\log\left(1+ie^{-\text{sech}^{-1}(cx)}\right)\right)+2\,\text{PolyLog}\left(2\right.\right.}{c}$$

input
```
Integrate[(a + b*ArcSech[c*x])^2,x]
```

output
```
a^2*x + (2*a*b*(c*x*ArcSech[c*x] - 2*ArcTan[Tanh[ArcSech[c*x]/2]]))/c + (I
*b^2*(ArcSech[c*x]*((-I)*c*x*ArcSech[c*x] + 2*Log[1 - I/E^ArcSech[c*x]] -
2*Log[1 + I/E^ArcSech[c*x]]) + 2*PolyLog[2, (-I)/E^ArcSech[c*x]] - 2*PolyL
og[2, I/E^ArcSech[c*x]]))/c
```

3.36.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6833, 5974, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^2 dx$$

$$\Big\downarrow \text{6833}$$

$$-\frac{\int cx\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2 d\text{sech}^{-1}(cx)}{c}$$

$$\Big\downarrow \text{5974}$$

$$-\frac{2b\int cx\left(a+b\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)-cx\left(a+b\text{sech}^{-1}(cx)\right)^2}{c}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-cx\left(a+b\text{sech}^{-1}(cx)\right)^2+2b\int \left(a+b\text{sech}^{-1}(cx)\right)\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(cx)}{c}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{-cx\left(a+b\text{sech}^{-1}(cx)\right)^2+2b\left(-ib\int \log\left(1-ie^{\text{sech}^{-1}(cx)}\right)d\text{sech}^{-1}(cx)+ib\int \log\left(1+ie^{\text{sech}^{-1}(cx)}\right)d\text{sech}^{-1}(cx)+\right.}{c}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{-cx\left(a+b\text{sech}^{-1}(cx)\right)^2+2b\left(-ib\int e^{-\text{sech}^{-1}(cx)}\log\left(1-ie^{\text{sech}^{-1}(cx)}\right)de^{\text{sech}^{-1}(cx)}+ib\int e^{-\text{sech}^{-1}(cx)}\log\left(1+ie^{\text{sec}}\right.\right.}{c}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{-cx(a+b\operatorname{sech}^{-1}(cx))^2+2b\left(2\arctan\left(e^{\operatorname{sech}^{-1}(cx)}\right)(a+b\operatorname{sech}^{-1}(cx))-ib\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(cx)}\right)+ib\operatorname{PolyI}}{c}$$

input
```
Int[(a + b*ArcSech[c*x])^2,x]
```

output
```
-((-(c*x*(a + b*ArcSech[c*x])^2) + 2*b*(2*(a + b*ArcSech[c*x])*ArcTan[E^Ar
cSech[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSech[c*x]] + I*b*PolyLog[2, I*E^Arc
Sech[c*x]]))/c)
```

3.36.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6833
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-c^(-1)   S
ubst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
 b, c, n}, x] && IGtQ[n, 0]
```

3.36.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.97

method	result
derivativedivides	$\frac{a^2cx+b^2\left(\operatorname{arcsech}(cx)^2cx+2i\operatorname{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\right)-2i\operatorname{arcsech}(cx)\ln\left(1-i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1\right.\right.\right.}{}$
default	$\frac{a^2cx+b^2\left(\operatorname{arcsech}(cx)^2cx+2i\operatorname{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\right)-2i\operatorname{arcsech}(cx)\ln\left(1-i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1\right.\right.\right.}{}$
parts	$a^2x+\frac{b^2\left(\operatorname{arcsech}(cx)^2cx+2i\operatorname{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\right)-2i\operatorname{arcsech}(cx)\ln\left(1-i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\right.\right.\right.}{c}$

input
```
int((a+b*arcsech(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2*c*x+b^2*(arcsech(c*x)^2*c*x+2*I*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2)))-2*I*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2)))+2*I*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)
))-2*I*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))))+2*a*b*(c*x*arc
sech(c*x)-arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))))
```

3.36.5 Fricas [F]

$$\int \left(a+b\operatorname{sech}^{-1}(cx)\right)^2\,dx=\int \left(b\operatorname{arsech}\left(cx\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2, x)
```

3.36.6 Sympy [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int (a+b\operatorname{asech}(cx))^2\,dx$$

input
```
integrate((a+b*asech(c*x))**2,x)
```

output
```
Integral((a + b*asech(c*x))**2, x)
```

3.36.7 Maxima [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int (b\operatorname{arsech}(cx)+a)^2\,dx$$

input
```
integrate((a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

output
```
(x*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^2 - integrate(-(c^2*x^2*log(c)^2
+ (c^2*x^2 - 1)*log(x)^2 + (c^2*x^2*log(c)^2 + (c^2*x^2 - 1)*log(x)^2 - lo
g(c)^2 + 2*(c^2*x^2*log(c) - log(c))*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1)
- 2*(c^2*x^2*log(c) + (c^2*x^2*(log(c) + 1) + (c^2*x^2 - 1)*log(x) - log(c
))*sqrt(c*x + 1)*sqrt(-c*x + 1) + (c^2*x^2 - 1)*log(x) - log(c))*log(sqrt(
c*x + 1)*sqrt(-c*x + 1) + 1) - log(c)^2 + 2*(c^2*x^2*log(c) - log(c))*log(
x))/(c^2*x^2 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(-c*x + 1) - 1), x))*b^2 +
a^2*x + 2*(c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*a*b/c
```

3.36.8 Giac [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int (b\operatorname{arsech}(cx)+a)^2\,dx$$

input
```
integrate((a+b*arcsech(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^2, x)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int (a + b\text{sech}^{-1}(cx))^2 \, dx = \int \left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

input
```
int((a + b*acosh(1/(c*x)))^2,x)
```

output
```
int((a + b*acosh(1/(c*x)))^2, x)
```

3.37 $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x} dx$

3.37.1 Optimal result

Integrand size = 14, antiderivative size = 83

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x} dx = \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3b} - \left(a+b\text{sech}^{-1}(cx)\right)^2 \log\left(1+e^{2\text{sech}^{-1}(cx)}\right) - b\left(a+b\text{sech}^{-1}(cx)\right) \text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right) + \frac{1}{2}b^2 \text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(cx)}\right)$$

output
```
1/3*(a+b*arcsech(c*x))^3/b-(a+b*arcsech(c*x))^2*ln(1+(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))^2)-b*(a+b*arcsech(c*x))*polylog(2,-(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))^2)+1/2*b^2*polylog(3,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))^2)
```

3.37.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.40

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x}\,dx = a^2\log(cx) + ab\left(-\operatorname{sech}^{-1}(cx)\left(\operatorname{sech}^{-1}(cx)+2\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)\right) + \operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(cx)}\right)\right) + b^2\left(-\frac{1}{3}\operatorname{sech}^{-1}(cx)^3 - \operatorname{sech}^{-1}(cx)^2\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right) + \operatorname{sech}^{-1}(cx)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(cx)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(3,-e^{-2\operatorname{sech}^{-1}(cx)}\right)\right)$$

input
```
Integrate[(a + b*ArcSech[c*x])^2/x,x]
```

output
```
a^2*Log[c*x] + a*b*(-(ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech
[c*x])])) + PolyLog[2, -E^(-2*ArcSech[c*x])]) + b^2*(-1/3*ArcSech[c*x]^3 -
 ArcSech[c*x]^2*Log[1 + E^(-2*ArcSech[c*x])] + ArcSech[c*x]*PolyLog[2, -E^
(-2*ArcSech[c*x])] + PolyLog[3, -E^(-2*ArcSech[c*x])]/2)
```

3.37.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6839, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x}\,dx$$

$$\Big\downarrow \text{ 6839}$$

$$-\int \sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2 d\operatorname{sech}^{-1}(cx)$$

$$\Big\downarrow \; 3042$$

$$-\int -i\left(a+b\,\text{sech}^{-1}(cx)\right)^2 \tan\left(i\,\text{sech}^{-1}(cx)\right) d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \; 26$$

$$i\int \left(a+b\,\text{sech}^{-1}(cx)\right)^2 \tan\left(i\,\text{sech}^{-1}(cx)\right) d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \; 4201$$

$$i\left(2i\int \frac{e^{2\text{sech}^{-1}(cx)}\left(a+b\,\text{sech}^{-1}(cx)\right)^2}{1+e^{2\text{sech}^{-1}(cx)}}d\text{sech}^{-1}(cx)-\frac{i\left(a+b\,\text{sech}^{-1}(cx)\right)^3}{3b}\right)$$

$$\Big\downarrow \; 2620$$

$$i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+b\,\text{sech}^{-1}(cx)\right)^2-b\int \left(a+b\,\text{sech}^{-1}(cx)\right)\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)d\text{sech}^{-1}(cx)\right)-\right.$$

$$\Big\downarrow \; 3011$$

$$i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+b\,\text{sech}^{-1}(cx)\right)^2-b\left(\frac{1}{2}b\int \text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right)d\text{sech}^{-1}(cx)-\frac{1}{2}\text{PolyLog}\right.\right.\right.$$

$$\Big\downarrow \; 2720$$

$$i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+b\,\text{sech}^{-1}(cx)\right)^2-b\left(\frac{1}{4}b\int e^{-2\text{sech}^{-1}(cx)}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right)de^{2\text{sech}^{-1}(cx)}-\right.\right.\right.$$

$$\Big\downarrow \; 7143$$

$$i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+b\,\text{sech}^{-1}(cx)\right)^2-b\left(\frac{1}{4}b\,\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcSech[c*x])^2/x,x]
```

output
```
I*(((-1/3*I)*(a + b*ArcSech[c*x])^3)/b + (2*I)*(((a + b*ArcSech[c*x])^2*Lo
g[1 + E^(2*ArcSech[c*x])])/2 - b*(-1/2*((a + b*ArcSech[c*x])*PolyLog[2, -E
^(2*ArcSech[c*x])]) + (b*PolyLog[3, -E^(2*ArcSech[c*x])])/4)))
```

3.37. $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x}\,dx$

3.37.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.37. $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.37.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.90

method	result
parts	$a^2 \ln(x) + b^2 \left(\frac{\operatorname{arcsech}(cx)^3}{3} - \operatorname{arcsech}(cx)^2 \ln\left(1 + \left(\frac{1}{cx} + \sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2\right) - \text{arcs}\right.$
derivativedivides	$a^2 \ln(cx) + b^2 \left(\frac{\operatorname{arcsech}(cx)^3}{3} - \operatorname{arcsech}(cx)^2 \ln\left(1 + \left(\frac{1}{cx} + \sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2\right) - \text{arc}\right.$
default	$a^2 \ln(cx) + b^2 \left(\frac{\operatorname{arcsech}(cx)^3}{3} - \operatorname{arcsech}(cx)^2 \ln\left(1 + \left(\frac{1}{cx} + \sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2\right) - \text{arc}\right.$

input
```
int((a+b*arcsech(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(x)+b^2*(1/3*arcsech(c*x)^3-arcsech(c*x)^2*ln(1+(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))^2)-arcsech(c*x)*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2))^2)+1/2*polylog(3,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^
2))+2*a*b*(1/2*arcsech(c*x)^2-arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2))^2)-1/2*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^
2))
```

3.37.5 Fricas [F]

$$\int \frac{(a + b\operatorname{sech}^{-1}(cx))^2}{x}\,dx = \int \frac{(b\,\operatorname{arsech}(cx) + a)^2}{x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2)/x, x)
```

3.37.6 Sympy [F]

$$\int \frac{\left(a + b\operatorname{sech}^{-1}(cx)\right)^2}{x}\,dx = \int \frac{(a + b\operatorname{asech}(cx))^2}{x}\,dx$$

input
```
integrate((a+b*asech(c*x))**2/x,x)
```

output
```
Integral((a + b*asech(c*x))**2/x, x)
```

3.37.7 Maxima [F]

$$\int \frac{\left(a + b\operatorname{sech}^{-1}(cx)\right)^2}{x}\,dx = \int \frac{(b\operatorname{arsech}(cx) + a)^2}{x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + integrate(b^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*
x))^2/x + 2*a*b*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x)
```

3.37.8 Giac [F]

$$\int \frac{\left(a + b\operatorname{sech}^{-1}(cx)\right)^2}{x}\,dx = \int \frac{(b\operatorname{arsech}(cx) + a)^2}{x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^2/x, x)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2}{x}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))^2/x,x)
```

output
```
int((a + b*acosh(1/(c*x)))^2/x, x)
```

3.38 $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx$

3.38.1 Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx=-\frac{2b^2}{x}+\frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{x}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x}$$

output
```
-2*b^2/x-(a+b*arcsech(c*x))^2/x+2*b*(c*x+1)*(a+b*arcsech(c*x))*((-c*x+1)/(
c*x+1))^(1/2)/x
```

3.38.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{a^2+2b^2-2ab\sqrt{\frac{1-cx}{1+cx}}(1+cx)-2b\left(-a+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)\text{sech}^{-1}(cx)+b^2\text{sech}^{-1}(cx)^2}{x}$$

input
```
Integrate[(a + b*ArcSech[c*x])^2/x^2,x]
```

output
```
-((a^2 + 2*b^2 - 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) - 2*b*(-a + b*S
qrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*x] + b^2*ArcSech[c*x]^2)/x)
```

3.38.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.30, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6839, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^2}dx$$

$$\Big\downarrow \text{6839}$$

$$-c\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$-c\int -i\left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{26}$$

$$ic\int \left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{3777}$$

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}-2ib\int \frac{a+b\text{sech}^{-1}(cx)}{cx}d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}-2ib\int \left(a+b\text{sech}^{-1}(cx)\right)\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3777}$$

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}-2ib\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{cx}-ib\int -\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{cx}d\text{sech}^{-1}(cx)\right)\right)$$

$$\Big\downarrow \text{26}$$

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}-2ib\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{cx}-b\int\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{cx}d\text{sech}^{-1}(cx)\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}-2ib\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{cx}-b\int-i\sin\left(i\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)\right)\right)$$

$$\Big\downarrow \text{26}$$

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}-2ib\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{cx}+ib\int\sin\left(i\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)\right)\right)$$

$$\Big\downarrow \text{3118}$$

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}-2ib\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{cx}-\frac{b}{cx}\right)\right)$$

input
```
Int[(a + b*ArcSech[c*x])^2/x^2,x]
```

output
```
I*c*((I*(a + b*ArcSech[c*x])^2)/(c*x) - (2*I)*b*(-(b/(c*x)) + (Sqrt[(1 - c
*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/(c*x)))
```

3.38.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.38.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(59) = 118$.

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.98

method	result
parts	$-\frac{a^2}{x}+b^2c\left(-\frac{\operatorname{arcsech}(cx)^2}{cx}+2\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}-\frac{2}{cx}\right)+2abc\left(-\frac{\operatorname{arcsech}(cx)}{cx}+\right.$
derivativedivides	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{cx}+2\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}-\frac{2}{cx}\right)+2ab\left(-\frac{\operatorname{arcsech}(cx)}{cx}+\right.\right.$
default	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{cx}+2\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}-\frac{2}{cx}\right)+2ab\left(-\frac{\operatorname{arcsech}(cx)}{cx}+\right.\right.$

input
```
int((a+b*arcsech(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+b^2*c*(-1/c/x*arcsech(c*x)^2+2*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2)*((
c*x+1)/c/x)^(1/2)-2/c/x)+2*a*b*c*(-1/c/x*arcsech(c*x)+(-(c*x-1)/c/x)^(1/2)
*((c*x+1)/c/x)^(1/2))
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(59) = 118$.

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.34

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx$$
$$=\frac{2\,abcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-b^2\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2-a^2-2\,b^2+2\left(b^2cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-ab\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{x}$$

input
```
integrate((a+b*arcsech(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
(2*a*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - b^2*log((c*x*sqrt(-(c^2*x^2 -
1)/(c^2*x^2)) + 1)/(c*x))^2 - a^2 - 2*b^2 + 2*(b^2*c*x*sqrt(-(c^2*x^2 - 1)
/(c^2*x^2)) - a*b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/x
```

3.38.6 Sympy [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx=\int \frac{(a+b\,\text{asech}\,(cx))^2}{x^2}\,dx$$

input
```
integrate((a+b*asech(c*x))**2/x**2,x)
```

output
```
Integral((a + b*asech(c*x))**2/x**2, x)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx=2\left(c\sqrt{\frac{1}{c^2x^2}-1}-\frac{\text{arsech}\,(cx)}{x}\right)ab+2\left(c\sqrt{\frac{1}{c^2x^2}-1}\,\text{arsech}\,(cx)-\frac{1}{x}\right)b^2-\frac{b^2\,\text{arsech}\,(cx)^2}{x}-\frac{a^2}{x}$$

input
```
integrate((a+b*arcsech(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
2*(c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*a*b + 2*(c*sqrt(1/(c^2*x^2) -
 1)*arcsech(c*x) - 1/x)*b^2 - b^2*arcsech(c*x)^2/x - a^2/x
```

3.38.8 Giac [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx = \int \frac{\left(b\,\text{arsech}\,(cx)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^2/x^2, x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx = \int \frac{\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2}{x^2}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))^2/x^2,x)
```

output
```
int((a + b*acosh(1/(c*x)))^2/x^2, x)
```

3.39 $\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x^3}\,dx$

3.39.1 Optimal result

Integrand size = 14, antiderivative size = 118

$$\begin{aligned}\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x^3}\,dx = & -\frac{b^2(1-cx)(1+cx)}{4x^2}-\frac{1}{2}abc^2\mathrm{sech}^{-1}(cx) \\ & -\frac{1}{4}b^2c^2\mathrm{sech}^{-1}(cx)^2+\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{2x^2} \\ & -\frac{(1-cx)(1+cx)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{2x^2}\end{aligned}$$

output
```
-1/4*b^2*(-c*x+1)*(c*x+1)/x^2-1/2*a*b*c^2*arcsech(c*x)-1/4*b^2*c^2*arcsech
(c*x)^2-1/2*(-c*x+1)*(c*x+1)*(a+b*arcsech(c*x))^2/x^2+1/2*b*(c*x+1)*(a+b*a
rcsech(c*x))*((-c*x+1)/(c*x+1))^(1/2)/x^2
```

3.39.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.55

$$\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x^3}\,dx$$
$$= \frac{-2a^2-b^2+2ab\sqrt{\frac{1-cx}{1+cx}}+2abcx\sqrt{\frac{1-cx}{1+cx}}+2b\left(-2a+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)\mathrm{sech}^{-1}(cx)+b^2(-2+c^2x^2)\,\mathrm{sech}^{-1}}{4x^2}$$

input
```
Integrate[(a + b*ArcSech[c*x])^2/x^3,x]
```

output
```
(-2*a^2 - b^2 + 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)] + 2*a*b*c*x*Sqrt[(1 - c*x)
/(1 + c*x)] + 2*b*(-2*a + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c
*x] + b^2*(-2 + c^2*x^2)*ArcSech[c*x]^2 - 2*a*b*c^2*x^2*Log[x] + 2*a*b*c^2
*x^2*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/(
4*x^2)
```

3.39.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6839, 5969, 3042, 25, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^3}dx$$

$$\Big\downarrow \text{ 6839}$$

$$-c^2\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{c^2x^2}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{ 5969}$$

$$-c^2\left(\frac{(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2x^2}-b\int \frac{(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{c^2x^2}d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-c^2\left(\frac{(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2x^2}-b\int -\left(\left(a+b\text{sech}^{-1}(cx)\right)\sin\left(i\text{sech}^{-1}(cx)\right)^2\right)d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 25}$$

$$-c^2\left(\frac{(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2x^2}+b\int \left(a+b\text{sech}^{-1}(cx)\right)\sin\left(i\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 3791}$$

$$-c^2\left(b\left(\frac{1}{2}\int \left(a+b\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)-\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{2c^2x^2}+\frac{b(1-cx)(cx+1)}{4c^2x^2}\right)+\frac{(1-c\right.$$

$$\Big\downarrow \text{17}$$

$$-c^2\left(\frac{(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2x^2}+b\left(-\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{2c^2x^2}+\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{4b}+\frac{b(1\right.\right.$$

input
```
Int[(a + b*ArcSech[c*x])^2/x^3,x]
```

output
```
-(c^2*(((1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(2*c^2*x^2) + b*((b*(1
 - c*x)*(1 + c*x))/(4*c^2*x^2) - (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a +
 b*ArcSech[c*x]))/(2*c^2*x^2) + (a + b*ArcSech[c*x])^2/(4*b))))
```

3.39.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.39.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.62

method	result
parts	$-\frac{a^2}{2x^2}+b^2c^2\left(-\frac{\operatorname{arcsech}(cx)^2}{2c^2x^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{2cx}+\frac{\operatorname{arcsech}(cx)^2}{4}-\frac{1}{4c^2x^2}\right)+2ab\,c^2\Bigg(-\frac{\text{ar}}{}$
derivativedivides	$c^2\Bigg(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{2c^2x^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{2cx}+\frac{\operatorname{arcsech}(cx)^2}{4}-\frac{1}{4c^2x^2}\right)+2ab\Bigg(-$
default	$c^2\Bigg(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{2c^2x^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{2cx}+\frac{\operatorname{arcsech}(cx)^2}{4}-\frac{1}{4c^2x^2}\right)+2ab\Bigg(-$

input
```
int((a+b*arcsech(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2+b^2*c^2*(-1/2/c^2/x^2*arcsech(c*x)^2+1/2*(-(c*x-1)/c/x)^(1/2)
*((c*x+1)/c/x)^(1/2)/c/x*arcsech(c*x)+1/4*arcsech(c*x)^2-1/4/c^2/x^2)+2*a*
b*c^2*(-1/2/c^2/x^2*arcsech(c*x)+1/4*(-(c*x-1)/c/x)^(1/2)/c/x*((c*x+1)/c/x
)^(1/2)*(arctanh(1/(-c^2*x^2+1)^(1/2))*c^2*x^2+(-c^2*x^2+1)^(1/2))/(-c^2*x
^2+1)^(1/2))
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.40

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^3}\,dx$$
$$=\frac{2\,abcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+(b^2c^2x^2-2\,b^2)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2-2\,a^2-b^2+2\left(abc^2x^2+b^2cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-2\,ab\right)}{4\,x^2}$$

input
```
integrate((a+b*arcsech(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
1/4*(2*a*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + (b^2*c^2*x^2 - 2*b^2)*log(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 - 2*a^2 - b^2 + 2*(a*b*c
^2*x^2 + b^2*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*a*b)*log((c*x*sqrt(-(c
^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/x^2
```

3.39.6 Sympy [F]

$$\int \frac{(a + b\operatorname{sech}^{-1}(cx))^2}{x^3}\,dx = \int \frac{(a + b\operatorname{asech}(cx))^2}{x^3}\,dx$$

input
```
integrate((a+b*asech(c*x))**2/x**3,x)
```

output
```
Integral((a + b*asech(c*x))**2/x**3, x)
```

3.39.7 Maxima [F]

$$\int \frac{(a + b\operatorname{sech}^{-1}(cx))^2}{x^3}\,dx = \int \frac{(b\operatorname{arsech}(cx) + a)^2}{x^3}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-1/4*a*b*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) -
 c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) - 1
) - 1))/c + 4*arcsech(c*x)/x^2) + b^2*integrate(log(sqrt(1/(c*x) + 1)*sqrt
(1/(c*x) - 1) + 1/(c*x))^2/x^3, x) - 1/2*a^2/x^2
```

3.39.8 Giac [F]

$$\int \frac{(a + b\operatorname{sech}^{-1}(cx))^2}{x^3}\,dx = \int \frac{(b\operatorname{arsech}(cx) + a)^2}{x^3}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^2/x^3, x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2}{x^3}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))^2/x^3,x)
```

output
```
int((a + b*acosh(1/(c*x)))^2/x^3, x)
```

3.40 $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^4} \, dx$

3.40.1 Optimal result

Integrand size = 14, antiderivative size = 122

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^4} \, dx = -\frac{2b^2}{27x^3}-\frac{4b^2c^2}{9x}+\frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{9x^3}+\frac{4bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{9x}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3x^3}$$

output
```
-2/27*b^2/x^3-4/9*b^2*c^2/x-1/3*(a+b*arcsech(c*x))^2/x^3+2/9*b*(c*x+1)*(a+
b*arcsech(c*x))*((-c*x+1)/(c*x+1))^(1/2)/x^3+4/9*b*c^2*(c*x+1)*(a+b*arcsec
h(c*x))*((-c*x+1)/(c*x+1))^(1/2)/x
```

3.40.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^4} \, dx = \frac{-9a^2-2b^2(1+6c^2x^2)+6ab\sqrt{\frac{1-cx}{1+cx}}(1+cx+2c^2x^2+2c^3x^3)+6b\left(-3a+b\sqrt{\frac{1-cx}{1+cx}}(1+cx+2c^2x^2+2c^3x\right.}{27x^3}$$

input
```
Integrate[(a + b*ArcSech[c*x])^2/x^4,x]
```

3.40. $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^4} \, dx$

output
```
(-9*a^2 - 2*b^2*(1 + 6*c^2*x^2) + 6*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x
 + 2*c^2*x^2 + 2*c^3*x^3) + 6*b*(-3*a + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c
*x + 2*c^2*x^2 + 2*c^3*x^3))*ArcSech[c*x] - 9*b^2*ArcSech[c*x]^2)/(27*x^3)
```

3.40.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6839, 5970, 3042, 3791, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^4}dx$$

$$\Big\downarrow \text{ 6839}$$

$$-c^3\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{c^3x^3}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{ 5970}$$

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\int \frac{a+b\text{sech}^{-1}(cx)}{c^3x^3}d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\int \left(a+b\text{sech}^{-1}(cx)\right)\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^3d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 3791}$$

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\left(\frac{2}{3}\int \frac{a+b\text{sech}^{-1}(cx)}{cx}d\text{sech}^{-1}(cx)+\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^3x^3}-\frac{b}{9c^3x^3}\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\left(\frac{2}{3}\int \left(a+b\text{sech}^{-1}(cx)\right)\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(cx)+\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a\right.}{3c^3x}\right.\right.$$

$$\Big\downarrow \text{ 3777}$$

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\left(\frac{2}{3}\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{cx}-ib\int -\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{cx}d\text{sech}^{-1}(cx)\right)+\right.\right.$$

$\big\downarrow$ 26

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\left(\frac{2}{3}\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{cx}-b\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{cx}d\text{sech}^{-1}(cx)\right)+\frac{\sqrt{}}{}\right.\right.$$

$\big\downarrow$ 3042

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\left(\frac{2}{3}\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{cx}-b\int -i\sin\left(i\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)\right)\right.\right.$$

$\big\downarrow$ 26

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\left(\frac{2}{3}\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{cx}+ib\int \sin\left(i\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)\right)+\right.\right.$$

$\big\downarrow$ 3118

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^3x^3}+\frac{2}{3}\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{cx}-\frac{b}{cx}\right.\right.\right.$$

input
```
Int[(a + b*ArcSech[c*x])^2/x^4,x]
```

output
```
-(c^3*((a + b*ArcSech[c*x])^2/(3*c^3*x^3) - (2*b*(-1/9*b/(c^3*x^3) + (Sqrt
[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/(3*c^3*x^3) + (2*(-(
b/(c*x)) + (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/(c*x
)))/3))/3))
```

3.40. $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^4}\,dx$

3.40.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.40. $\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^4}\,dx$

3.40.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.57

method	result
parts	$-\frac{a^2}{3x^3}+b^2c^3\left(-\frac{\operatorname{arcsech}(cx)^2}{3c^3x^3}+\frac{4\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{9}+\frac{2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{9c^2x^2}-\frac{4}{9cx}-\right.$
derivativedivides	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{3c^3x^3}+\frac{4\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{9}+\frac{2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{9c^2x^2}-\frac{4}{9cx}\right.\right.$
default	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{3c^3x^3}+\frac{4\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{9}+\frac{2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{9c^2x^2}-\frac{4}{9cx}\right.\right.$

input
```
int((a+b*arcsech(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3+b^2*c^3*(-1/3/c^3/x^3*arcsech(c*x)^2+4/9*arcsech(c*x)*(-(c*x-
1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+2/9/c^2/x^2*(-(c*x-1)/c/x)^(1/2)*((c*x+1
)/c/x)^(1/2)*arcsech(c*x)-4/9/c/x-2/27/c^3/x^3)+2*a*b*c^3*(-1/3/c^3/x^3*ar
csech(c*x)+1/9*(-(c*x-1)/c/x)^(1/2)/c^2/x^2*((c*x+1)/c/x)^(1/2)*(2*c^2*x^2
+1))
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.48

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^4}\,dx =$$

$$-\frac{12\,b^2c^2x^2+9\,b^2\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2+9\,a^2+2\,b^2+6\left(3\,ab-\left(2\,b^2c^3x^3+b^2cx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{-\right.}{}}{}\right.}{27\,x^3}$$

input
```
integrate((a+b*arcsech(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
-1/27*(12*b^2*c^2*x^2 + 9*b^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)
/(c*x))^2 + 9*a^2 + 2*b^2 + 6*(3*a*b - (2*b^2*c^3*x^3 + b^2*c*x)*sqrt(-(c^
2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))
 - 6*(2*a*b*c^3*x^3 + a*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^3
```

3.40.6 Sympy [F]

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^2}{x^4}\,dx = \int \frac{(a + b\,\text{asech}\,(cx))^2}{x^4}\,dx$$

input
```
integrate((a+b*asech(c*x))**2/x**4,x)
```

output
```
Integral((a + b*asech(c*x))**2/x**4, x)
```

3.40.7 Maxima [F]

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^2}{x^4}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
2/9*a*b*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3
*arcsech(c*x)/x^3) + b^2*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1)
 + 1/(c*x))^2/x^4, x) - 1/3*a^2/x^3
```

3.40.8 Giac [F]

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^2}{x^4}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^2/x^4, x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\mathrm{sech}^{-1}(cx)\right)^2}{x^4} \, dx = \int \frac{\left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2}{x^4} \, dx$$

input
```
int((a + b*acosh(1/(c*x)))^2/x^4,x)
```

output
```
int((a + b*acosh(1/(c*x)))^2/x^4, x)
```

3.41 $\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^5}\,dx$

3.41.1 Optimal result

Integrand size = 14, antiderivative size = 151

$$\begin{aligned}\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^5}\,dx ={}& -\frac{b^2}{32x^4}-\frac{3b^2c^2}{32x^2}+\frac{3}{16}abc^4\operatorname{sech}^{-1}(cx)+\frac{3}{32}b^2c^4\operatorname{sech}^{-1}(cx)^2\\ &+\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{8x^4}\\ &+\frac{3bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16x^2}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{4x^4}\end{aligned}$$

output
```
-1/32*b^2/x^4-3/32*b^2*c^2/x^2+3/16*a*b*c^4*arcsech(c*x)+3/32*b^2*c^4*arcs
ech(c*x)^2-1/4*(a+b*arcsech(c*x))^2/x^4+1/8*b*(c*x+1)*(a+b*arcsech(c*x))*(
(-c*x+1)/(c*x+1))^(1/2)/x^4+3/16*b*c^2*(c*x+1)*(a+b*arcsech(c*x))*((-c*x+1
)/(c*x+1))^(1/2)/x^2
```

3.41.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.77

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^5}\,dx$$

$$= \frac{-8a^2-b^2-3b^2c^2x^2+4ab\sqrt{\frac{1-cx}{1+cx}}+4abcx\sqrt{\frac{1-cx}{1+cx}}+6abc^2x^2\sqrt{\frac{1-cx}{1+cx}}+6abc^3x^3\sqrt{\frac{1-cx}{1+cx}}+2b\Big(-8a+b\sqrt{\frac{1-cx}{1+cx}}}{}$$

input
```
Integrate[(a + b*ArcSech[c*x])^2/x^5,x]
```

output
```
(-8*a^2 - b^2 - 3*b^2*c^2*x^2 + 4*a*b*Sqrt[(1 - c*x)/(1 + c*x)] + 4*a*b*c*
x*Sqrt[(1 - c*x)/(1 + c*x)] + 6*a*b*c^2*x^2*Sqrt[(1 - c*x)/(1 + c*x)] + 6*
a*b*c^3*x^3*Sqrt[(1 - c*x)/(1 + c*x)] + 2*b*(-8*a + b*Sqrt[(1 - c*x)/(1 +
c*x)]*(2 + 2*c*x + 3*c^2*x^2 + 3*c^3*x^3))*ArcSech[c*x] + b^2*(-8 + 3*c^4*
x^4)*ArcSech[c*x]^2 - 6*a*b*c^4*x^4*Log[x] + 6*a*b*c^4*x^4*Log[1 + Sqrt[(1
 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/(32*x^4)
```

3.41.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6839, 5970, 3042, 3791, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^5}dx$$

$$\Big\downarrow \text{6839}$$

$$-c^4\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{c^4x^4}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{5970}$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\int \frac{a+b\text{sech}^{-1}(cx)}{c^4x^4}d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\int \left(a+b\text{sech}^{-1}(cx)\right)\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^4d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3791}$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\left(\frac{3}{4}\int \frac{a+b\text{sech}^{-1}(cx)}{c^2x^2}d\text{sech}^{-1}(cx)+\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{4c^4x^4}-\frac{b}{16c^4x^4}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\left(\frac{3}{4}\int\left(a+b\text{sech}^{-1}(cx)\right)\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^2d\text{sech}^{-1}(cx)+\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a\right.}{4c^4\right.}$$

$$\Big\downarrow \text{3791}$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\left(\frac{3}{4}\left(\frac{1}{2}\int\left(a+b\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)+\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{2c^2x^2}-\frac{}{4}\right.\right.\right.$$

$$\Big\downarrow \text{17}$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{4c^4x^4}+\frac{3}{4}\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{2c^2x^2}+\frac{(a}{}\right.\right.\right.$$

input
```
Int[(a + b*ArcSech[c*x])^2/x^5,x]
```

output
```
-(c^4*((a + b*ArcSech[c*x])^2/(4*c^4*x^4) - (b*(-1/16*b/(c^4*x^4) + (Sqrt[
(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/(4*c^4*x^4) + (3*(-1/
4*b/(c^2*x^2) + (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))
/(2*c^2*x^2) + (a + b*ArcSech[c*x])^2/(4*b)))/4))/2))
```

3.41.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.41. $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^5}\,dx$

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.41.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.74

method	result
parts	$-\frac{a^2}{4x^4}+b^2c^4\left(-\frac{\operatorname{arcsech}(cx)^2}{4c^4x^4}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{8c^3x^3}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{16cx}+\frac{3\operatorname{arcsech}(}{32}\right.$
derivativedivides	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{4c^4x^4}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{8c^3x^3}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{16cx}+3\operatorname{arc}\right.\right.$
default	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{4c^4x^4}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{8c^3x^3}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{16cx}+3\operatorname{arc}\right.\right.$

input
```
int((a+b*arcsech(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/x^4+b^2*c^4*(-1/4/c^4/x^4*arcsech(c*x)^2+1/8/c^3/x^3*(-(c*x-1)/c/
x)^(1/2)*((c*x+1)/c/x)^(1/2)*arcsech(c*x)+3/16*(-(c*x-1)/c/x)^(1/2)*((c*x+
1)/c/x)^(1/2)/c/x*arcsech(c*x)+3/32*arcsech(c*x)^2-1/32/c^4/x^4-3/32/c^2/x
^2)+2*a*b*c^4*(-1/4/c^4/x^4*arcsech(c*x)+1/32*(-(c*x-1)/c/x)^(1/2)/c^3/x^3
*((c*x+1)/c/x)^(1/2)*(3*arctanh(1/(-c^2*x^2+1)^(1/2))*c^4*x^4+3*(-c^2*x^2+
1)^(1/2)*c^2*x^2+2*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^5}\,dx =$$
$$-\frac{3\,b^2c^2x^2 - (3\,b^2c^4x^4 - 8\,b^2)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2 + 8\,a^2 + b^2 - 2\left(3\,abc^4x^4 - 8\,ab + (3\,b^2c^3x^3 + 2\,b^2cx)\right.}{32\,x^4}$$

input
```
integrate((a+b*arcsech(c*x))^2/x^5,x, algorithm="fricas")
```

output
```
-1/32*(3*b^2*c^2*x^2 - (3*b^2*c^4*x^4 - 8*b^2)*log((c*x*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) + 1)/(c*x))^2 + 8*a^2 + b^2 - 2*(3*a*b*c^4*x^4 - 8*a*b + (3*b
^2*c^3*x^3 + 2*b^2*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^
2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(3*a*b*c^3*x^3 + 2*a*b*c*x)*sqrt(-(c
^2*x^2 - 1)/(c^2*x^2)))/x^4
```

3.41.6 Sympy [F]

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^5}\,dx = \int \frac{(a+b\operatorname{asech}(cx))^2}{x^5}\,dx$$

input
```
integrate((a+b*asech(c*x))**2/x**5,x)
```

output
```
Integral((a + b*asech(c*x))**2/x**5, x)
```

3.41.7 Maxima [F]

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^5}\,dx = \int \frac{(b\operatorname{arsech}(cx)+a)^2}{x^5}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^2/x^5,x, algorithm="maxima")
```

output
```
1/32*a*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) - 3*c^5*log(c*x*sqrt(1
/(c^2*x^2) - 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) - 1)^(3/2) - 5*c^6*x*sqrt
(1/(c^2*x^2) - 1))/(c^4*x^4*(1/(c^2*x^2) - 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) -
 1) + 1))/c - 16*arcsech(c*x)/x^4) + b^2*integrate(log(sqrt(1/(c*x) + 1)*s
qrt(1/(c*x) - 1) + 1/(c*x))^2/x^5, x) - 1/4*a^2/x^4
```

3.41.8 Giac [F]

$$\int \frac{(a + b\operatorname{sech}^{-1}(cx))^2}{x^5}\,dx = \int \frac{(b\operatorname{arsech}(cx) + a)^2}{x^5}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^2/x^5,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^2/x^5, x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\operatorname{sech}^{-1}(cx))^2}{x^5}\,dx = \int \frac{\left(a + b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^2}{x^5}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))^2/x^5,x)
```

output
```
int((a + b*acosh(1/(c*x)))^2/x^5, x)
```

3.42 $\int x^3 (a + b\text{sech}^{-1}(cx))^3 \, dx$

3.42.1 Optimal result

Integrand size = 14, antiderivative size = 223

$$\begin{aligned}\int x^3 (a + b\text{sech}^{-1}(cx))^3 \, dx = {} & \frac{b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4c^4} - \frac{b^2x^2(a+b\text{sech}^{-1}(cx))}{4c^2} \\ & - \frac{b(a+b\text{sech}^{-1}(cx))^2}{2c^4} - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\,(a+b\text{sech}^{-1}(cx))^2}{2c^4} \\ & - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\,(a+b\text{sech}^{-1}(cx))^2}{4c^2} \\ & + \frac{1}{4}x^4(a+b\text{sech}^{-1}(cx))^3 \\ & + \frac{b^2(a+b\text{sech}^{-1}(cx))\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)}{c^4} \\ & + \frac{b^3\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right)}{2c^4}\end{aligned}$$

output
```
-1/4*b^2*x^2*(a+b*arcsech(c*x))/c^2-1/2*b*(a+b*arcsech(c*x))^2/c^4+1/4*x^4
*(a+b*arcsech(c*x))^3+b^2*(a+b*arcsech(c*x))*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2))^2)/c^4+1/2*b^3*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))^2)/c^4+1/4*b^3*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/c^4-1/2*b*(c*x+1
)*(a+b*arcsech(c*x))^2*((-c*x+1)/(c*x+1))^(1/2)/c^4-1/4*b*x^2*(c*x+1)*(a+b
*arcsech(c*x))^2*((-c*x+1)/(c*x+1))^(1/2)/c^2
```

3.42.2 Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.51

$$\int x^3\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx$$
$$=\frac{1}{4}\left(a^3x^4+b^3x^4\text{sech}^{-1}(cx)^3+a^2b\left(-\frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(2+c^2x^2\right)}{c^4}+3x^4\text{sech}^{-1}(cx)\right)\right.$$
$$+\frac{ab^2\left(-c^2x^2-2\sqrt{\frac{1-cx}{1+cx}}\left(2+2cx+c^2x^2+c^3x^3\right)\text{sech}^{-1}(cx)+3c^4x^4\text{sech}^{-1}(cx)^2+4\log\left(\frac{1}{cx}\right)\right)}{c^4}$$
$$-\frac{b^3\left(-\sqrt{\frac{1-cx}{1+cx}}(1+cx)+\left(-2+2\sqrt{\frac{1-cx}{1+cx}}+2cx\sqrt{\frac{1-cx}{1+cx}}+c^2x^2\sqrt{\frac{1-cx}{1+cx}}+c^3x^3\sqrt{\frac{1-cx}{1+cx}}\right)\text{sech}^{-1}(cx)^2+\text{sech}^{-}}{c^4}$$

input
```
Integrate[x^3*(a + b*ArcSech[c*x])^3,x]
```

output
```
(a^3*x^4 + b^3*x^4*ArcSech[c*x]^3 + a^2*b*(-((Sqrt[(1 - c*x)/(1 + c*x)]*(1
 + c*x)*(2 + c^2*x^2))/c^4) + 3*x^4*ArcSech[c*x]) + (a*b^2*(-(c^2*x^2) - 2
*Sqrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x + c^2*x^2 + c^3*x^3)*ArcSech[c*x] +
3*c^4*x^4*ArcSech[c*x]^2 + 4*Log[1/(c*x)]))/c^4 - (b^3*(-(Sqrt[(1 - c*x)/(
1 + c*x)]*(1 + c*x)) + (-2 + 2*Sqrt[(1 - c*x)/(1 + c*x)] + 2*c*x*Sqrt[(1 -
 c*x)/(1 + c*x)] + c^2*x^2*Sqrt[(1 - c*x)/(1 + c*x)] + c^3*x^3*Sqrt[(1 - c
*x)/(1 + c*x)])*ArcSech[c*x]^2 + ArcSech[c*x]*(c^2*x^2 - 4*Log[1 + E^(-2*A
rcSech[c*x])]) + 2*PolyLog[2, -E^(-2*ArcSech[c*x])]))/c^4)/4
```

3.42.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {6839, 5974, 3042, 4674, 3042, 4254, 24, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx$$
$$\Big\downarrow \text{6839}$$

$$-\frac{\int c^4x^4\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^3d\text{sech}^{-1}(cx)}{c^4}$$

$$\Big\downarrow \text{5974}$$

$$-\frac{\frac{3}{4}b\int c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^3}{c^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{4}b\int\left(a+b\text{sech}^{-1}(cx)\right)^2\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^4d\text{sech}^{-1}(cx)}{c^4}$$

$$\Big\downarrow \text{4674}$$

$$-\frac{\frac{3}{4}b\left(\frac{2}{3}\int c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)-\frac{1}{3}b^2\int c^2x^2d\text{sech}^{-1}(cx)+\frac{1}{3}bc^2x^2\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}c^2x^2\sqrt{\frac{1-c}{cx+}}\right.}{c^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\int\left(a+b\text{sech}^{-1}(cx)\right)^2\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^2d\text{sech}^{-1}(cx)-\frac{1}{3}b^2\int\csc\left(i\text{sech}^{-}\right.\right.}{c^4}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\int\left(a+b\text{sech}^{-1}(cx)\right)^2\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^2d\text{sech}^{-1}(cx)-\frac{1}{3}ib^2\int 1d\left(-i\sqrt{\frac{1}{c}}\right.\right.}{c^4}$$

$$\Big\downarrow \text{24}$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\int\left(a+b\text{sech}^{-1}(cx)\right)^2\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^2d\text{sech}^{-1}(cx)+\frac{1}{3}bc^2x^2\left(a+b\text{sec}\right.\right.}{c^4}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2-2ib\int -i\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(\right.\right.\right.}{c}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{3}{4}b\left(\frac{2}{3}\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2-2b\int\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)\right)+\frac{1}{3}bc^2x^2\left(a+\right.\right.}{c^4}$$

$$\Big\downarrow \text{3042}$$

3.42. $\int x^3\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2-2b\int -i\left(a+b\text{sech}^{-1}(cx)\right)\tan\left(i\text{sech}^{-1}\right.\right.\right.}{}$$

$\downarrow$ 26

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2+2ib\int\left(a+b\text{sech}^{-1}(cx)\right)\tan\left(i\text{sech}^{-1}(\right.\right.\right.}{c}$$

$\downarrow$ 4201

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2+2ib\left(2i\int\frac{e^{2\text{sech}^{-1}(cx)}\left(a+b\text{sech}^{-1}(cx)\right)}{1+e^{2\text{sech}^{-1}(cx)}}ds\right.\right.\right.}{}$$

$\downarrow$ 2620

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2+2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+\right.\right.\right.\right.\right.}{}$$

$\downarrow$ 2715

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2+2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+\right.\right.\right.\right.\right.}{}$$

$\downarrow$ 2838

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{1}{3}bc^2x^2\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}c^2x^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2+\frac{2}{3}\left(\sqrt{\right.\right.}{}$$

input
```
Int[x^3*(a + b*ArcSech[c*x])^3,x]
```

output
```
-((-1/4*(c^4*x^4*(a + b*ArcSech[c*x])^3) + (3*b*(-1/3*(b^2*Sqrt[(1 - c*x)/
(1 + c*x)]*(1 + c*x)) + (b*c^2*x^2*(a + b*ArcSech[c*x]))/3 + (c^2*x^2*Sqrt
[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/3 + (2*(Sqrt[(1 -
c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2 + (2*I)*b*(((-1/2*I)*(a +
 b*ArcSech[c*x])^2)/b + (2*I)*(((a + b*ArcSech[c*x])*Log[1 + E^(2*ArcSech[
c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSech[c*x])])/4))))/3))/4)/c^4)
```

3.42. $\int x^3\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx$

3.42.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.42.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.08

method	result
derivativedivides	$\frac{\frac{a^3c^4x^4}{4}+b^3\left(\frac{\operatorname{arcsech}(cx)^3c^4x^4}{4}-\frac{\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,c^3x^3}{4}-\frac{\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx}{2}-\frac{c^2x^2\operatorname{arcsech}(cx)}{4}+\sqrt{}\right.}{}$
default	$\frac{\frac{a^3c^4x^4}{4}+b^3\left(\frac{\operatorname{arcsech}(cx)^3c^4x^4}{4}-\frac{\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,c^3x^3}{4}-\frac{\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx}{2}-\frac{c^2x^2\operatorname{arcsech}(cx)}{4}+\sqrt{}\right.}{}$
parts	$\frac{a^3x^4}{4}+\frac{b^3\left(\frac{\operatorname{arcsech}(cx)^3c^4x^4}{4}-\frac{\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,c^3x^3}{4}-\frac{\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx}{2}-\frac{c^2x^2\operatorname{arcsech}(cx)}{4}+\sqrt{}\right.}{}$

input
```
int(x^3*(a+b*arcsech(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/4*a^3*c^4*x^4+b^3*(1/4*arcsech(c*x)^3*c^4*x^4-1/4*arcsech(c*x)^2*
(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c^3*x^3-1/2*arcsech(c*x)^2*(-(c*x
-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-1/4*c^2*x^2*arcsech(c*x)+1/4*(-(c*x
-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)-1/2*arcsech(c*x)^2-1/4+arcsech(c*x)
*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+1/2*polylog(2,-(1/c/x+(-
1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))+3*a*b^2*(-1/3*arcsech(c*x)+1/4*arcsech
(c*x)^2*c^4*x^4-1/6*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*
c^3*x^3-1/3*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-1/12
*c^2*x^2+1/3*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))+3*b*a^2*(1/
4*c^4*x^4*arcsech(c*x)-1/12*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(
c^2*x^2+2)))
```

3.42.5 Fricas [F]

$$\int x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)^3\,dx=\int\left(b\operatorname{arsech}\left(cx\right)+a\right)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arcsech(c*x)^3 + 3*a*b^2*x^3*arcsech(c*x)^2 + 3*a^2*b*x^3
*arcsech(c*x) + a^3*x^3, x)
```

3.42.6 Sympy [F]

$$\int x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)^3\,dx=\int x^3\left(a+b\operatorname{asech}\left(cx\right)\right)^3\,dx$$

input
```
integrate(x**3*(a+b*asech(c*x))**3,x)
```

output
```
Integral(x**3*(a + b*asech(c*x))**3, x)
```

3.42.7 Maxima [F]

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^3 dx = \int (b \operatorname{arsech}(cx)+a)^3 x^3 dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

output
```
1/4*a^3*x^4 + 1/4*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) -
 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*a^2*b + integrate(b^3*x^3*log(sqrt(1/(c*x
) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^3 + 3*a*b^2*x^3*log(sqrt(1/(c*x) + 1)*
sqrt(1/(c*x) - 1) + 1/(c*x))^2, x)
```

3.42.8 Giac [F]

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^3 dx = \int (b \operatorname{arsech}(cx)+a)^3 x^3 dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^3*x^3, x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^3 dx = \int x^3 \left(a+b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)^3 dx$$

input
```
int(x^3*(a + b*acosh(1/(c*x)))^3,x)
```

output
```
int(x^3*(a + b*acosh(1/(c*x)))^3, x)
```

3.43 $\int x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3\,dx$

3.43.1 Optimal result

Integrand size = 14, antiderivative size = 242

$$\begin{aligned}\int x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3\,dx=&-\frac{b^2x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{c^2}-\frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{2c^2}\\&+\frac{1}{3}x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3\\&-\frac{b\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2\arctan\left(e^{\mathrm{sech}^{-1}(cx)}\right)}{c^3}\\&+\frac{b^3\arctan\left(\frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{cx}\right)}{c^3}\\&+\frac{ib^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\mathrm{PolyLog}\left(2,-ie^{\mathrm{sech}^{-1}(cx)}\right)}{c^3}\\&-\frac{ib^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\mathrm{PolyLog}\left(2,ie^{\mathrm{sech}^{-1}(cx)}\right)}{c^3}\\&-\frac{ib^3\,\mathrm{PolyLog}\left(3,-ie^{\mathrm{sech}^{-1}(cx)}\right)}{c^3}+\frac{ib^3\,\mathrm{PolyLog}\left(3,ie^{\mathrm{sech}^{-1}(cx)}\right)}{c^3}\end{aligned}$$

output
```
-b^2*x*(a+b*arcsech(c*x))/c^2+1/3*x^3*(a+b*arcsech(c*x))^3-b*(a+b*arcsech(
c*x))^2*arctan(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/c^3+b^3*arctan((c*x
+1)*((-c*x+1)/(c*x+1))^(1/2)/c/x)/c^3+I*b^2*(a+b*arcsech(c*x))*polylog(2,-
I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-I*b^2*(a+b*arcsech(c*x))*p
olylog(2,I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-I*b^3*polylog(3,-
I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3+I*b^3*polylog(3,I*(1/c/x+(
-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-1/2*b*x*(c*x+1)*(a+b*arcsech(c*x))^2
*((-c*x+1)/(c*x+1))^(1/2)/c^2
```

3.43.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.82

$$\int x^2 \left(a+b\operatorname{sech}^{-1}(cx)\right)^3 dx$$
$$= \frac{2a^3c^3x^3 - 3a^2bcx\sqrt{\frac{1-cx}{1+cx}}(1+cx) + 6a^2bc^3x^3\operatorname{sech}^{-1}(cx) + 3ia^2b\log\left(-2icx + 2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right) - 6ab^2\left(c\right.}{}$$

input
```
Integrate[x^2*(a + b*ArcSech[c*x])^3,x]
```

output
```
(2*a^3*c^3*x^3 - 3*a^2*b*c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + 6*a^2*b
*c^3*x^3*ArcSech[c*x] + (3*I)*a^2*b*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 +
 c*x)]*(1 + c*x)] - 6*a*b^2*(c*x + c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)
*ArcSech[c*x] - c^3*x^3*ArcSech[c*x]^2 - I*ArcSech[c*x]*Log[1 - I/E^ArcSec
h[c*x]] + I*ArcSech[c*x]*Log[1 + I/E^ArcSech[c*x]] - I*PolyLog[2, (-I)/E^A
rcSech[c*x]] + I*PolyLog[2, I/E^ArcSech[c*x]]) - b^3*(6*c*x*ArcSech[c*x] +
 3*c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*ArcSech[c*x]^2 - 2*c^3*x^3*ArcS
ech[c*x]^3 - (3*I)*((-4*I)*ArcTan[Tanh[ArcSech[c*x]/2]] + ArcSech[c*x]^2*L
og[1 - I/E^ArcSech[c*x]] - ArcSech[c*x]^2*Log[1 + I/E^ArcSech[c*x]] + 2*Ar
cSech[c*x]*PolyLog[2, (-I)/E^ArcSech[c*x]] - 2*ArcSech[c*x]*PolyLog[2, I/E
^ArcSech[c*x]] + 2*PolyLog[3, (-I)/E^ArcSech[c*x]] - 2*PolyLog[3, I/E^ArcS
ech[c*x]])))/(6*c^3)
```

3.43.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6839, 5974, 3042, 4674, 3042, 4257, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\text{sech}^{-1}(cx)\right)^3 dx$$

$$\Big\downarrow \text{6839}$$

$$-\frac{\int c^3x^3\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^3d\text{sech}^{-1}(cx)}{c^3}$$

$$\Big\downarrow \text{5974}$$

$$-\frac{b\int c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^3+b\int\left(a+b\text{sech}^{-1}(cx)\right)^2\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^3d\text{sech}^{-1}(cx)}{c^3}$$

$$\Big\downarrow \text{4674}$$

$$-\frac{b\left(\frac{1}{2}\int cx\left(a+b\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)+b^2\left(-\int cxd\text{sech}^{-1}(cx)\right)+bcx\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{2}cx\sqrt{\frac{1-cx}{cx+1}}(cx+1\right.}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^3+b\left(\frac{1}{2}\int\left(a+b\text{sech}^{-1}(cx)\right)^2\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(cx)+b^2\left(-\int\csc\left(i\text{sech}^{-1}\right.\right.}{c^3}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^3+b\left(\frac{1}{2}\int\left(a+b\text{sech}^{-1}(cx)\right)^2\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(cx)+bcx\left(a+b\text{sech}^{-1}(c\right.\right.}{c^3}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^3+b\left(\frac{1}{2}\left(-2ib\int\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-ie^{\text{sech}^{-1}(cx)}\right)d\text{sech}^{-1}(cx)+2ib\int\left(a+b\text{sec}\right.\right.\right.}{}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^3+b\left(\frac{1}{2}\left(2ib\left(b\int\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)d\text{sech}^{-1}(cx)-\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)\right.\right.\right.}{}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^3+b\left(\frac{1}{2}\left(2ib\left(b\int e^{-\text{sech}^{-1}(cx)}\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)de^{\text{sech}^{-1}(cx)}-\text{PolyLog}\left(2,-ie\right.\right.\right.\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^3+b\left(\frac{1}{2}\left(2\arctan\left(e^{\text{sech}^{-1}(cx)}\right)\left(a+b\text{sech}^{-1}(cx)\right)^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{sech}^{-1}(cx)}\right)\right.\right.\right.}{}$$

input
```
Int[x^2*(a + b*ArcSech[c*x])^3,x]
```

output
```
-((-1/3*(c^3*x^3*(a + b*ArcSech[c*x])^3) + b*(b*c*x*(a + b*ArcSech[c*x]) +
 (c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/2 - b^2*
ArcTan[(Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(c*x)] + (2*(a + b*ArcSech[c*
x])^2*ArcTan[E^ArcSech[c*x]] + (2*I)*b*(-((a + b*ArcSech[c*x])*PolyLog[2,
(-I)*E^ArcSech[c*x]]) + b*PolyLog[3, (-I)*E^ArcSech[c*x]]) - (2*I)*b*(-((a
 + b*ArcSech[c*x])*PolyLog[2, I*E^ArcSech[c*x]]) + b*PolyLog[3, I*E^ArcSec
h[c*x]]))/2))/c^3)
```

3.43. $\int x^2\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx$

3.43.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

3.43. $\int x^2 \left(a + b\,\text{sech}^{-1}(cx)\right)^3 \, dx$

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.43.4 Maple [F]

$$\int x^2 (a + b\ \mathrm{arcsech}\left(cx\right))^3\, dx$$

input
```
int(x^2*(a+b*arcsech(c*x))^3,x)
```

output
```
int(x^2*(a+b*arcsech(c*x))^3,x)
```

3.43.5 Fricas [F]

$$\int x^2 \left(a + b\mathrm{sech}^{-1}(cx)\right)^3\, dx = \int (b\,\mathrm{arsech}\left(cx\right) + a)^3 x^2\, dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arcsech(c*x)^3 + 3*a*b^2*x^2*arcsech(c*x)^2 + 3*a^2*b*x^2
*arcsech(c*x) + a^3*x^2, x)
```

3.43.6 Sympy [F]

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^3 \,dx = \int x^2 (a + b \operatorname{asech}{(cx)})^3\, dx$$

input
```
integrate(x**2*(a+b*asech(c*x))**3,x)
```

output
```
Integral(x**2*(a + b*asech(c*x))**3, x)
```

3.43.7 Maxima [F]

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^3 \,dx = \int (b\,\text{arsech}\,(cx) + a)^3 x^2\, dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

output
```
1/3*a^3*x^3 + 1/2*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^
2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*a^2*b + integra
te(b^3*x^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^3 + 3*a*b^2*
x^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2, x)
```

3.43.8 Giac [F]

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^3 \,dx = \int (b\,\text{arsech}\,(cx) + a)^3 x^2\, dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^3*x^2, x)
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int x^2 \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

input
```
int(x^2*(a + b*acosh(1/(c*x)))^3,x)
```

output
```
int(x^2*(a + b*acosh(1/(c*x)))^3, x)
```

3.44 $\int x\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx$

3.44.1 Optimal result

Integrand size = 12, antiderivative size = 126

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=-\frac{3b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2}-\frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3b^2\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)}{c^2}+\frac{3b^3\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right)}{2c^2}$$

output
```
-3/2*b*(a+b*arcsech(c*x))^2/c^2+1/2*x^2*(a+b*arcsech(c*x))^3+3*b^2*(a+b*ar
csech(c*x))*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/c^2+3/2*b^3*p
olylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/c^2-3/2*b*(c*x+1)*(a
+b*arcsech(c*x))^2*((-c*x+1)/(c*x+1))^(1/2)/c^2
```

3.44.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.74

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx$$
$$=\frac{-3b^2\left(-ac^2x^2+b\left(-1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)\right)\text{sech}^{-1}(cx)^2+b^3c^2x^2\text{sech}^{-1}(cx)^3+3b\text{sech}^{-1}(cx)\left(a\left(ac^2x^2\right.\right.}{}$$

input
```
Integrate[x*(a + b*ArcSech[c*x])^3,x]
```

output
```
(-3*b^2*(-(a*c^2*x^2) + b*(-1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 -
c*x)/(1 + c*x)]))*ArcSech[c*x]^2 + b^3*c^2*x^2*ArcSech[c*x]^3 + 3*b*ArcSec
h[c*x]*(a*(a*c^2*x^2 - 2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)) + 2*b^2*Lo
g[1 + E^(-2*ArcSech[c*x])]) + a*(a*(a*c^2*x^2 - 3*b*Sqrt[(1 - c*x)/(1 + c*
x)]*(1 + c*x)) + 6*b^2*Log[1/(c*x)]) - 3*b^3*PolyLog[2, -E^(-2*ArcSech[c*x
])])/(2*c^2)
```

3.44.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.917$, Rules used = {6839, 5974, 3042, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx$$
$$\Big\downarrow \text{6839}$$
$$-\frac{\int c^2x^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^3d\text{sech}^{-1}(cx)}{c^2}$$
$$\Big\downarrow \text{5974}$$
$$-\frac{\frac{3}{2}b\int c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^3}{c^2}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{2}b\int \left(a+b\text{sech}^{-1}(cx)\right)^2\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^2 d\text{sech}^{-1}(cx)}{c^2}$$

$\Big\downarrow$ 4672

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{2}b\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2-2ib\int -i\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx\right.\right.}{c^2}$$

$\Big\downarrow$ 26

$$-\frac{\frac{3}{2}b\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2-2b\int \sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)\right)-\frac{1}{2}c^2x^2\left(a+bsec\right.}{c^2}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{2}b\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2-2b\int -i\left(a+b\text{sech}^{-1}(cx)\right)\tan\left(i\text{sech}^{-1}(c\right.\right.}{c^2}$$

$\Big\downarrow$ 26

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{2}b\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2+2ib\int \left(a+b\text{sech}^{-1}(cx)\right)\tan\left(i\text{sech}^{-1}(cx\right.\right.}{c^2}$$

$\Big\downarrow$ 4201

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{2}b\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2+2ib\left(2i\int \frac{e^{2\text{sech}^{-1}(cx)}\left(a+b\text{sech}^{-1}(cx)\right)}{1+e^{2\text{sech}^{-1}(cx)}}d\text{sech}\right.\right.}{c^2}$$

$\Big\downarrow$ 2620

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{2}b\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2+2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+bse\right.\right.\right.\right.}{c^2}$$

$\Big\downarrow$ 2715

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{2}b\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2+2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+bse\right.\right.\right.\right.}{c^2}$$

$\Big\downarrow$ 2838

3.44. $\int x\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3}{2}b\left(\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2+2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+b\text{se\right.\right.\right.}{c^2}$$

input
```
Int[x*(a + b*ArcSech[c*x])^3,x]
```

output
```
-((-1/2*(c^2*x^2*(a + b*ArcSech[c*x])^3) + (3*b*(Sqrt[(1 - c*x)/(1 + c*x)]
*(1 + c*x)*(a + b*ArcSech[c*x])^2 + (2*I)*b*(((-1/2*I)*(a + b*ArcSech[c*x]
)^2)/b + (2*I)*(((a + b*ArcSech[c*x])*Log[1 + E^(2*ArcSech[c*x])])/2 + (b*
PolyLog[2, -E^(2*ArcSech[c*x])])/4))))/2)/c^2)
```

3.44.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.44.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.52

method	result
derivativedivides	$\frac{\frac{a^3c^2x^2}{2}+b^3\left(\frac{\operatorname{arcsech}(cx)^2\left(c^2x^2\operatorname{arcsech}(cx)-3\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+3\right)}{2}-3\operatorname{arcsech}(cx)^2+3\operatorname{arcsech}(cx)\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\cdots}\right.\right.\right.}{\cdots}$
default	$\frac{\frac{a^3c^2x^2}{2}+b^3\left(\frac{\operatorname{arcsech}(cx)^2\left(c^2x^2\operatorname{arcsech}(cx)-3\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+3\right)}{2}-3\operatorname{arcsech}(cx)^2+3\operatorname{arcsech}(cx)\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\cdots}\right.\right.\right.}{\cdots}$
parts	$\frac{a^3x^2}{2}+\frac{b^3\left(\frac{\operatorname{arcsech}(cx)^2\left(c^2x^2\operatorname{arcsech}(cx)-3\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+3\right)}{2}-3\operatorname{arcsech}(cx)^2+3\operatorname{arcsech}(cx)\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\cdots}\right.\right.\right.}{c^2}$

input
```
int(x*(a+b*arcsech(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/2*a^3*c^2*x^2+b^3*(1/2*arcsech(c*x)^2*(c^2*x^2*arcsech(c*x)-3*(-(
c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+3)-3*arcsech(c*x)^2+3*arcsech(c*
x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+3/2*polylog(2,-(1/c/x+
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))+3*a*b^2*(-2*arcsech(c*x)+1/2*arcsech
(c*x)*(c^2*x^2*arcsech(c*x)-2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)
+2)+ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))+3*b*a^2*(1/2*c^2*x^2
*arcsech(c*x)-1/2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)))
```

3.44.5 Fricas [F]

$$\int x\left(a+b\operatorname{sech}^{-1}(cx)\right)^3\,dx=\int\left(b\operatorname{arsech}\left(cx\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arcsech(c*x)^3 + 3*a*b^2*x*arcsech(c*x)^2 + 3*a^2*b*x*arcse
ch(c*x) + a^3*x, x)
```

3.44.6 Sympy [F]

$$\int x\left(a+b\operatorname{sech}^{-1}(cx)\right)^3\,dx=\int x\left(a+b\operatorname{asech}\left(cx\right)\right)^3\,dx$$

input
```
integrate(x*(a+b*asech(c*x))**3,x)
```

output
```
Integral(x*(a + b*asech(c*x))**3, x)
```

3.44.7 Maxima [F]

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=\int (b\,\text{arsech}\,(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

output
```
3/2*a*b^2*x^2*arcsech(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arcsech(c*x) - x*sqr
t(1/(c^2*x^2) - 1)/c)*a^2*b - 3*(x*sqrt(1/(c^2*x^2) - 1)*arcsech(c*x)/c +
log(x)/c^2)*a*b^2 + b^3*integrate(x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1
) + 1/(c*x))^3, x)
```

3.44.8 Giac [F]

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=\int (b\,\text{arsech}\,(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^3*x, x)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=\int x\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

input
```
int(x*(a + b*acosh(1/(c*x)))^3,x)
```

output
```
int(x*(a + b*acosh(1/(c*x)))^3, x)
```

3.45 $\int \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx$

3.45.1 Optimal result

Integrand size = 10, antiderivative size = 140

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = x\left(a+b\text{sech}^{-1}(cx)\right)^3-\frac{6b\left(a+b\text{sech}^{-1}(cx)\right)^2\arctan\left(e^{\text{sech}^{-1}(cx)}\right)}{c}+\frac{6ib^2\left(a+b\text{sech}^{-1}(cx)\right)\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)}{c}-\frac{6ib^2\left(a+b\text{sech}^{-1}(cx)\right)\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(cx)}\right)}{c}-\frac{6ib^3\,\text{PolyLog}\left(3,-ie^{\text{sech}^{-1}(cx)}\right)}{c}+\frac{6ib^3\,\text{PolyLog}\left(3,ie^{\text{sech}^{-1}(cx)}\right)}{c}$$

output
```
x*(a+b*arcsech(c*x))^3-6*b*(a+b*arcsech(c*x))^2*arctan(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))/c+6*I*b^2*(a+b*arcsech(c*x))*polylog(2,-I*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c-6*I*b^2*(a+b*arcsech(c*x))*polylog(2,I*(1
/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c-6*I*b^3*polylog(3,-I*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c+6*I*b^3*polylog(3,I*(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2)))/c
```

3.45.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 282 vs. 2(140) = 280.

Time = 0.57 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.01

$$\int \left(a+b\operatorname{sech}^{-1}(cx)\right)^3 dx = a^3x+3a^2bx\operatorname{sech}^{-1}(cx)-\frac{3a^2b\arctan\left(\frac{cx\sqrt{\frac{1-cx}{1+cx}}}{-1+cx}\right)}{c}$$
$$+\frac{3iab^2\left(\operatorname{sech}^{-1}(cx)\left(-icx\operatorname{sech}^{-1}(cx)+2\log\left(1-ie^{-\operatorname{sech}^{-1}(cx)}\right)-2\log\left(1+ie^{-\operatorname{sech}^{-1}(cx)}\right)\right)+2\operatorname{PolyLog}\right.}{c}$$
$$+\frac{b^3\left(cx\operatorname{sech}^{-1}(cx)^3-3i\left(-\operatorname{sech}^{-1}(cx)^2\left(\log\left(1-ie^{-\operatorname{sech}^{-1}(cx)}\right)-\log\left(1+ie^{-\operatorname{sech}^{-1}(cx)}\right)\right)-2\operatorname{sech}^{-1}(cx)\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSech[c*x])^3,x]
```

output
```
a^3*x + 3*a^2*b*x*ArcSech[c*x] - (3*a^2*b*ArcTan[(c*x*Sqrt[(1 - c*x)/(1 +
c*x)])/(-1 + c*x)])/c + ((3*I)*a*b^2*(ArcSech[c*x]*((-I)*c*x*ArcSech[c*x]
+ 2*Log[1 - I/E^ArcSech[c*x]] - 2*Log[1 + I/E^ArcSech[c*x]]) + 2*PolyLog[2
, (-I)/E^ArcSech[c*x]] - 2*PolyLog[2, I/E^ArcSech[c*x]]))/c + (b^3*(c*x*Ar
cSech[c*x]^3 - (3*I)*(-(ArcSech[c*x]^2*(Log[1 - I/E^ArcSech[c*x]] - Log[1
+ I/E^ArcSech[c*x]])) - 2*ArcSech[c*x]*(PolyLog[2, (-I)/E^ArcSech[c*x]] -
PolyLog[2, I/E^ArcSech[c*x]]) - 2*(PolyLog[3, (-I)/E^ArcSech[c*x]] - PolyL
og[3, I/E^ArcSech[c*x]]))))/c
```

3.45.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6833, 5974, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\operatorname{sech}^{-1}(cx)\right)^3 dx$$

$$\Big\downarrow \text{ 6833}$$

$$-\frac{\int cx\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^3d\text{sech}^{-1}(cx)}{c}$$

$\Big\downarrow$ 5974

$$-\frac{3b\int cx\left(a+b\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)-cx\left(a+b\text{sech}^{-1}(cx)\right)^3}{c}$$

$\Big\downarrow$ 3042

$$-\frac{-cx\left(a+b\text{sech}^{-1}(cx)\right)^3+3b\int\left(a+b\text{sech}^{-1}(cx)\right)^2\csc\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(cx)}{c}$$

$\Big\downarrow$ 4668

$$-\frac{-cx\left(a+b\text{sech}^{-1}(cx)\right)^3+3b\left(-2ib\int\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-ie^{\text{sech}^{-1}(cx)}\right)d\text{sech}^{-1}(cx)+2ib\int\left(a+b\text{sech}^{-1}(\right.\right.}{c}$$

$\Big\downarrow$ 3011

$$-\frac{-cx\left(a+b\text{sech}^{-1}(cx)\right)^3+3b\left(2ib\left(b\int\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)d\text{sech}^{-1}(cx)-\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)(a+\right.\right.}{}$$

$\Big\downarrow$ 2720

$$-\frac{-cx\left(a+b\text{sech}^{-1}(cx)\right)^3+3b\left(2ib\left(b\int e^{-\text{sech}^{-1}(cx)}\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)de^{\text{sech}^{-1}(cx)}-\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}\right.\right.}{}$$

$\Big\downarrow$ 7143

$$-\frac{-cx\left(a+b\text{sech}^{-1}(cx)\right)^3+3b\left(2\arctan\left(e^{\text{sech}^{-1}(cx)}\right)\left(a+b\text{sech}^{-1}(cx)\right)^2+2ib\left(b\text{PolyLog}\left(3,-ie^{\text{sech}^{-1}(cx)}\right)-\text{Po}\right.\right.}{}$$

input
```
Int[(a + b*ArcSech[c*x])^3,x]
```

output
```
-((-(c*x*(a + b*ArcSech[c*x])^3) + 3*b*(2*(a + b*ArcSech[c*x])^2*ArcTan[E^
ArcSech[c*x]] + (2*I)*b*(-((a + b*ArcSech[c*x])*PolyLog[2, (-I)*E^ArcSech[
c*x]]) + b*PolyLog[3, (-I)*E^ArcSech[c*x]]) - (2*I)*b*(-((a + b*ArcSech[c*
x])*PolyLog[2, I*E^ArcSech[c*x]]) + b*PolyLog[3, I*E^ArcSech[c*x]])))/c)
```

3.45. $\int\left(a+b\text{sech}^{-1}(cx)\right)^3dx$

3.45.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6833
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-c^(-1)   S
ubst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
 b, c, n}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.45.4 Maple [F]

$$\int (a+b\,\mathrm{arcsech}\,(cx))^3\,dx$$

input
```
int((a+b*arcsech(c*x))^3,x)
```

output
```
int((a+b*arcsech(c*x))^3,x)
```

3.45.5 Fricas [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arsech}\,(cx)+a)^3\,dx$$

input
```
integrate((a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c*x
) + a^3, x)
```

3.45.6 Sympy [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=\int (a+b\,\mathrm{asech}\,(cx))^3\,dx$$

input
```
integrate((a+b*asech(c*x))**3,x)
```

output
```
Integral((a + b*asech(c*x))**3, x)
```

3.45.7 Maxima [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=\int (b\,\text{arsech}\,(cx)+a)^3\,dx$$

input
```
integrate((a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

output
```
b^3*x*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^3 + a^3*x + 3*(c*x*arcsech(c*x
) - arctan(sqrt(1/(c^2*x^2) - 1)))*a^2*b/c - integrate(-(b^3*log(c)^3 - 3*
a*b^2*log(c)^2 - (b^3*c^2*x^2 - b^3)*log(x)^3 - (b^3*c^2*log(c)^3 - 3*a*b^
2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)
*x^2 + (b^3*log(c) - a*b^2 - (b^3*c^2*(log(c) + 1) - a*b^2*c^2)*x^2 - (b^3
*c^2*x^2 - b^3)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^3*c^2*x^2 - b^3)
*log(x))*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^2 + 3*(b^3*log(c) - a*b^2 -
 (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + (b^3*log(c)^3 - 3*a*b^2*log(
c)^2 - (b^3*c^2*x^2 - b^3)*log(x)^3 - (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(
c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(
x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) - (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*
log(c))*x^2)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - 3*(b^3*log(c)^2 - 2*a*
b^2*log(c) - (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 - (b^3*c^2*x^2 -
b^3)*log(x)^2 + (b^3*log(c)^2 - 2*a*b^2*log(c) - (b^3*c^2*log(c)^2 - 2*a*b
^2*c^2*log(c))*x^2 - (b^3*c^2*x^2 - b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2
- (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) +
 2*(b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*log(sqr
t(c*x + 1)*sqrt(-c*x + 1) + 1) + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) - (b^3*c
^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log(x))/(c^2*x^2 + (c^2*x^2 - 1)*sq
rt(c*x + 1)*sqrt(-c*x + 1) - 1), x)
```

3.45.8 Giac [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=\int (b\,\text{arsech}\,(cx)+a)^3\,dx$$

input
```
integrate((a+b*arcsech(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^3, x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int (a + b\text{sech}^{-1}(cx))^3 \, dx = \int \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^3 dx$$

input
```
int((a + b*acosh(1/(c*x)))^3,x)
```

output
```
int((a + b*acosh(1/(c*x)))^3, x)
```

3.46 $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x}\,dx$

3.46.1 Optimal result

Integrand size = 14, antiderivative size = 114

$$\begin{aligned}\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x}\,dx =& \frac{\left(a+b\text{sech}^{-1}(cx)\right)^4}{4b} - \left(a+b\text{sech}^{-1}(cx)\right)^3\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)\\ &-\frac{3}{2}b\left(a+b\text{sech}^{-1}(cx)\right)^2\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right)\\ &+\frac{3}{2}b^2\left(a+b\text{sech}^{-1}(cx)\right)\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(cx)}\right)\\ &-\frac{3}{4}b^3\,\text{PolyLog}\left(4,-e^{2\text{sech}^{-1}(cx)}\right)\end{aligned}$$

output
```
1/4*(a+b*arcsech(c*x))^4/b-(a+b*arcsech(c*x))^3*ln(1+(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))^2)-3/2*b*(a+b*arcsech(c*x))^2*polylog(2,-(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+3/2*b^2*(a+b*arcsech(c*x))*polylog(3,-(1/c/
x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-3/4*b^3*polylog(4,-(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))^2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.60

$$\int \frac{(a+b\text{sech}^{-1}(cx))^3}{x}\,dx = \frac{1}{4}\Big(-6a^2b\text{sech}^{-1}(cx)^2 - 4ab^2\text{sech}^{-1}(cx)^3 - b^3\text{sech}^{-1}(cx)^4 - 12a^2b\text{sech}^{-1}(cx)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right) - 12ab^2\text{sech}^{-1}(cx)^2\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right) - 4b^3\text{sech}^{-1}(cx)^3\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right) + 4a^3\log(cx) + 6b(a+b\text{sech}^{-1}(cx))^2\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right) + 6b^2(a+b\text{sech}^{-1}(cx))\,\text{PolyLog}\left(3,-e^{-2\text{sech}^{-1}(cx)}\right) + 3b^3\,\text{PolyLog}\left(4,-e^{-2\text{sech}^{-1}(cx)}\right)\Big)$$

input
```
Integrate[(a + b*ArcSech[c*x])^3/x,x]
```

output
```
(-6*a^2*b*ArcSech[c*x]^2 - 4*a*b^2*ArcSech[c*x]^3 - b^3*ArcSech[c*x]^4 - 1
2*a^2*b*ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - 12*a*b^2*ArcSech[c*x]^
2*Log[1 + E^(-2*ArcSech[c*x])] - 4*b^3*ArcSech[c*x]^3*Log[1 + E^(-2*ArcSec
h[c*x])] + 4*a^3*Log[c*x] + 6*b*(a + b*ArcSech[c*x])^2*PolyLog[2, -E^(-2*A
rcSech[c*x])] + 6*b^2*(a + b*ArcSech[c*x])*PolyLog[3, -E^(-2*ArcSech[c*x])
] + 3*b^3*PolyLog[4, -E^(-2*ArcSech[c*x])])/4
```

3.46.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6839, 3042, 26, 4201, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{sech}^{-1}(cx))^3}{x}\,dx$$

$$\Big\downarrow \text{ 6839}$$

$$-\int \sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^3d\text{sech}^{-1}(cx)$$

$\Big\downarrow$ 3042

$$-\int -i\left(a+b\text{sech}^{-1}(cx)\right)^3\tan\left(i\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)$$

$\Big\downarrow$ 26

$$i\int\left(a+b\text{sech}^{-1}(cx)\right)^3\tan\left(i\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)$$

$\Big\downarrow$ 4201

$$i\left(2i\int\frac{e^{2\text{sech}^{-1}(cx)}\left(a+b\text{sech}^{-1}(cx)\right)^3}{1+e^{2\text{sech}^{-1}(cx)}}d\text{sech}^{-1}(cx)-\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^4}{4b}\right)$$

$\Big\downarrow$ 2620

$$i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+b\text{sech}^{-1}(cx)\right)^3-\frac{3}{2}b\int\left(a+b\text{sech}^{-1}(cx)\right)^2\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)d\text{sech}^{-1}(cx)\right)\right.$$

$\Big\downarrow$ 3011

$$i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+b\text{sech}^{-1}(cx)\right)^3-\frac{3}{2}b\left(b\int\left(a+b\text{sech}^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right)d\text{sech}^{-1}\right.\right.\right.$$

$\Big\downarrow$ 7163

$$i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+b\text{sech}^{-1}(cx)\right)^3-\frac{3}{2}b\left(b\left(\frac{1}{2}\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(cx)}\right)\left(a+b\text{sech}^{-1}(cx)\right)-\frac{1}{2}b\int\right.\right.\right.\right.$$

$\Big\downarrow$ 2720

$$i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+b\text{sech}^{-1}(cx)\right)^3-\frac{3}{2}b\left(b\left(\frac{1}{2}\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(cx)}\right)\left(a+b\text{sech}^{-1}(cx)\right)-\frac{1}{4}b\int\right.\right.\right.\right.$$

$\Big\downarrow$ 7143

$$i\left(2i\left(\frac{1}{2}\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+b\text{sech}^{-1}(cx)\right)^3-\frac{3}{2}b\left(b\left(\frac{1}{2}\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(cx)}\right)\left(a+b\text{sech}^{-1}(cx)\right)-\frac{1}{4}b\,\text{P}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcSech[c*x])^3/x,x]
```

3.46. $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x}\,dx$

output
```
I*(((-1/4*I)*(a + b*ArcSech[c*x])^4)/b + (2*I)*(((a + b*ArcSech[c*x])^3*Lo
g[1 + E^(2*ArcSech[c*x])])/2 - (3*b*(-1/2*((a + b*ArcSech[c*x])^2*PolyLog[
2, -E^(2*ArcSech[c*x])]) + b*(((a + b*ArcSech[c*x])*PolyLog[3, -E^(2*ArcSe
ch[c*x])])/2 - (b*PolyLog[4, -E^(2*ArcSech[c*x])])/4)))/2))
```

3.46.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.46. $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x}\,dx$

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.46.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(206) = 412$.

Time = 0.58 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.76

method	result
parts	$a^3 \ln(x) + b^3 \left(\frac{\operatorname{arcsech}(cx)^4}{4} - \operatorname{arcsech}(cx)^3 \ln\left(1 + \left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\right)^2\right) - \frac{3 \operatorname{arcs}}{}\right.$
derivativedivides	$a^3 \ln(cx) + b^3 \left(\frac{\operatorname{arcsech}(cx)^4}{4} - \operatorname{arcsech}(cx)^3 \ln\left(1 + \left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\right)^2\right) - \frac{3 \operatorname{ar}}{}\right.$
default	$a^3 \ln(cx) + b^3 \left(\frac{\operatorname{arcsech}(cx)^4}{4} - \operatorname{arcsech}(cx)^3 \ln\left(1 + \left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\right)^2\right) - \frac{3 \operatorname{ar}}{}\right.$

input
```
int((a+b*arcsech(c*x))^3/x,x,method=_RETURNVERBOSE)
```

output
```
a^3*ln(x)+b^3*(1/4*arcsech(c*x)^4-arcsech(c*x)^3*ln(1+(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))^2)-3/2*arcsech(c*x)^2*polylog(2,-(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))^2)+3/2*arcsech(c*x)*polylog(3,-(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))^2)-3/4*polylog(4,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/
2))^2))+3*a*b^2*(1/3*arcsech(c*x)^3-arcsech(c*x)^2*ln(1+(1/c/x+(-1+1/c/x)^
(1/2)*(1+1/c/x)^(1/2))^2)-arcsech(c*x)*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2))^2)+1/2*polylog(3,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)
)^2))+3*b*a^2*(1/2*arcsech(c*x)^2-arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))^2)-1/2*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/
2))^2))
```

3.46.5 Fricas [F]

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x}\,dx = \int \frac{\left(b\operatorname{arsech}\left(cx\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c*
x) + a^3)/x, x)
```

3.46.6 Sympy [F]

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x}\,dx = \int \frac{\left(a+b\operatorname{asech}\left(cx\right)\right)^3}{x}\,dx$$

input
```
integrate((a+b*asech(c*x))**3/x,x)
```

output
```
Integral((a + b*asech(c*x))**3/x, x)
```

3.46.7 Maxima [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + integrate(b^3*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*
x))^3/x + 3*a*b^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2/x +
 3*a^2*b*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x)
```

3.46.8 Giac [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^3/x, x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x}\,dx = \int \frac{\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))^3/x,x)
```

output
```
int((a + b*acosh(1/(c*x)))^3/x, x)
```

3.47 $\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^2} dx$

3.47.1 Optimal result

Integrand size = 14, antiderivative size = 102

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^2} dx = \frac{6b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{x} - \frac{6b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x}$$

output
```
-6*b^2*(a+b*arcsech(c*x))/x-(a+b*arcsech(c*x))^3/x+6*b^3*(c*x+1)*((-c*x+1)
/(c*x+1))^(1/2)/x+3*b*(c*x+1)*(a+b*arcsech(c*x))^2*((-c*x+1)/(c*x+1))^(1/2
)/x
```

3.47.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^2} dx = -\frac{a^3+6ab^2-3a^2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)-6b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)+3b\left(a^2+2b^2-2ab\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)\operatorname{sech}^{-1}(cx)}{x}$$

input
```
Integrate[(a + b*ArcSech[c*x])^3/x^2,x]
```

output
```
-((a^3 + 6*a*b^2 - 3*a^2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) - 6*b^3*Sqr
t[(1 - c*x)/(1 + c*x)]*(1 + c*x) + 3*b*(a^2 + 2*b^2 - 2*a*b*Sqrt[(1 - c*x)
/(1 + c*x)]*(1 + c*x))*ArcSech[c*x] - 3*b^2*(-a + b*Sqrt[(1 - c*x)/(1 + c*
x)]*(1 + c*x))*ArcSech[c*x]^2 + b^3*ArcSech[c*x]^3)/x)
```

3.47.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6839, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^2}dx$$

$$\Big\downarrow \text{6839}$$

$$-c\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^3}{cx}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$-c\int -i\left(a+b\text{sech}^{-1}(cx)\right)^3\sin\left(i\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{26}$$

$$ic\int \left(a+b\text{sech}^{-1}(cx)\right)^3\sin\left(i\text{sech}^{-1}(cx)\right)d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{3777}$$

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^3}{cx}-3ib\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^3}{cx}-3ib\int \left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3777}$$

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^3}{cx}-3ib\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}-2ib\int -\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{cx}\right.\right.$$

$\downarrow$ 26

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^3}{cx}-3ib\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}-2b\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{cx}d\text{se}\right.\right.$$

$\downarrow$ 3042

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^3}{cx}-3ib\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}-2b\int -i\left(a+b\text{sech}^{-1}(cx)\right)\sin\left(i\text{sech}^{-1}(cx)\right)\right.\right.$$

$\downarrow$ 26

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^3}{cx}-3ib\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}+2ib\int \left(a+b\text{sech}^{-1}(cx)\right)\sin\left(i\text{sech}^{-1}(cx)\right)d\right.\right.$$

$\downarrow$ 3777

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^3}{cx}-3ib\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}+2ib\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)}{cx}-ib\int \frac{1}{cx}d\text{sech}^{-1}\right.\right.\right.$$

$\downarrow$ 3042

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^3}{cx}-3ib\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}+2ib\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)}{cx}-ib\int \sin\left(i\text{sech}^{-}\right.\right.\right.\right.$$

$\downarrow$ 3117

$$ic\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)^3}{cx}-3ib\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}+2ib\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)}{cx}-\frac{ib\sqrt{\frac{1-cx}{cx+1}}(cx+}{cx}\right.\right.\right.$$

input
```
Int[(a + b*ArcSech[c*x])^3/x^2,x]
```

3.47. $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^2}\,dx$

output
```
I*c*((I*(a + b*ArcSech[c*x])^3)/(c*x) - (3*I)*b*((Sqrt[(1 - c*x)/(1 + c*x)
]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(c*x) + (2*I)*b*(((-I)*b*Sqrt[(1 - c*x
)/(1 + c*x)]*(1 + c*x))/(c*x) + (I*(a + b*ArcSech[c*x]))/(c*x))))
```

3.47.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.47.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(98) = 196.

Time = 0.54 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.21

method	result
parts	$-\frac{a^3}{x}+b^3c\left(-\frac{\operatorname{arcsech}(cx)^3}{cx}+3\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\operatorname{arcsech}(cx)^2-\frac{6\operatorname{arcsech}(cx)}{cx}+6\sqrt{-\frac{cx-1}{cx}}\,\sqrt{}\right.$
derivativedivides	$c\left(-\frac{a^3}{cx}+b^3\left(-\frac{\operatorname{arcsech}(cx)^3}{cx}+3\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\operatorname{arcsech}(cx)^2-\frac{6\operatorname{arcsech}(cx)}{cx}+6\sqrt{-\frac{cx-1}{cx}}\,\sqrt{}\right.\right.$
default	$c\left(-\frac{a^3}{cx}+b^3\left(-\frac{\operatorname{arcsech}(cx)^3}{cx}+3\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\operatorname{arcsech}(cx)^2-\frac{6\operatorname{arcsech}(cx)}{cx}+6\sqrt{-\frac{cx-1}{cx}}\,\sqrt{}\right.\right.$

input
```
int((a+b*arcsech(c*x))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3/x+b^3*c*(-1/c/x*arcsech(c*x)^3+3*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(
1/2)*arcsech(c*x)^2-6/c/x*arcsech(c*x)+6*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x
)^(1/2))+3*a*b^2*c*(-1/c/x*arcsech(c*x)^2+2*arcsech(c*x)*(-(c*x-1)/c/x)^(1
/2)*((c*x+1)/c/x)^(1/2)-2/c/x)+3*b*a^2*c*(-1/c/x*arcsech(c*x)+(-(c*x-1)/c/
x)^(1/2)*((c*x+1)/c/x)^(1/2))
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(98) = 196.

Time = 0.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.24

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^2}\,dx =$$
$$-\frac{b^3\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^3 - 3\,(a^2b+2\,b^3)cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + a^3 + 6\,ab^2 - 3\left(b^3cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - ab^2\right)\log\left(cx\sqrt{\cdots}\right.}{x}$$

input
```
integrate((a+b*arcsech(c*x))^3/x^2,x, algorithm="fricas")
```

output
```
-(b^3*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^3 - 3*(a^2*b + 2
*b^3)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + a^3 + 6*a*b^2 - 3*(b^3*c*x*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) - a*b^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
) + 1)/(c*x))^2 - 3*(2*a*b^2*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - a^2*b -
2*b^3)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/x
```

3.47.6 Sympy [F]

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^2}\,dx = \int \frac{(a+b\operatorname{asech}(cx))^3}{x^2}\,dx$$

input
```
integrate((a+b*asech(c*x))**3/x**2,x)
```

output
```
Integral((a + b*asech(c*x))**3/x**2, x)
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.41

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^2}\,dx = -\frac{b^3\operatorname{arsech}(cx)^3}{x} + 3\left(c\sqrt{\frac{1}{c^2x^2}-1}-\frac{\operatorname{arsech}(cx)}{x}\right)a^2b + 6\left(c\sqrt{\frac{1}{c^2x^2}-1}\operatorname{arsech}(cx)-\frac{1}{x}\right)ab^2 + 3\left(c\sqrt{\frac{1}{c^2x^2}-1}\operatorname{arsech}(cx)^2+2c\sqrt{\frac{1}{c^2x^2}-1}-\frac{2\operatorname{arsech}(cx)}{x}\right)b^3 - \frac{3ab^2\operatorname{arsech}(cx)^2}{x}-\frac{a^3}{x}$$

input
```
integrate((a+b*arcsech(c*x))^3/x^2,x, algorithm="maxima")
```

output
```
-b^3*arcsech(c*x)^3/x + 3*(c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*a^2*b
 + 6*(c*sqrt(1/(c^2*x^2) - 1)*arcsech(c*x) - 1/x)*a*b^2 + 3*(c*sqrt(1/(c^2
*x^2) - 1)*arcsech(c*x)^2 + 2*c*sqrt(1/(c^2*x^2) - 1) - 2*arcsech(c*x)/x)*
b^3 - 3*a*b^2*arcsech(c*x)^2/x - a^3/x
```

3.47.8 Giac [F]

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^2}\,dx = \int \frac{(b\operatorname{arsech}(cx)+a)^3}{x^2}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^3/x^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^3/x^2, x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x^2}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))^3/x^2,x)
```

output
```
int((a + b*acosh(1/(c*x)))^3/x^2, x)
```

3.48 $\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x^3}\,dx$

3.48.1 Optimal result

Integrand size = 14, antiderivative size = 163

$$\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x^3}\,dx = \frac{3b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{8x^2}-\frac{3}{8}b^3c^2\mathrm{sech}^{-1}(cx) - \frac{3b^2(1-cx)(1+cx)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4x^2} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{4x^2} - \frac{1}{4}c^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3-\frac{(1-cx)(1+cx)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{2x^2}$$

output
```
-3/8*b^3*c^2*arcsech(c*x)-3/4*b^2*(-c*x+1)*(c*x+1)*(a+b*arcsech(c*x))/x^2-
1/4*c^2*(a+b*arcsech(c*x))^3-1/2*(-c*x+1)*(c*x+1)*(a+b*arcsech(c*x))^3/x^2
+3/8*b^3*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/x^2+3/4*b*(c*x+1)*(a+b*arcsech(c
*x))^2*((-c*x+1)/(c*x+1))^(1/2)/x^2
```

3.48.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.50

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^3}\,dx$$
$$= \frac{-4a^3-6ab^2+3b(2a^2+b^2)\sqrt{\frac{1-cx}{1+cx}}(1+cx)-6b\left(2a^2+b^2-2ab\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)\operatorname{sech}^{-1}(cx)+6b^2\left(b\sqrt{\frac{1-}{1+}}\right.}{}$$

input
```
Integrate[(a + b*ArcSech[c*x])^3/x^3,x]
```

output
```
(-4*a^3 - 6*a*b^2 + 3*b*(2*a^2 + b^2)*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)
- 6*b*(2*a^2 + b^2 - 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*
x] + 6*b^2*(b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + a*(-2 + c^2*x^2))*ArcS
ech[c*x]^2 + 2*b^3*(-2 + c^2*x^2)*ArcSech[c*x]^3 - 3*b*(2*a^2 + b^2)*c^2*x
^2*Log[x] + 3*b*(2*a^2 + b^2)*c^2*x^2*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] +
c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/(8*x^2)
```

3.48.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6839, 5969, 3042, 25, 3792, 17, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^3}\,dx$$

$$\Big\downarrow \text{6839}$$

$$-c^2\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{c^2x^2}d\operatorname{sech}^{-1}(cx)$$

$$\Big\downarrow \text{5969}$$

$$-c^2\left(\frac{(1-cx)(cx+1)\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{2c^2x^2}-\frac{3}{2}b\int \frac{(1-cx)(cx+1)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{c^2x^2}d\operatorname{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^2\left(\frac{(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^3}{2c^2x^2}-\frac{3}{2}b\int -\left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)\right)$$

$\downarrow$ 25

$$-c^2\left(\frac{(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^3}{2c^2x^2}+\frac{3}{2}b\int \left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)\right)$$

$\downarrow$ 3792

$$-c^2\left(\frac{3}{2}b\left(\frac{1}{2}\int \left(a+b\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)+\frac{1}{2}b^2\int -\frac{(1-cx)(cx+1)}{c^2x^2}d\text{sech}^{-1}(cx)+\frac{b(1-cx)(cx+1)\left(a+\right.}{2c^2x^2}\right.\right.$$

$\downarrow$ 17

$$-c^2\left(\frac{3}{2}b\left(\frac{1}{2}b^2\int -\frac{(1-cx)(cx+1)}{c^2x^2}d\text{sech}^{-1}(cx)-\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2x^2}+\frac{b(1-cx)(cx+1)\left(a+b\right.}{2c^2x^2}\right.\right.$$

$\downarrow$ 25

$$-c^2\left(\frac{3}{2}b\left(-\frac{1}{2}b^2\int \frac{(1-cx)(cx+1)}{c^2x^2}d\text{sech}^{-1}(cx)-\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2x^2}+\frac{b(1-cx)(cx+1)\left(a+b\right.}{2c^2x^2}\right.\right.$$

$\downarrow$ 3042

$$-c^2\left(\frac{(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^3}{2c^2x^2}+\frac{3}{2}b\left(-\frac{1}{2}b^2\int -\sin\left(i\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)-\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a\right.}{2c^2x}\right.\right.$$

$\downarrow$ 25

$$-c^2\left(\frac{(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^3}{2c^2x^2}+\frac{3}{2}b\left(\frac{1}{2}b^2\int \sin\left(i\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)-\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+bs\right.}{2c^2x^2}\right.\right.$$

$\downarrow$ 3115

$$-c^2\left(\frac{3}{2}b\left(\frac{1}{2}b^2\left(\frac{1}{2}\int 1d\text{sech}^{-1}(cx)-\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{2c^2x^2}\right)-\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2x^2}+\frac{b(1-cx)(cx+1)}{2c}\right.\right.$$

3.48. $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^3}dx$

$\downarrow$ 24

$$-c^2\left(\frac{3}{2}b\left(-\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2x^2}+\frac{b(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{2c^2x^2}+\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{6b}+\right.\right.$$

input
```
Int[(a + b*ArcSech[c*x])^3/x^3,x]
```

output
```
-(c^2*(((1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x])^3)/(2*c^2*x^2) + (3*b*((b
^2*(-1/2*(Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(c^2*x^2) + ArcSech[c*x]/2)
)/2 + (b*(1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x]))/(2*c^2*x^2) - (Sqrt[(1
- c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(2*c^2*x^2) + (a + b*A
rcSech[c*x])^3/(6*b)))/2))
```

3.48.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.48.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(147) = 294$.

Time = 0.50 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.97

method	result
derivativedivides	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(-\frac{\operatorname{arcsech}(cx)^3}{2c^2x^2}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{4cx}+\frac{\operatorname{arcsech}(cx)^3}{4}-\frac{3\operatorname{arcsech}(cx)}{4c^2x^2}+\frac{3\sqrt{}}{}\right.\right.$
default	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(-\frac{\operatorname{arcsech}(cx)^3}{2c^2x^2}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{4cx}+\frac{\operatorname{arcsech}(cx)^3}{4}-\frac{3\operatorname{arcsech}(cx)}{4c^2x^2}+\frac{3\sqrt{}}{}\right.\right.$
parts	$-\frac{a^3}{2x^2}+b^3c^2\left(-\frac{\operatorname{arcsech}(cx)^3}{2c^2x^2}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{4cx}+\frac{\operatorname{arcsech}(cx)^3}{4}-\frac{3\operatorname{arcsech}(cx)}{4c^2x^2}+\frac{3\sqrt{-\frac{cx}{}}}{}\right.$

input
```
int((a+b*arcsech(c*x))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a^3/c^2/x^2+b^3*(-1/2/c^2/x^2*arcsech(c*x)^3+3/4*(-(c*x-1)/c/x)^
(1/2)*((c*x+1)/c/x)^(1/2)/c/x*arcsech(c*x)^2+1/4*arcsech(c*x)^3-3/4/c^2/x^
2*arcsech(c*x)+3/8*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)/c/x+3/8*arcsec
h(c*x))+3*a*b^2*(-1/2/c^2/x^2*arcsech(c*x)^2+1/2*(-(c*x-1)/c/x)^(1/2)*((c*
x+1)/c/x)^(1/2)/c/x*arcsech(c*x)+1/4*arcsech(c*x)^2-1/4/c^2/x^2)+3*b*a^2*(
-1/2/c^2/x^2*arcsech(c*x)+1/4*(-(c*x-1)/c/x)^(1/2)/c/x*((c*x+1)/c/x)^(1/2)
*(arctanh(1/(-c^2*x^2+1)^(1/2))*c^2*x^2+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(
1/2)))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.66

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^3}\,dx$$

$$= \frac{2\left(b^3c^2x^2-2\,b^3\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^3+3\left(2\,a^2b+b^3\right)cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-4\,a^3-6\,ab^2+6\left(ab^2c^2x^2+b^3cx\sqrt{-\cdots}\right.}{\cdots}$$

input
```
integrate((a+b*arcsech(c*x))^3/x^3,x, algorithm="fricas")
```

output
```
1/8*(2*(b^3*c^2*x^2 - 2*b^3)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/
(c*x))^3 + 3*(2*a^2*b + b^3)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 4*a^3 -
6*a*b^2 + 6*(a*b^2*c^2*x^2 + b^3*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*a*
b^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 + 3*(4*a*b^2*c*
x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + (2*a^2*b + b^3)*c^2*x^2 - 4*a^2*b - 2*b
^3)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/x^2
```

3.48.6 Sympy [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\text{asech}\,(cx)\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*asech(c*x))**3/x**3,x)
```

output
```
Integral((a + b*asech(c*x))**3/x**3, x)
```

3.48.7 Maxima [F]

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^3}{x^3}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)^3}{x^3}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^3/x^3,x, algorithm="maxima")
```

output
```
-3/8*a^2*b*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1)
 - c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) -
 1) - 1))/c + 4*arcsech(c*x)/x^2) - 1/2*a^3/x^2 + integrate(b^3*log(sqrt(1
/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^3/x^3 + 3*a*b^2*log(sqrt(1/(c*x)
+ 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2/x^3, x)
```

3.48.8 Giac [F]

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^3}{x^3}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)^3}{x^3}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^3/x^3, x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^3}{x^3}\,dx = \int \frac{\left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x^3}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))^3/x^3,x)
```

output
```
int((a + b*acosh(1/(c*x)))^3/x^3, x)
```

3.49 $\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^4}\,dx$

3.49.1 Optimal result

Integrand size = 14, antiderivative size = 213

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^4}\,dx = \frac{14b^3c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{9x}+\frac{2b^3\left(\frac{1-cx}{1+cx}\right)^{3/2}(1+cx)^3}{27x^3} - \frac{2b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{9x^3}-\frac{4b^2c^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{3x^3} + \frac{2bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{3x}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{3x^3}$$

output
```
2/27*b^3*((-c*x+1)/(c*x+1))^(3/2)*(c*x+1)^3/x^3-2/9*b^2*(a+b*arcsech(c*x))
/x^3-4/3*b^2*c^2*(a+b*arcsech(c*x))/x-1/3*(a+b*arcsech(c*x))^3/x^3+14/9*b^
3*c^2*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/x+1/3*b*(c*x+1)*(a+b*arcsech(c*x))^
2*((-c*x+1)/(c*x+1))^(1/2)/x^3+2/3*b*c^2*(c*x+1)*(a+b*arcsech(c*x))^2*((-c
*x+1)/(c*x+1))^(1/2)/x
```

3.49.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.20

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^4}\,dx$$
$$= \frac{-9a^3-6ab^2(1+6c^2x^2)+9a^2b\sqrt{\frac{1-cx}{1+cx}}(1+cx+2c^2x^2+2c^3x^3)+2b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx+20c^2x^2+20c^3x^3)-}{}$$

input
```
Integrate[(a + b*ArcSech[c*x])^3/x^4,x]
```

output
```
(-9*a^3 - 6*a*b^2*(1 + 6*c^2*x^2) + 9*a^2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
 c*x + 2*c^2*x^2 + 2*c^3*x^3) + 2*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x +
 20*c^2*x^2 + 20*c^3*x^3) - 3*b*(9*a^2 + 2*b^2*(1 + 6*c^2*x^2) - 6*a*b*Sqr
t[(1 - c*x)/(1 + c*x)]*(1 + c*x + 2*c^2*x^2 + 2*c^3*x^3))*ArcSech[c*x] + 9
*b^2*(-3*a + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x + 2*c^2*x^2 + 2*c^3*x^3)
)*ArcSech[c*x]^2 - 9*b^3*ArcSech[c*x]^3)/(27*x^3)
```

3.49.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.29, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6839, 5970, 3042, 3792, 3042, 3113, 2009, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^4}\,dx$$
$$\Big\downarrow \ 6839$$
$$-c^3\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^3}{c^3x^3}d\text{sech}^{-1}(cx)$$
$$\Big\downarrow \ 5970$$
$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3c^3x^3}-b\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{c^3x^3}d\text{sech}^{-1}(cx)\right)$$

$\downarrow$ 3042

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3c^3x^3}-b\int \left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^3d\text{sech}^{-1}(cx)\right)$$

$\downarrow$ 3792

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3c^3x^3}-b\left(\frac{2}{3}\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}d\text{sech}^{-1}(cx)+\frac{2}{9}b^2\int \frac{1}{c^3x^3}d\text{sech}^{-1}(cx)-\frac{2b(a+b\text{sech}^{-1}(}{9c^3x^3}\right.\right.$$

$\downarrow$ 3042

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3c^3x^3}-b\left(\frac{2}{3}\int \left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(cx)+\frac{2}{9}b^2\int \sin\left(i\text{sech}^{-1}(\right.\right.\right.$$

$\downarrow$ 3113

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3c^3x^3}-b\left(\frac{2}{3}\int \left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(cx)+\frac{2}{9}ib^2\int \left(\frac{(1-cx)(c}{c^2x^2}\right.\right.\right.$$

$\downarrow$ 2009

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3c^3x^3}-b\left(\frac{2}{3}\int \left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(cx)-\frac{2b(a+b\text{sech}^{-1}(cx)}{9c^3x^3}\right.\right.$$

$\downarrow$ 3777

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3c^3x^3}-b\left(\frac{2}{3}\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}-2ib\int -\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(c\right.}{cx}\right.\right.\right.$$

$\downarrow$ 26

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3c^3x^3}-b\left(\frac{2}{3}\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}-2b\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{cx}\right.\right.\right.$$

$\downarrow$ 3042

3.49. $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^4}\,dx$

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3c^3x^3}-b\left(\frac{2}{3}\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}-2b\int -i\left(a+b\text{sech}^{-1}(cx)\right)\sin\left(i\text{sech}^{-1}(c\right.\right.\right.\right.$$

$$\Big\downarrow 26$$

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3c^3x^3}-b\left(\frac{2}{3}\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}+2ib\int\left(a+b\text{sech}^{-1}(cx)\right)\sin\left(i\text{sech}^{-1}(cx\right.\right.\right.\right.$$

$$\Big\downarrow 3777$$

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3c^3x^3}-b\left(\frac{2}{3}\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}+2ib\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)}{cx}-ib\int\frac{1}{cx}d\text{sech}\right.\right.\right.\right.$$

$$\Big\downarrow 3042$$

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3c^3x^3}-b\left(\frac{2}{3}\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{cx}+2ib\left(\frac{i\left(a+b\text{sech}^{-1}(cx)\right)}{cx}-ib\int\sin\left(i\text{sec}\right.\right.\right.\right.\right.$$

$$\Big\downarrow 3117$$

$$-c^3\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3c^3x^3}-b\left(-\frac{2b\left(a+b\text{sech}^{-1}(cx)\right)}{9c^3x^3}+\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{3c^3x^3}+\frac{2}{3}\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1\right.\right.\right.$$

input
```
Int[(a + b*ArcSech[c*x])^3/x^4,x]
```

output
```
-(c^3*((a + b*ArcSech[c*x])^3/(3*c^3*x^3) - b*(((2*I)/9)*b^2*(((-I)*Sqrt[(
1 - c*x)/(1 + c*x)]*(1 + c*x))/(c*x) - ((I/3)*((1 - c*x)/(1 + c*x))^(3/2)*
(1 + c*x)^3)/(c^3*x^3)) - (2*b*(a + b*ArcSech[c*x]))/(9*c^3*x^3) + (Sqrt[(
1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(3*c^3*x^3) + (2*((S
qrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(c*x) + (2*I)*b
*(((-I)*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(c*x) + (I*(a + b*ArcSech[c
*x]))/(c*x))))/3)))
```

3.49. $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^4}\,dx$

3.49.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.49. $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^4}\,dx$

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.49.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(191) = 382$.

Time = 0.83 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.82

method	result
derivativedivides	$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arcsech}(cx)^3}{3c^3x^3}+\frac{2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{3c^2x^2}-\frac{4\ a\right.\right.$
default	$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arcsech}(cx)^3}{3c^3x^3}+\frac{2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{3c^2x^2}-\frac{4\ a\right.\right.$
parts	$-\frac{a^3}{3x^3}+b^3c^3\left(-\frac{\operatorname{arcsech}(cx)^3}{3c^3x^3}+\frac{2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{3c^2x^2}-\frac{4\ \operatorname{arcse}}{3c}\right.$

input
```
int((a+b*arcsech(c*x))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/3*a^3/c^3/x^3+b^3*(-1/3/c^3/x^3*arcsech(c*x)^3+2/3*(-(c*x-1)/c/x)^
(1/2)*((c*x+1)/c/x)^(1/2)*arcsech(c*x)^2+1/3/c^2/x^2*(-(c*x-1)/c/x)^(1/2)*
((c*x+1)/c/x)^(1/2)*arcsech(c*x)^2-4/3/c/x*arcsech(c*x)+40/27*(-(c*x-1)/c/
x)^(1/2)*((c*x+1)/c/x)^(1/2)-2/9/c^3/x^3*arcsech(c*x)+2/27*(-(c*x-1)/c/x)^
(1/2)*((c*x+1)/c/x)^(1/2)/c^2/x^2)+3*a*b^2*(-1/3/c^3/x^3*arcsech(c*x)^2+4/
9*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+2/9/c^2/x^2*(-(c*x
-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*arcsech(c*x)-4/9/c/x-2/27/c^3/x^3)+3*b*
a^2*(-1/3/c^3/x^3*arcsech(c*x)+1/9*(-(c*x-1)/c/x)^(1/2)/c^2/x^2*((c*x+1)/c
/x)^(1/2)*(2*c^2*x^2+1)))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^4}\,dx =$$

$$-\frac{36\,ab^2c^2x^2+9\,b^3\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^3+9\,a^3+6\,ab^2+9\left(3\,ab^2-\left(2\,b^3c^3x^3+b^3cx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}\right)\log\left(\frac{cx}{}\right.}{}$$

input
```
integrate((a+b*arcsech(c*x))^3/x^4,x, algorithm="fricas")
```

output
```
-1/27*(36*a*b^2*c^2*x^2 + 9*b^3*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) +
1)/(c*x))^3 + 9*a^3 + 6*a*b^2 + 9*(3*a*b^2 - (2*b^3*c^3*x^3 + b^3*c*x)*sqr
t(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/
(c*x))^2 + 3*(12*b^3*c^2*x^2 + 9*a^2*b + 2*b^3 - 6*(2*a*b^2*c^3*x^3 + a*b^
2*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2)) + 1)/(c*x)) - (2*(9*a^2*b + 20*b^3)*c^3*x^3 + (9*a^2*b + 2*b^3)*c*x)*
sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^3
```

3.49.6 Sympy [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^4}\,dx = \int \frac{(a+b\,\text{asech}\,(cx))^3}{x^4}\,dx$$

input
```
integrate((a+b*asech(c*x))**3/x**4,x)
```

output
```
Integral((a + b*asech(c*x))**3/x**4, x)
```

3.49.7 Maxima [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^4}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^3/x^4,x, algorithm="maxima")
```

output
```
1/3*a^2*b*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c -
 3*arcsech(c*x)/x^3) - 1/3*a^3/x^3 + integrate(b^3*log(sqrt(1/(c*x) + 1)*s
qrt(1/(c*x) - 1) + 1/(c*x))^3/x^4 + 3*a*b^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(
c*x) - 1) + 1/(c*x))^2/x^4, x)
```

3.49.8 Giac [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^4}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^3/x^4,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^3/x^4, x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^4}\,dx=\int \frac{\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x^4}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))^3/x^4,x)
```

output
```
int((a + b*acosh(1/(c*x)))^3/x^4, x)
```

3.50 $\int \frac{(a+b\text{sech}^{-1}(cx))^3}{x^5} dx$

3.50.1 Optimal result

Integrand size = 14, antiderivative size = 242

$$\int \frac{(a+b\text{sech}^{-1}(cx))^3}{x^5} dx = \frac{3b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{128x^4} + \frac{45b^3c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{256x^2} + \frac{45}{256}b^3c^4\text{sech}^{-1}(cx) - \frac{3b^2(a+b\text{sech}^{-1}(cx))}{32x^4} - \frac{9b^2c^2(a+b\text{sech}^{-1}(cx))}{32x^2} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a+b\text{sech}^{-1}(cx))^2}{16x^4} + \frac{9bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a+b\text{sech}^{-1}(cx))^2}{32x^2} + \frac{3}{32}c^4(a+b\text{sech}^{-1}(cx))^3 - \frac{(a+b\text{sech}^{-1}(cx))^3}{4x^4}$$

output
```
45/256*b^3*c^4*arcsech(c*x)-3/32*b^2*(a+b*arcsech(c*x))/x^4-9/32*b^2*c^2*(
a+b*arcsech(c*x))/x^2+3/32*c^4*(a+b*arcsech(c*x))^3-1/4*(a+b*arcsech(c*x))
^3/x^4+3/128*b^3*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/x^4+45/256*b^3*c^2*(c*x+
1)*((-c*x+1)/(c*x+1))^(1/2)/x^2+3/16*b*(c*x+1)*(a+b*arcsech(c*x))^2*((-c*x
+1)/(c*x+1))^(1/2)/x^4+9/32*b*c^2*(c*x+1)*(a+b*arcsech(c*x))^2*((-c*x+1)/(
c*x+1))^(1/2)/x^2
```

3.50.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.37

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^5}\,dx$$
$$= \frac{-8a(8a^2+3b^2)-72ab^2c^2x^2+3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(8a^2(2+3c^2x^2)+b^2(2+15c^2x^2)\right)-24b\Big(8a^2+b^2(1+3}{}$$

input
```
Integrate[(a + b*ArcSech[c*x])^3/x^5,x]
```

output
```
(-8*a*(8*a^2 + 3*b^2) - 72*a*b^2*c^2*x^2 + 3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(
1 + c*x)*(8*a^2*(2 + 3*c^2*x^2) + b^2*(2 + 15*c^2*x^2)) - 24*b*(8*a^2 + b^
2*(1 + 3*c^2*x^2) - 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x + 3*c^2*x^2
 + 3*c^3*x^3))*ArcSech[c*x] + 24*b^2*(b*Sqrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c
*x + 3*c^2*x^2 + 3*c^3*x^3) + a*(-8 + 3*c^4*x^4))*ArcSech[c*x]^2 + 8*b^3*(
-8 + 3*c^4*x^4)*ArcSech[c*x]^3 - 9*b*(8*a^2 + 5*b^2)*c^4*x^4*Log[x] + 9*b*
(8*a^2 + 5*b^2)*c^4*x^4*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 -
c*x)/(1 + c*x)]])/(256*x^4)
```

3.50.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6839, 5970, 3042, 3792, 3042, 3115, 3042, 3115, 24, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^5}\,dx$$
$$\Big\downarrow \text{6839}$$
$$-c^4\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^3}{c^4x^4}d\text{sech}^{-1}(cx)$$
$$\Big\downarrow \text{5970}$$
$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{c^4x^4}d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \; 3042$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\int \left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^4 d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \; 3792$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{c^2x^2}d\text{sech}^{-1}(cx)+\frac{1}{8}b^2\int \frac{1}{c^4x^4}d\text{sech}^{-1}(cx)-\frac{b\left(a+b\text{sech}^{-1}\right.}{8c^4x^4}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int \left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^2 d\text{sech}^{-1}(cx)+\frac{1}{8}b^2\int \sin\left(i\text{sech}\right.\right.\right.$$

$$\Big\downarrow \; 3115$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int \left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^2 d\text{sech}^{-1}(cx)+\frac{1}{8}b^2\left(\frac{3}{4}\int \frac{1}{c^2x^2}d\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int \left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^2 d\text{sech}^{-1}(cx)+\frac{1}{8}b^2\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx}{4c^4x^4}\right.\right.\right.$$

$$\Big\downarrow \; 3115$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int \left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^2 d\text{sech}^{-1}(cx)+\frac{1}{8}b^2\left(\frac{3}{4}\left(\frac{1}{2}\int 1d\right.\right.\right.\right.$$

$$\Big\downarrow \; 24$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int \left(a+b\text{sech}^{-1}(cx)\right)^2\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^2 d\text{sech}^{-1}(cx)-\frac{b\left(a+b\text{sech}^{-1}(cx\right.}{8c^4x^4}\right.\right.$$

$$\Big\downarrow \; 3792$$

3.50. $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^5}\,dx$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(\frac{1}{2}\int\left(a+b\text{sech}^{-1}(cx)\right)^2d\text{sech}^{-1}(cx)+\frac{1}{2}b^2\int\frac{1}{c^2x^2}d\text{sech}^{-1}(cx)-\frac{b(a+b\text{sec}}{2c^2}\right.\right.\right.$$

$$\downarrow 17$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(\frac{1}{2}b^2\int\frac{1}{c^2x^2}d\text{sech}^{-1}(cx)+\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2x^2}-\frac{b(a+b\text{sech}}{2c^2x}\right.\right.\right.$$

$$\downarrow 3042$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(\frac{1}{2}b^2\int\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)^2d\text{sech}^{-1}(cx)+\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(c\right.}{2c^2x^2}\right.\right.\right.$$

$$\downarrow 3115$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(\frac{1}{2}b^2\left(\frac{1}{2}\int1d\text{sech}^{-1}(cx)+\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{2c^2x^2}\right)+\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}\right.}{2c^2x^2}\right.\right.\right.$$

$$\downarrow 24$$

$$-c^4\left(\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2x^2}-\frac{b\left(a+b\text{sech}^{-1}(cx)\right)}{2c^2x^2}+\frac{\left(a+b\text{sech}^{-1}(cx\right.}{6b}\right.\right.\right.$$

input
```
Int[(a + b*ArcSech[c*x])^3/x^5,x]
```

output
```
-(c^4*((a + b*ArcSech[c*x])^3/(4*c^4*x^4) - (3*b*((b^2*((Sqrt[(1 - c*x)/(1
 + c*x)]*(1 + c*x))/(4*c^4*x^4) + (3*((Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)
)/(2*c^2*x^2) + ArcSech[c*x]/2))/4))/8 - (b*(a + b*ArcSech[c*x]))/(8*c^4*x
^4) + (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(4*c^4*
x^4) + (3*((b^2*((Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(2*c^2*x^2) + ArcSe
ch[c*x]/2))/2 - (b*(a + b*ArcSech[c*x]))/(2*c^2*x^2) + (Sqrt[(1 - c*x)/(1
+ c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(2*c^2*x^2) + (a + b*ArcSech[c*x
])^3/(6*b)))/4))/4))
```

3.50. $\int\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^5}dx$

3.50.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.50. $\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^5}\,dx$

3.50.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. 2(216) = 432.

Time = 0.99 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.00

method	result
derivativedivides	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\operatorname{arcsech}(cx)^3}{4c^4x^4}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{16c^3x^3}+\frac{9\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{32cx}+\frac{3}{}\right.\right.$
default	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\operatorname{arcsech}(cx)^3}{4c^4x^4}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{16c^3x^3}+\frac{9\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{32cx}+\frac{3}{}\right.\right.$
parts	$-\frac{a^3}{4x^4}+b^3c^4\left(-\frac{\operatorname{arcsech}(cx)^3}{4c^4x^4}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{16c^3x^3}+\frac{9\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{32cx}+\frac{3\operatorname{arcse}}{}\right.$

input
```
int((a+b*arcsech(c*x))^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
c^4*(-1/4*a^3/c^4/x^4+b^3*(-1/4/c^4/x^4*arcsech(c*x)^3+3/16/c^3/x^3*(-(c*x
-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*arcsech(c*x)^2+9/32*(-(c*x-1)/c/x)^(1/2
)*((c*x+1)/c/x)^(1/2)/c/x*arcsech(c*x)^2+3/32*arcsech(c*x)^3-3/32/c^4/x^4*
arcsech(c*x)+3/128*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)/c^3/x^3+45/256
*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)/c/x+45/256*arcsech(c*x)-9/32/c^2
/x^2*arcsech(c*x))+3*a*b^2*(-1/4/c^4/x^4*arcsech(c*x)^2+1/8/c^3/x^3*(-(c*x
-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*arcsech(c*x)+3/16*(-(c*x-1)/c/x)^(1/2)*
((c*x+1)/c/x)^(1/2)/c/x*arcsech(c*x)+3/32*arcsech(c*x)^2-1/32/c^4/x^4-3/32
/c^2/x^2)+3*b*a^2*(-1/4/c^4/x^4*arcsech(c*x)+1/32*(-(c*x-1)/c/x)^(1/2)/c^3
/x^3*((c*x+1)/c/x)^(1/2)*(3*arctanh(1/(-c^2*x^2+1)^(1/2))*c^4*x^4+3*(-c^2*
x^2+1)^(1/2)*c^2*x^2+2*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.45

$$\int\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^5}\,dx=$$

$$-\frac{72\,ab^2c^2x^2-8\left(3\,b^3c^4x^4-8\,b^3\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^3+64\,a^3+24\,ab^2-24\left(3\,ab^2c^4x^4-8\,ab^2+\left(3\,b^3c^3\right.\right.}{}$$

input
```
integrate((a+b*arcsech(c*x))^3/x^5,x, algorithm="fricas")
```

output
```
-1/256*(72*a*b^2*c^2*x^2 - 8*(3*b^3*c^4*x^4 - 8*b^3)*log((c*x*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)) + 1)/(c*x))^3 + 64*a^3 + 24*a*b^2 - 24*(3*a*b^2*c^4*x^4
 - 8*a*b^2 + (3*b^3*c^3*x^3 + 2*b^3*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*l
og((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 - 3*(3*(8*a^2*b + 5*b
^3)*c^4*x^4 - 24*b^3*c^2*x^2 - 64*a^2*b - 8*b^3 + 16*(3*a*b^2*c^3*x^3 + 2*
a*b^2*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c
^2*x^2)) + 1)/(c*x)) - 3*(3*(8*a^2*b + 5*b^3)*c^3*x^3 + 2*(8*a^2*b + b^3)*
c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^4
```

3.50.6 Sympy [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^5}\,dx=\int \frac{\left(a+b\operatorname{asech}\left(cx\right)\right)^3}{x^5}\,dx$$

input
```
integrate((a+b*asech(c*x))**3/x**5,x)
```

output
```
Integral((a + b*asech(c*x))**3/x**5, x)
```

3.50.7 Maxima [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^5}\,dx=\int \frac{\left(b\operatorname{arsech}\left(cx\right)+a\right)^3}{x^5}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^3/x^5,x, algorithm="maxima")
```

output
```
3/64*a^2*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) - 3*c^5*log(c*x*sqrt
(1/(c^2*x^2) - 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) - 1)^(3/2) - 5*c^6*x*sq
rt(1/(c^2*x^2) - 1))/(c^4*x^4*(1/(c^2*x^2) - 1)^2 - 2*c^2*x^2*(1/(c^2*x^2)
 - 1) + 1))/c - 16*arcsech(c*x)/x^4) - 1/4*a^3/x^4 + integrate(b^3*log(sqr
t(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^3/x^5 + 3*a*b^2*log(sqrt(1/(c*
x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2/x^5, x)
```

3.50.8 Giac [F]

$$\int \frac{(a+b\text{sech}^{-1}(cx))^3}{x^5}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arcsech(c*x))^3/x^5,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^3/x^5, x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{sech}^{-1}(cx))^3}{x^5}\,dx=\int \frac{\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x^5}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))^3/x^5,x)
```

output
```
int((a + b*acosh(1/(c*x)))^3/x^5, x)
```

3.51 $\int \frac{x}{a+b\text{sech}^{-1}(cx)} dx$

3.51.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{a+b\text{sech}^{-1}(cx)} dx = \text{Int}\left(\frac{x}{a+b\text{sech}^{-1}(cx)}, x\right)$$

output
```
Unintegrable(x/(a+b*arcsech(c*x)),x)
```

3.51.2 Mathematica [N/A]

Not integrable

Time = 2.99 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\text{sech}^{-1}(cx)} dx = \int \frac{x}{a+b\text{sech}^{-1}(cx)} dx$$

input
```
Integrate[x/(a + b*ArcSech[c*x]),x]
```

output
```
Integrate[x/(a + b*ArcSech[c*x]), x]
```

3.51.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a + b\text{sech}^{-1}(cx)} \, dx$$

$$\Big\downarrow \text{6865}$$

$$\int \frac{x}{a + b\text{sech}^{-1}(cx)} dx$$

input
```
Int[x/(a + b*ArcSech[c*x]),x]
```

output
```
$Aborted
```

3.51.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.51.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{a + b \,\mathrm{arcsech}\,(cx)} dx$$

input
```
int(x/(a+b*arcsech(c*x)),x)
```

output
```
int(x/(a+b*arcsech(c*x)),x)
```

3.51.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{x}{b\,\text{arsech}\,(cx)+a}\,dx$$

input
```
integrate(x/(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
integral(x/(b*arcsech(c*x) + a), x)
```

3.51.6 Sympy [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{x}{a+b\,\text{asech}\,(cx)}\,dx$$

input
```
integrate(x/(a+b*asech(c*x)),x)
```

output
```
Integral(x/(a + b*asech(c*x)), x)
```

3.51.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{x}{b\,\text{arsech}\,(cx)+a}\,dx$$

input
```
integrate(x/(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
integrate(x/(b*arcsech(c*x) + a), x)
```

3.51.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a + b\text{sech}^{-1}(cx)} \, dx = \int \frac{x}{b \operatorname{arsech}(cx) + a} \, dx$$

input
```
integrate(x/(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate(x/(b*arcsech(c*x) + a), x)
```

3.51.9 Mupad [N/A]

Not integrable

Time = 4.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{a + b\text{sech}^{-1}(cx)} \, dx = \int \frac{x}{a + b \operatorname{acosh}\left(\frac{1}{c x}\right)} \, dx$$

input
```
int(x/(a + b*acosh(1/(c*x))),x)
```

output
```
int(x/(a + b*acosh(1/(c*x))), x)
```

3.52 $\int \frac{1}{a+b\text{sech}^{-1}(cx)} dx$

3.52.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{a+b\text{sech}^{-1}(cx)} dx = \text{Int}\left(\frac{1}{a+b\text{sech}^{-1}(cx)}, x\right)$$

output
```
Unintegrable(1/(a+b*arcsech(c*x)),x)
```

3.52.2 Mathematica [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\text{sech}^{-1}(cx)} dx = \int \frac{1}{a+b\text{sech}^{-1}(cx)} dx$$

input
```
Integrate[(a + b*ArcSech[c*x])^(-1),x]
```

output
```
Integrate[(a + b*ArcSech[c*x])^(-1), x]
```

3.52.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\text{sech}^{-1}(cx)} \, dx$$
$$\Big\downarrow \text{ 6865}$$
$$\int \frac{1}{a+b\text{sech}^{-1}(cx)}dx$$

input
```
Int[(a + b*ArcSech[c*x])^(-1),x]
```

output
```
$Aborted
```

3.52.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.52.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b \,\mathrm{arcsech}\left(c x \right)}d x$$

input
```
int(1/(a+b*arcsech(c*x)),x)
```

output
```
int(1/(a+b*arcsech(c*x)),x)
```

3.52.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{1}{b\,\text{arsech}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*arcsech(c*x) + a), x)
```

3.52.6 Sympy [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{1}{a+b\,\text{asech}\,(cx)}\,dx$$

input
```
integrate(1/(a+b*asech(c*x)),x)
```

output
```
Integral(1/(a + b*asech(c*x)), x)
```

3.52.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{1}{b\,\text{arsech}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arcsech(c*x) + a), x)
```

3.52.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\mathrm{sech}^{-1}(cx)}\,dx = \int \frac{1}{b\,\mathrm{arsech}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(b*arcsech(c*x) + a), x)
```

3.52.9 Mupad [N/A]

Not integrable

Time = 3.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{a+b\mathrm{sech}^{-1}(cx)}\,dx = \int \frac{1}{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}\,dx$$

input
```
int(1/(a + b*acosh(1/(c*x))),x)
```

output
```
int(1/(a + b*acosh(1/(c*x))), x)
```

3.53 $\int \frac{1}{x\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}\,dx$

3.53.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}\,dx=\mathrm{Int}\left(\frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)},x\right)$$

output
```
Unintegrable(1/x/(a+b*arcsech(c*x)),x)
```

3.53.2 Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcSech[c*x])),x]
```

output
```
Integrate[1/(x*(a + b*ArcSech[c*x])), x]
```

3.53.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx$$

$$\Big\downarrow \text{6865}$$

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}dx$$

input
```
Int[1/(x*(a + b*ArcSech[c*x])),x]
```

output
```
$Aborted
```

3.53.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.53.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\,\operatorname{arcsech}\left(cx\right)\right)}dx$$

input
```
int(1/x/(a+b*arcsech(c*x)),x)
```

output
```
int(1/x/(a+b*arcsech(c*x)),x)
```

3.53.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\text{arsech}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x*arcsech(c*x) + a*x), x)
```

3.53.6 Sympy [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{x\,(a+b\,\text{asech}\,(cx))}\,dx$$

input
```
integrate(1/x/(a+b*asech(c*x)),x)
```

output
```
Integral(1/(x*(a + b*asech(c*x))), x)
```

3.53.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\text{arsech}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsech(c*x) + a)*x), x)
```

3.53.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\text{arsech}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((b*arcsech(c*x) + a)*x), x)
```

3.53.9 Mupad [N/A]

Not integrable

Time = 3.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{x\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

input
```
int(1/(x*(a + b*acosh(1/(c*x)))),x)
```

output
```
int(1/(x*(a + b*acosh(1/(c*x)))), x)
```

3.54 $\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)}\,dx$

3.54.1 Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)}\,dx = \frac{c\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b}-\frac{c\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}$$

output
```
-c*cosh(a/b)*Shi(a/b+arcsech(c*x))/b+c*Chi(a/b+arcsech(c*x))*sinh(a/b)/b
```

3.54.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)}\,dx = \frac{c\left(\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)}{b}$$

input
```
Integrate[1/(x^2*(a + b*ArcSech[c*x])),x]
```

output
```
(c*(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/
b + ArcSech[c*x]]))/b
```

3.54.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6839, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b\text{sech}^{-1}(cx)\right)} \, dx$$

$$\Big\downarrow \text{ 6839}$$

$$-c\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{cx\left(a+b\text{sech}^{-1}(cx)\right)}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{ 3042}$$

$$-c\int -\frac{i\sin\left(i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{ 26}$$

$$ic\int \frac{\sin\left(i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{ 3784}$$

$$ic\left(\cosh\left(\frac{a}{b}\right)\int \frac{i\sinh\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 26}$$

$$ic\left(i\cosh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$ic\left(i\cosh\left(\frac{a}{b}\right)\int -\frac{i\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 26}$$

$$ic\left(\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3779}$$

$$ic\left(\frac{i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}-i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3782}$$

$$ic\left(\frac{i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}-\frac{i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}\right)$$

input
```
Int[1/(x^2*(a + b*ArcSech[c*x])),x]
```

output
```
I*c*(((-I)*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/b + (I*Cosh[a/b]*Si
nhIntegral[a/b + ArcSech[c*x]])/b)
```

3.54.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.54.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$c\left(-\frac{\mathrm{e}^{\frac{a}{b}}\operatorname{Ei}_1\left(\frac{a}{b}+\operatorname{arcsech}(cx)\right)}{2b}+\frac{\mathrm{e}^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsech}(cx)-\frac{a}{b}\right)}{2b}\right)$	54
default	$c\left(-\frac{\mathrm{e}^{\frac{a}{b}}\operatorname{Ei}_1\left(\frac{a}{b}+\operatorname{arcsech}(cx)\right)}{2b}+\frac{\mathrm{e}^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsech}(cx)-\frac{a}{b}\right)}{2b}\right)$	54

input
```
int(1/x^2/(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
c*(-1/2/b*exp(a/b)*Ei(1,a/b+arcsech(c*x))+1/2/b*exp(-a/b)*Ei(1,-arcsech(c*
x)-a/b))
```

3.54.5 Fricas [F]

$$\int \frac{1}{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\operatorname{arsech}(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^2*arcsech(c*x) + a*x^2), x)
```

3.54.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\left(a+b\operatorname{asech}\left(cx\right)\right)}\,dx$$

input
```
integrate(1/x**2/(a+b*asech(c*x)),x)
```

output
```
Integral(1/(x**2*(a + b*asech(c*x))), x)
```

3.54.7 Maxima [F]

$$\int \frac{1}{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\operatorname{arsech}\left(cx\right)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsech(c*x) + a)*x^2), x)
```

3.54.8 Giac [F]

$$\int \frac{1}{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\operatorname{arsech}\left(cx\right)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((b*arcsech(c*x) + a)*x^2), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a + b\text{sech}^{-1}(cx)\right)} \, dx = \int \frac{1}{x^2 \left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)} \, dx$$

input
```
int(1/(x^2*(a + b*acosh(1/(c*x)))),x)
```

output
```
int(1/(x^2*(a + b*acosh(1/(c*x)))), x)
```

3.55 $\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)}\,dx$

3.55.1 Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)}\,dx = \frac{c^2\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2b} - \frac{c^2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{2b}$$

output
```
-1/2*c^2*cosh(2*a/b)*Shi(2*a/b+2*arcsech(c*x))/b+1/2*c^2*Chi(2*a/b+2*arcse
ch(c*x))*sinh(2*a/b)/b
```

3.55.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)}\,dx = \frac{c^2\left(\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)\right)}{2b}$$

input
```
Integrate[1/(x^3*(a + b*ArcSech[c*x])),x]
```

output
```
(c^2*(CoshIntegral[(2*a)/b + 2*ArcSech[c*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/b]
*SinhIntegral[(2*a)/b + 2*ArcSech[c*x]]))/(2*b)
```

3.55.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6839, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b\text{sech}^{-1}(cx)\right)} \, dx$$

$$\Big\downarrow \text{6839}$$

$$-c^2 \int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{c^2x^2 \left(a+b\text{sech}^{-1}(cx)\right)}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{5971}$$

$$-c^2 \int \frac{\sinh \left(2\text{sech}^{-1}(cx)\right)}{2 \left(a+b\text{sech}^{-1}(cx)\right)}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}c^2 \int \frac{\sinh \left(2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}c^2 \int -\frac{i \sin \left(2i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}ic^2 \int \frac{\sin \left(2i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{3784}$$

$$\frac{1}{2}ic^2\left(\cosh\left(\frac{2a}{b}\right)\int \frac{i \sinh \left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh \left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx\right.$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}ic^2\left(i\cosh\left(\frac{2a}{b}\right)\int \frac{\sinh \left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh \left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx\right.$$

$$\downarrow 3042$$

$$\frac{1}{2}ic^2\left(i\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sec}$$

$$\downarrow 26$$

$$\frac{1}{2}ic^2\left(\cosh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}($$

$$\downarrow 3779$$

$$\frac{1}{2}ic^2\left(\frac{i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b}-i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)$$

$$\downarrow 3782$$

$$\frac{1}{2}ic^2\left(\frac{i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b}-\frac{i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b}\right)$$

input
```
Int[1/(x^3*(a + b*ArcSech[c*x])),x]
```

output
```
(I/2)*c^2*(((-I)*CoshIntegral[(2*a)/b + 2*ArcSech[c*x]]*Sinh[(2*a)/b])/b +
 (I*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSech[c*x]])/b)
```

3.55.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.55.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$c^{2}\left(-\frac{\mathrm{e}^{\frac{2a}{b}}\operatorname{Ei}_{1}\left(\frac{2a}{b}+2\,\mathrm{arcsech}(cx)\right)}{4b}+\frac{\mathrm{e}^{-\frac{2a}{b}}\operatorname{Ei}_{1}\left(-2\,\mathrm{arcsech}(cx)-\frac{2a}{b}\right)}{4b}\right)$	60
default	$c^{2}\left(-\frac{\mathrm{e}^{\frac{2a}{b}}\operatorname{Ei}_{1}\left(\frac{2a}{b}+2\,\mathrm{arcsech}(cx)\right)}{4b}+\frac{\mathrm{e}^{-\frac{2a}{b}}\operatorname{Ei}_{1}\left(-2\,\mathrm{arcsech}(cx)-\frac{2a}{b}\right)}{4b}\right)$	60

input
```
int(1/x^3/(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/4/b*exp(2*a/b)*Ei(1,2*a/b+2*arcsech(c*x))+1/4/b*exp(-2*a/b)*Ei(1,-
2*arcsech(c*x)-2*a/b))
```

3.55.5 Fricas [F]

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\,\text{arsech}\,(cx)+a)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^3*arcsech(c*x) + a*x^3), x)
```

3.55.6 Sympy [F]

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)}\,dx = \int \frac{1}{x^3\left(a+b\,\text{asech}\,(cx)\right)}\,dx$$

input
```
integrate(1/x**3/(a+b*asech(c*x)),x)
```

output
```
Integral(1/(x**3*(a + b*asech(c*x))), x)
```

3.55.7 Maxima [F]

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\,\text{arsech}\,(cx)+a)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsech(c*x) + a)*x^3), x)
```

3.55.8 Giac [F]

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\text{arsech}\,(cx)+a)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((b*arcsech(c*x) + a)*x^3), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{x^3\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

input
```
int(1/(x^3*(a + b*acosh(1/(c*x)))),x)
```

output
```
int(1/(x^3*(a + b*acosh(1/(c*x)))), x)
```

3.56 $\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)}\,dx$

3.56.1 Optimal result

Integrand size = 14, antiderivative size = 117

$$\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)}\,dx = \frac{c^3\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4b} + \frac{c^3\text{Chi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{4b} - \frac{c^3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{4b} - \frac{c^3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)}{4b}$$

output
```
-1/4*c^3*cosh(a/b)*Shi(a/b+arcsech(c*x))/b-1/4*c^3*cosh(3*a/b)*Shi(3*a/b+3
*arcsech(c*x))/b+1/4*c^3*Chi(a/b+arcsech(c*x))*sinh(a/b)/b+1/4*c^3*Chi(3*a
/b+3*arcsech(c*x))*sinh(3*a/b)/b
```

3.56.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)}\,dx = -\frac{c^3\left(-\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-\text{Chi}\left(3\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right.}{4b}$$

input
```
Integrate[1/(x^4*(a + b*ArcSech[c*x])),x]
```

output
```
-1/4*(c^3*(-(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b]) - CoshIntegral[3*
(a/b + ArcSech[c*x])]*Sinh[(3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSech
[c*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSech[c*x])]))/b
```

3.56.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6839, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)} \, dx$$

$$\Big\downarrow \text{6839}$$

$$-c^3 \int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{c^3x^3 \left(a+b\text{sech}^{-1}(cx)\right)}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{5971}$$

$$-c^3 \int \left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{4cx \left(a+b\text{sech}^{-1}(cx)\right)}+\frac{\sinh \left(3\text{sech}^{-1}(cx)\right)}{4 \left(a+b\text{sech}^{-1}(cx)\right)}\right)d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{2009}$$

$$-c^3 \left(-\frac{\sinh \left(\frac{a}{b}\right) \text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{4b}-\frac{\sinh \left(\frac{3a}{b}\right) \text{Chi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)}{4b}+\frac{\cosh \left(\frac{a}{b}\right) \text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{4b}+\text{cosl}\right.$$

input
```
Int[1/(x^4*(a + b*ArcSech[c*x])),x]
```

output
```
-(c^3*(-1/4*(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/b - (CoshIntegral
[(3*a)/b + 3*ArcSech[c*x]]*Sinh[(3*a)/b])/(4*b) + (Cosh[a/b]*SinhIntegral[
a/b + ArcSech[c*x]])/(4*b) + (Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSe
ch[c*x]])/(4*b)))
```

3.56.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.56.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94

method	result
derivativedivides	$c^3\left(-\frac{e^{\frac{3a}{b}}\operatorname{Ei}_1\left(\frac{3a}{b}+3\operatorname{arcsech}(cx)\right)}{8b}-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\frac{a}{b}+\operatorname{arcsech}(cx)\right)}{8b}+\frac{e^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsech}(cx)-\frac{a}{b}\right)}{8b}+\frac{e^{-\frac{3a}{b}}\operatorname{Ei}_1\left(-3\operatorname{arc}\right.}{8b}\right.$
default	$c^3\left(-\frac{e^{\frac{3a}{b}}\operatorname{Ei}_1\left(\frac{3a}{b}+3\operatorname{arcsech}(cx)\right)}{8b}-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\frac{a}{b}+\operatorname{arcsech}(cx)\right)}{8b}+\frac{e^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsech}(cx)-\frac{a}{b}\right)}{8b}+\frac{e^{-\frac{3a}{b}}\operatorname{Ei}_1\left(-3\operatorname{arc}\right.}{8b}\right.$

input
```
int(1/x^4/(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/8/b*exp(3*a/b)*Ei(1,3*a/b+3*arcsech(c*x))-1/8/b*exp(a/b)*Ei(1,a/b+
arcsech(c*x))+1/8/b*exp(-a/b)*Ei(1,-arcsech(c*x)-a/b)+1/8/b*exp(-3*a/b)*Ei
(1,-3*arcsech(c*x)-3*a/b))
```

3.56.5 Fricas [F]

$$\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\text{arsech}\,(cx)+a)x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^4*arcsech(c*x) + a*x^4), x)
```

3.56.6 Sympy [F]

$$\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{x^4\left(a+b\,\text{asech}\,(cx)\right)}\,dx$$

input
```
integrate(1/x**4/(a+b*asech(c*x)),x)
```

output
```
Integral(1/(x**4*(a + b*asech(c*x))), x)
```

3.56.7 Maxima [F]

$$\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\text{arsech}\,(cx)+a)x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsech(c*x) + a)*x^4), x)
```

3.56.8 Giac [F]

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)} \, dx = \int \frac{1}{(b \operatorname{arsech}(cx)+a)x^4} \, dx$$

input
```
integrate(1/x^4/(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((b*arcsech(c*x) + a)*x^4), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)} \, dx = \int \frac{1}{x^4 \left(a+b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)} \, dx$$

input
```
int(1/(x^4*(a + b*acosh(1/(c*x)))),x)
```

output
```
int(1/(x^4*(a + b*acosh(1/(c*x)))), x)
```

3.57 $\int \frac{x}{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}\,dx$

3.57.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}\,dx = \text{Int}\left(\frac{x}{\left(a+b\,\text{sech}^{-1}(cx)\right)^2},x\right)$$

output
```
Unintegrable(x/(a+b*arcsech(c*x))^2,x)
```

3.57.2 Mathematica [N/A]

Not integrable

Time = 19.99 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}\,dx = \int \frac{x}{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}\,dx$$

input
```
Integrate[x/(a + b*ArcSech[c*x])^2,x]
```

output
```
Integrate[x/(a + b*ArcSech[c*x])^2, x]
```

3.57.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$$
$$\Big\downarrow \text{ 6865}$$
$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^2}dx$$

input
```
Int[x/(a + b*ArcSech[c*x])^2,x]
```

output
```
$Aborted
```

3.57.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.57.4 Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+b\,\operatorname{arcsech}\left(cx\right)\right)^{2}}dx$$

input
```
int(x/(a+b*arcsech(c*x))^2,x)
```

output
```
int(x/(a+b*arcsech(c*x))^2,x)
```

3.57.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{x}{(a+b\text{sech}^{-1}(cx))^2}\,dx = \int \frac{x}{(b\,\text{arsech}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

output
```
integral(x/(b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2), x)
```

3.57.6 Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\text{sech}^{-1}(cx))^2}\,dx = \int \frac{x}{(a+b\,\text{asech}\,(cx))^2}\,dx$$

input
```
integrate(x/(a+b*asech(c*x))**2,x)
```

output
```
Integral(x/(a + b*asech(c*x))**2, x)
```

3.57.7 Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 546, normalized size of antiderivative = 45.50

$$\int \frac{x}{(a+b\text{sech}^{-1}(cx))^2}\,dx = \int \frac{x}{(b\,\text{arsech}\,(cx)+a)^2}\,dx$$

input
```
integrate(x/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x + (c^2*x^3 - x)*x)/((b^2*c^
2*log(c) - a*b*c^2)*x^2 - b^2*log(c) - (b^2*log(c) + b^2*log(x) - a*b)*sqr
t(c*x + 1)*sqrt(-c*x + 1) + a*b - (b^2*c^2*x^2 - sqrt(c*x + 1)*sqrt(-c*x +
 1)*b^2 - b^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^2*c^2*x^2 - b^2)
*log(x)) + integrate((2*(2*c^2*x^2 - 1)*(c*x + 1)*(c*x - 1)*x + (3*c^4*x^4
 - 8*c^2*x^2 + 4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x + 2*(c^4*x^4 - 2*c^2*x^2
+ 1)*x)/((b^2*c^4*log(c) - a*b*c^4)*x^4 - (b^2*log(c) + b^2*log(x) - a*b)*
(c*x + 1)*(c*x - 1) - 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - 2*((
b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b + (b^2*c^2*x^2 - b^2)*log
(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - a*b - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 - (
c*x + 1)*(c*x - 1)*b^2 - 2*(b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1
) + b^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^2*c^4*x^4 - 2*b^2*c^2*
x^2 + b^2)*log(x)), x)
```

3.57.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^2} \, dx = \int \frac{x}{\left(b\,\text{arsech}\,(cx)+a\right)^2} \, dx$$

input
```
integrate(x/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x/(b*arcsech(c*x) + a)^2, x)
```

3.57.9 Mupad [N/A]

Not integrable

Time = 4.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^2} \, dx = \int \frac{x}{\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2} \, dx$$

input
```
int(x/(a + b*acosh(1/(c*x)))^2,x)
```

output
```
int(x/(a + b*acosh(1/(c*x)))^2, x)
```

3.58 $\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2} dx$

3.58.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2} dx = \text{Int}\left(\frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}, x\right)$$

output
```
Unintegrable(1/(a+b*arcsech(c*x))^2,x)
```

3.58.2 Mathematica [N/A]

Not integrable

Time = 76.82 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2} dx = \int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2} dx$$

input
```
Integrate[(a + b*ArcSech[c*x])^(-2),x]
```

output
```
Integrate[(a + b*ArcSech[c*x])^(-2), x]
```

3.58.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2} \, dx$$
$$\Big\downarrow \ 6865$$
$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}dx$$

input
```
Int[(a + b*ArcSech[c*x])^(-2),x]
```

output
```
$Aborted
```

3.58.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.58.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\,\operatorname{arcsech}\left(c x\right)\right)^{2}}d x$$

input
```
int(1/(a+b*arcsech(c*x))^2,x)
```

output
```
int(1/(a+b*arcsech(c*x))^2,x)
```

3.58.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \frac{1}{\left(a + b\text{sech}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\text{arsech}\,(cx) + a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2), x)
```

3.58.6 Sympy [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a + b\text{sech}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(a + b\,\text{asech}\,(cx)\right)^2}\,dx$$

input
```
integrate(1/(a+b*asech(c*x))**2,x)
```

output
```
Integral((a + b*asech(c*x))**(-2), x)
```

3.58.7 Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 535, normalized size of antiderivative = 53.50

$$\int \frac{1}{\left(a + b\text{sech}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\text{arsech}\,(cx) + a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*log(
c) - a*b*c^2)*x^2 - b^2*log(c) - (b^2*log(c) + b^2*log(x) - a*b)*sqrt(c*x
+ 1)*sqrt(-c*x + 1) + a*b - (b^2*c^2*x^2 - sqrt(c*x + 1)*sqrt(-c*x + 1)*b^
2 - b^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^2*c^2*x^2 - b^2)*log(x
)) + integrate((c^4*x^4 - 2*c^2*x^2 + (3*c^2*x^2 - 1)*(c*x + 1)*(c*x - 1)
+ (2*c^4*x^4 - 5*c^2*x^2 + 2)*sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/((b^2*c^4*
log(c) - a*b*c^4)*x^4 - (b^2*log(c) + b^2*log(x) - a*b)*(c*x + 1)*(c*x - 1
) - 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - 2*((b^2*c^2*log(c) - a
*b*c^2)*x^2 - b^2*log(c) + a*b + (b^2*c^2*x^2 - b^2)*log(x))*sqrt(c*x + 1)
*sqrt(-c*x + 1) - a*b - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 - (c*x + 1)*(c*x - 1)
*b^2 - 2*(b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) + b^2)*log(sqrt(
c*x + 1)*sqrt(-c*x + 1) + 1) + (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(x))
, x)
```

3.58.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^(-2), x)
```

3.58.9 Mupad [N/A]

Not integrable

Time = 3.96 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

input
```
int(1/(a + b*acosh(1/(c*x)))^2,x)
```

output
```
int(1/(a + b*acosh(1/(c*x)))^2, x)
```

3.59 $\int \frac{1}{x\left(a+b\operatorname{sech}^{-1}(cx)\right)^2} dx$

3.59.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\operatorname{sech}^{-1}(cx)\right)^2} dx = \operatorname{Int}\left(\frac{1}{x\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}, x\right)$$

output
```
Unintegrable(1/x/(a+b*arcsech(c*x))^2,x)
```

3.59.2 Mathematica [N/A]

Not integrable

Time = 7.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\operatorname{sech}^{-1}(cx)\right)^2} dx = \int \frac{1}{x\left(a+b\operatorname{sech}^{-1}(cx)\right)^2} dx$$

input
```
Integrate[1/(x*(a + b*ArcSech[c*x])^2),x]
```

output
```
Integrate[1/(x*(a + b*ArcSech[c*x])^2), x]
```

3.59.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left(a+b\text{sech}^{-1}(cx)\right)^2} \, dx$$

$$\Big\downarrow \ 6865$$

$$\int \frac{1}{x \left(a+b\text{sech}^{-1}(cx)\right)^2}dx$$

input
```
Int[1/(x*(a + b*ArcSech[c*x])^2),x]
```

output
```
$Aborted
```

3.59.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.59.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \left(a+b \,\operatorname{arcsech}\left(c x\right)\right)^{2}}d x$$

input
```
int(1/x/(a+b*arcsech(c*x))^2,x)
```

output
```
int(1/x/(a+b*arcsech(c*x))^2,x)
```

3.59.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{arsech}\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x*arcsech(c*x)^2 + 2*a*b*x*arcsech(c*x) + a^2*x), x)
```

3.59.6 Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\left(a+b\,\text{asech}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(1/x/(a+b*asech(c*x))**2,x)
```

output
```
Integral(1/(x*(a + b*asech(c*x))**2), x)
```

3.59.7 Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 544, normalized size of antiderivative = 38.86

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{arsech}\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*x^2
- b^2)*x*log(x) - (b^2*x*log(x) + (b^2*log(c) - a*b)*x)*sqrt(c*x + 1)*sqrt
(-c*x + 1) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x + (sqrt
(c*x + 1)*sqrt(-c*x + 1)*b^2*x - (b^2*c^2*x^2 - b^2)*x)*log(sqrt(c*x + 1)*
sqrt(-c*x + 1) + 1)) + integrate(-(2*(c*x + 1)*(c*x - 1)*c^2*x^2 + (c^4*x^
4 - 2*c^2*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1))/((b^2*x*log(x) + (b^2*log(c)
- a*b)*x)*(c*x + 1)*(c*x - 1) - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x*log(
x) + 2*((b^2*c^2*x^2 - b^2)*x*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b
^2*log(c) + a*b)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((b^2*c^4*log(c) - a*b*
c^4)*x^4 - 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)*x - ((c*x
+ 1)*(c*x - 1)*b^2*x + 2*(b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*
x - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1
) + 1)), x)
```

3.59.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{arsech}\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*arcsech(c*x) + a)^2*x), x)
```

3.59.9 Mupad [N/A]

Not integrable

Time = 3.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

input
```
int(1/(x*(a + b*acosh(1/(c*x)))^2),x)
```

output
```
int(1/(x*(a + b*acosh(1/(c*x)))^2), x)
```

3.60 $\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2} dx$

3.60.1 Optimal result

Integrand size = 14, antiderivative size = 86

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2} dx = \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{bx\left(a+b\text{sech}^{-1}(cx)\right)} - \frac{c\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b^2} + \frac{c\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b^2}$$

output
```
-c*Chi(a/b+arcsech(c*x))*cosh(a/b)/b^2+c*Shi(a/b+arcsech(c*x))*sinh(a/b)/b
^2+(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/b/x/(a+b*arcsech(c*x))
```

3.60.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2} dx = \frac{\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{x\left(a+b\text{sech}^{-1}(cx)\right)} - c\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right) + c\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b^2}$$

input
```
Integrate[1/(x^2*(a + b*ArcSech[c*x])^2),x]
```

output
```
((b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x*(a + b*ArcSech[c*x])) - c*Cosh
[a/b]*CoshIntegral[a/b + ArcSech[c*x]] + c*Sinh[a/b]*SinhIntegral[a/b + Ar
cSech[c*x]])/b^2
```

3.60.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6839, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2} \, dx$$

$$\Big\downarrow \text{6839}$$

$$-c\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{cx\left(a+b\text{sech}^{-1}(cx)\right)^2}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$-c\int -\frac{i\sin\left(i\text{sech}^{-1}(cx)\right)}{\left(a+b\text{sech}^{-1}(cx)\right)^2}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{26}$$

$$ic\int \frac{\sin\left(i\text{sech}^{-1}(cx)\right)}{\left(a+b\text{sech}^{-1}(cx)\right)^2}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{3778}$$

$$ic\left(\frac{i\int \frac{1}{cx\left(a+b\text{sech}^{-1}(cx)\right)}d\text{sech}^{-1}(cx)}{b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{bcx\left(a+b\text{sech}^{-1}(cx)\right)}\right)$$

$$\Big\downarrow \text{3042}$$

$$ic\left(\frac{i\int \frac{\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)}{b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{bcx\left(a+b\text{sech}^{-1}(cx)\right)}\right)$$

$$\Big\downarrow \text{3784}$$

$$ic\left(\frac{i\left(\cosh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)+i\sinh\left(\frac{a}{b}\right)\int\frac{i\sinh\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx}{bcx\left(a+b\text{sech}\right.}\right.$$

$$\Big\downarrow \text{26}$$

$$ic\left(\frac{i\left(\cosh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-\sinh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+}{bcx\left(a+b\text{sech}^{-}\right.}\right.$$

$$\Big\downarrow \text{3042}$$

$$ic\left(\frac{i\left(\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-\sinh\left(\frac{a}{b}\right)\int-\frac{i\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}}{bcx\left(a+b\right.}\right.$$

$$\Big\downarrow \text{26}$$

$$ic\left(\frac{i\left(\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)+i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}(}{bcx\left(a+b\text{se}\right.}\right.$$

$$\Big\downarrow \text{3779}$$

$$ic\left(\frac{i\left(-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{bcx\left(a+b\text{sech}^{-1}(cx)\right)}\right)$$

$$\Big\downarrow \text{3782}$$

3.60. $\displaystyle\int\frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$

$$ic\left(\frac{i\left(\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}\right)}{b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{bcx\left(a+b\text{sech}^{-1}(cx)\right)}\right)$$

input
```
Int[1/(x^2*(a + b*ArcSech[c*x])^2),x]
```

output
```
I*c*(((-I)*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(b*c*x*(a + b*ArcSech[c*x]
)) + (I*((Cosh[a/b]*CoshIntegral[a/b + ArcSech[c*x]])/b - (Sinh[a/b]*SinhI
ntegral[a/b + ArcSech[c*x]])/b))/b)
```

3.60.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.60.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.91

method	result
derivativedivides	$c\left(\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}-1}{2cxb(a+b\,\mathrm{arcsech}(cx))}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\frac{a}{b}+\mathrm{arcsech}(cx)\right)}{2b^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+1}{2bcx(a+b\,\mathrm{arcsech}(cx))}+\frac{\mathrm{e}^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\,\mathrm{arcsech}(cx)-\frac{a}{b}\right)}{2b^2}\right)$
default	$c\left(\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}-1}{2cxb(a+b\,\mathrm{arcsech}(cx))}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\frac{a}{b}+\mathrm{arcsech}(cx)\right)}{2b^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+1}{2bcx(a+b\,\mathrm{arcsech}(cx))}+\frac{\mathrm{e}^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\,\mathrm{arcsech}(cx)-\frac{a}{b}\right)}{2b^2}\right)$

input
```
int(1/x^2/(a+b*arcsech(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
c*(1/2*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)-1)/c/x/b/(a+b*arcsech
(c*x))+1/2/b^2*exp(a/b)*Ei(1,a/b+arcsech(c*x))+1/2/b*((-(c*x-1)/c/x)^(1/2)
*c*x*((c*x+1)/c/x)^(1/2)+1)/c/x/(a+b*arcsech(c*x))+1/2/b^2*exp(-a/b)*Ei(1,
-arcsech(c*x)-a/b))
```

3.60.5 Fricas [F]

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{arsech}\left(cx\right)+a\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^2*arcsech(c*x)^2 + 2*a*b*x^2*arcsech(c*x) + a^2*x^2), x)
```

3.60.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\operatorname{asech}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(1/x**2/(a+b*asech(c*x))**2,x)
```

output
```
Integral(1/(x**2*(a + b*asech(c*x))**2), x)
```

3.60.7 Maxima [F]

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{arsech}\left(cx\right)+a\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*x^2
- b^2)*x^2*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^
2 - (b^2*x^2*log(x) + (b^2*log(c) - a*b)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)
 + (sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*x^2 - (b^2*c^2*x^2 - b^2)*x^2)*log(sq
rt(c*x + 1)*sqrt(-c*x + 1) + 1)) + integrate(-(c^4*x^4 - 2*c^2*x^2 - (c^2*
x^2 + 1)*(c*x + 1)*(c*x - 1) - (c^2*x^2 - 2)*sqrt(c*x + 1)*sqrt(-c*x + 1)
+ 1)/((b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x^2*log(x) - (b^2*x^2*log(x) + (
b^2*log(c) - a*b)*x^2)*(c*x + 1)*(c*x - 1) + ((b^2*c^4*log(c) - a*b*c^4)*x
^4 - 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)*x^2 - 2*((b^2*c^
2*x^2 - b^2)*x^2*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a
*b)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) + ((c*x + 1)*(c*x - 1)*b^2*x^2 + 2*(
b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2 - (b^2*c^4*x^4 - 2*b^2
*c^2*x^2 + b^2)*x^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

3.60.8 Giac [F]

$$\int \frac{1}{x^2 \left(a + b\text{sech}^{-1}(cx)\right)^2} \, dx = \int \frac{1}{(b \operatorname{arsech}(cx) + a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*arcsech(c*x) + a)^2*x^2), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a + b\text{sech}^{-1}(cx)\right)^2} \, dx = \int \frac{1}{x^2 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)^2} \, dx$$

input
```
int(1/(x^2*(a + b*acosh(1/(c*x)))^2),x)
```

output
```
int(1/(x^2*(a + b*acosh(1/(c*x)))^2), x)
```

3.61 $\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$

3.61.1 Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=-\frac{c^2\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b^2}+\frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{2b\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{c^2\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b^2}$$

output
```
-c^2*Chi(2*a/b+2*arcsech(c*x))*cosh(2*a/b)/b^2+c^2*Shi(2*a/b+2*arcsech(c*x
))*sinh(2*a/b)/b^2+1/2*c^2*sinh(2*arcsech(c*x))/b/(a+b*arcsech(c*x))
```

3.61.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\frac{\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{x^2\left(a+b\text{sech}^{-1}(cx)\right)}-c^2\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)+c^2\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)}{b^2}$$

input
```
Integrate[1/(x^3*(a + b*ArcSech[c*x])^2),x]
```

output
```
((b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x^2*(a + b*ArcSech[c*x])) - c^2*
Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSech[c*x])] + c^2*Sinh[(2*a)/b]*Sin
hIntegral[2*(a/b + ArcSech[c*x])])/b^2
```

3.61.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6839, 5971, 27, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b\text{sech}^{-1}(cx)\right)^2} \, dx$$

$$\Big\downarrow \text{ 6839}$$

$$-c^2 \int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{c^2x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{ 5971}$$

$$-c^2 \int \frac{\sinh \left(2\text{sech}^{-1}(cx)\right)}{2 \left(a+b\text{sech}^{-1}(cx)\right)^2}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{2}c^2 \int \frac{\sinh \left(2\text{sech}^{-1}(cx)\right)}{\left(a+b\text{sech}^{-1}(cx)\right)^2}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{2}c^2 \int -\frac{i \sin \left(2i\text{sech}^{-1}(cx)\right)}{\left(a+b\text{sech}^{-1}(cx)\right)^2}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{2}ic^2 \int \frac{\sin \left(2i\text{sech}^{-1}(cx)\right)}{\left(a+b\text{sech}^{-1}(cx)\right)^2}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{2}ic^2\left(\frac{2i\int \frac{\cosh\left(2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)}{b}-\frac{i\sinh\left(2\text{sech}^{-1}(cx)\right)}{b\left(a+b\text{sech}^{-1}(cx)\right)}\right)$$

$\downarrow$ 3042

$$\frac{1}{2}ic^2\left(\frac{2i\int \frac{\sin\left(2i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)}{b}-\frac{i\sinh\left(2\text{sech}^{-1}(cx)\right)}{b\left(a+b\text{sech}^{-1}(cx)\right)}\right)$$

$\downarrow$ 3784

$$\frac{1}{2}ic^2\left(\frac{2i\left(\cosh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)+i\sinh\left(\frac{2a}{b}\right)\int \frac{i\sinh\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}-\frac{i\sinh}{b\,(a}\right.$$

$\downarrow$ 26

$$\frac{1}{2}ic^2\left(\frac{2i\left(\cosh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-\sinh\left(\frac{2a}{b}\right)\int \frac{\sinh\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}-\frac{i\sinh(}{b\,(a+}\right.$$

$\downarrow$ 3042

$$\frac{1}{2}ic^2\left(\frac{2i\left(\cosh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-\sinh\left(\frac{2a}{b}\right)\int -\frac{i\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}-\frac{i}{b}\right.$$

$\downarrow$ 26

$$\frac{1}{2}ic^2\left(\frac{2i\left(\cosh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)+i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}-\frac{i\,\text{si}}{b\,(}\right.$$

$\downarrow$ 3779

3.61. $\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$

$$\frac{1}{2}ic^2\left(\frac{2i\left(-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b}+\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}-\frac{i\sinh\left(2\text{sech}^{-1}(cx)\right)}{b\left(a+b\text{sech}^{-1}(cx)\right)}\right.$$

$$\Big\downarrow \text{3782}$$

$$\frac{1}{2}ic^2\left(\frac{2i\left(\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b}\right)}{b}-\frac{i\sinh\left(2\text{sech}^{-1}(cx)\right)}{b\left(a+b\text{sech}^{-1}(cx)\right)}\right)$$

input
```
Int[1/(x^3*(a + b*ArcSech[c*x])^2),x]
```

output
```
(I/2)*c^2*(((-I)*Sinh[2*ArcSech[c*x]])/(b*(a + b*ArcSech[c*x])) + ((2*I)*(
(Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSech[c*x]])/b - (Sinh[(2*a)/b]*
SinhIntegral[(2*a)/b + 2*ArcSech[c*x]])/b))/b)
```

3.61.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.61.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(83) = 166$.

Time = 0.76 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.19

method	result
derivativedivides	$c^2 \left(\frac{2\sqrt{-\frac{cx-1}{cx}}\, cx \sqrt{\frac{cx+1}{cx}} + c^2x^2 - 2}{4c^2x^2 b(a+b\,\text{arcsech}(cx))} + \frac{e^{\frac{2a}{b}}\, \text{Ei}_1\left(\frac{2a}{b} + 2\,\text{arcsech}(cx)\right)}{2b^2} - \frac{c^2x^2 - 2 - 2\sqrt{-\frac{cx-1}{cx}}\, cx\sqrt{\frac{cx+1}{cx}}}{4b\,c^2x^2(a+b\,\text{arcsech}(cx))} + \frac{e^{-\frac{2a}{b}}\, \text{Ei}_1(\right.$
default	$c^2 \left(\frac{2\sqrt{-\frac{cx-1}{cx}}\, cx \sqrt{\frac{cx+1}{cx}} + c^2x^2 - 2}{4c^2x^2 b(a+b\,\text{arcsech}(cx))} + \frac{e^{\frac{2a}{b}}\, \text{Ei}_1\left(\frac{2a}{b} + 2\,\text{arcsech}(cx)\right)}{2b^2} - \frac{c^2x^2 - 2 - 2\sqrt{-\frac{cx-1}{cx}}\, cx\sqrt{\frac{cx+1}{cx}}}{4b\,c^2x^2(a+b\,\text{arcsech}(cx))} + \frac{e^{-\frac{2a}{b}}\, \text{Ei}_1(\right.$

input
```
int(1/x^3/(a+b*arcsech(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(1/4*(2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+c^2*x^2-2)/c^2/x^
2/b/(a+b*arcsech(c*x))+1/2/b^2*exp(2*a/b)*Ei(1,2*a/b+2*arcsech(c*x))-1/4/b
*(c^2*x^2-2-2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2))/c^2/x^2/(a+b*a
rcsech(c*x))+1/2/b^2*exp(-2*a/b)*Ei(1,-2*arcsech(c*x)-2*a/b))
```

3.61.5 Fricas [F]

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\text{arsech}\left(cx\right)+a\right)^2x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^3*arcsech(c*x)^2 + 2*a*b*x^3*arcsech(c*x) + a^2*x^3), x)
```

3.61.6 Sympy [F]

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x^3\left(a+b\,\text{asech}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(1/x**3/(a+b*asech(c*x))**2,x)
```

output
```
Integral(1/(x**3*(a + b*asech(c*x))**2), x)
```

3.61.7 Maxima [F]

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\text{arsech}\left(cx\right)+a\right)^2x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*x^2
- b^2)*x^3*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^
3 - (b^2*x^3*log(x) + (b^2*log(c) - a*b)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1)
 + (sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*x^3 - (b^2*c^2*x^2 - b^2)*x^3)*log(sq
rt(c*x + 1)*sqrt(-c*x + 1) + 1)) + integrate(-(2*c^4*x^4 - 4*c^2*x^2 - 2*(
c*x + 1)*(c*x - 1) + (c^4*x^4 - 4*c^2*x^2 + 4)*sqrt(c*x + 1)*sqrt(-c*x + 1
) + 2)/((b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x^3*log(x) + ((b^2*c^4*log(c)
- a*b*c^4)*x^4 - 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)*x^3
- (b^2*x^3*log(x) + (b^2*log(c) - a*b)*x^3)*(c*x + 1)*(c*x - 1) - 2*((b^2*
c^2*x^2 - b^2)*x^3*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) +
 a*b)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1) + ((c*x + 1)*(c*x - 1)*b^2*x^3 + 2
*(b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^3 - (b^2*c^4*x^4 - 2*b
^2*c^2*x^2 + b^2)*x^3)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

3.61.8 Giac [F]

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^2x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*arcsech(c*x) + a)^2*x^3), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\,\text{acosh}\left(\frac{1}{cx}\right)\right)^2}\,dx$$

input
```
int(1/(x^3*(a + b*acosh(1/(c*x)))^2),x)
```

output
```
int(1/(x^3*(a + b*acosh(1/(c*x)))^2), x)
```

3.62 $\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$

3.62.1 Optimal result

Integrand size = 14, antiderivative size = 190

$$\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx = \frac{c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4bx\left(a+b\text{sech}^{-1}(cx)\right)} - \frac{c^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{4b^2} - \frac{3c^3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)}{4b^2} + \frac{c^3\sinh\left(3\text{sech}^{-1}(cx)\right)}{4b\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{c^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{4b^2} + \frac{3c^3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)}{4b^2}$$

output
```
-1/4*c^3*Chi(a/b+arcsech(c*x))*cosh(a/b)/b^2-3/4*c^3*Chi(3*a/b+3*arcsech(c
*x))*cosh(3*a/b)/b^2+1/4*c^3*Shi(a/b+arcsech(c*x))*sinh(a/b)/b^2+3/4*c^3*S
hi(3*a/b+3*arcsech(c*x))*sinh(3*a/b)/b^2+1/4*c^3*sinh(3*arcsech(c*x))/b/(a
+b*arcsech(c*x))+1/4*c^2*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/b/x/(a+b*arcsech
(c*x))
```

3.62.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^2} \, dx$$
$$= \frac{4b\sqrt{\frac{1-cx}{1+cx}}+4bcx\sqrt{\frac{1-cx}{1+cx}}-c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)-3c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)$$

input
```
Integrate[1/(x^4*(a + b*ArcSech[c*x])^2),x]
```

output
```
(4*b*Sqrt[(1 - c*x)/(1 + c*x)] + 4*b*c*x*Sqrt[(1 - c*x)/(1 + c*x)] - c^3*x
^3*(a + b*ArcSech[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcSech[c*x]] - 3*c^3
*x^3*(a + b*ArcSech[c*x])*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSech[c*x]
)] + a*c^3*x^3*Sinh[a/b]*SinhIntegral[a/b + ArcSech[c*x]] + b*c^3*x^3*ArcS
ech[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSech[c*x]] + 3*a*c^3*x^3*Sinh[(3*
a)/b]*SinhIntegral[3*(a/b + ArcSech[c*x])] + 3*b*c^3*x^3*ArcSech[c*x]*Sinh
[(3*a)/b]*SinhIntegral[3*(a/b + ArcSech[c*x])])/(4*b^2*x^3*(a + b*ArcSech[
c*x]))
```

3.62.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6839, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^2} \, dx$$
$$\Big\downarrow \text{ 6839}$$
$$-c^3 \int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}d\text{sech}^{-1}(cx)$$
$$\Big\downarrow \text{ 5971}$$

$$-c^3\int \left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{4cx\left(a+b\text{sech}^{-1}(cx)\right)^2}+\frac{\sinh\left(3\text{sech}^{-1}(cx)\right)}{4\left(a+b\text{sech}^{-1}(cx)\right)^2}\right)d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{2009}$$

$$-c^3\left(\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{4b^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)}{4b^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{4b^2}-\frac{3\sin\right.$$

input
```
Int[1/(x^4*(a + b*ArcSech[c*x])^2),x]
```

output
```
-(c^3*(-1/4*(Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(b*c*x*(a + b*ArcSech[c*
x])) + (Cosh[a/b]*CoshIntegral[a/b + ArcSech[c*x]])/(4*b^2) + (3*Cosh[(3*a
)/b]*CoshIntegral[(3*a)/b + 3*ArcSech[c*x]])/(4*b^2) - Sinh[3*ArcSech[c*x]
]/(4*b*(a + b*ArcSech[c*x])) - (Sinh[a/b]*SinhIntegral[a/b + ArcSech[c*x]]
)/(4*b^2) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSech[c*x]])/(4*b^
2)))
```

3.62.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.62.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(176) = 352$.

Time = 0.94 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.21

method	result
derivativedivides	$c^3\left(-\frac{\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}\,c^3x^3-4\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}-3c^2x^2+4}{8c^3x^3b(a+b\,\mathrm{arcsech}(cx))}+\frac{3\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(\frac{3a}{b}+3\,\mathrm{arcsech}(cx)\right)}{8b^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\ldots}}{8cxb(a+b\,\mathrm{arcse}\ldots}\right.$
default	$c^3\left(-\frac{\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}\,c^3x^3-4\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}-3c^2x^2+4}{8c^3x^3b(a+b\,\mathrm{arcsech}(cx))}+\frac{3\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(\frac{3a}{b}+3\,\mathrm{arcsech}(cx)\right)}{8b^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\ldots}}{8cxb(a+b\,\mathrm{arcse}\ldots}\right.$

input
```
int(1/x^4/(a+b*arcsech(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/8*(((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x^3-4*(-(c*x-1)/c/
x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)-3*c^2*x^2+4)/c^3/x^3/b/(a+b*arcsech(c*x))
+3/8/b^2*exp(3*a/b)*Ei(1,3*a/b+3*arcsech(c*x))+1/8*((-(c*x-1)/c/x)^(1/2)*c
*x*((c*x+1)/c/x)^(1/2)-1)/c/x/b/(a+b*arcsech(c*x))+1/8/b^2*exp(a/b)*Ei(1,a
/b+arcsech(c*x))+1/8/b*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+1)/c/
x/(a+b*arcsech(c*x))+1/8/b^2*exp(-a/b)*Ei(1,-arcsech(c*x)-a/b)-1/8/b*(((c*
x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x^3-4*(-(c*x-1)/c/x)^(1/2)*c*x*((
c*x+1)/c/x)^(1/2)+3*c^2*x^2-4)/c^3/x^3/(a+b*arcsech(c*x))+3/8/b^2*exp(-3*a
/b)*Ei(1,-3*arcsech(c*x)-3*a/b))
```

3.62.5 Fricas [F]

$$\int \frac{1}{x^4\left(a+b\mathrm{sech}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\mathrm{arsech}\left(cx\right)+a\right)^2x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^4*arcsech(c*x)^2 + 2*a*b*x^4*arcsech(c*x) + a^2*x^4), x)
```

3.62.6 Sympy [F]

$$\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x^4\left(a+b\operatorname{asech}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(1/x**4/(a+b*asech(c*x))**2,x)
```

output
```
Integral(1/(x**4*(a + b*asech(c*x))**2), x)
```

3.62.7 Maxima [F]

$$\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{arsech}\left(cx\right)+a\right)^2x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*x^2
- b^2)*x^4*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^
4 - (b^2*x^4*log(x) + (b^2*log(c) - a*b)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)
 + (sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*x^4 - (b^2*c^2*x^2 - b^2)*x^4)*log(sq
rt(c*x + 1)*sqrt(-c*x + 1) + 1)) - integrate((3*c^4*x^4 - 6*c^2*x^2 + (c^2
*x^2 - 3)*(c*x + 1)*(c*x - 1) + (2*c^4*x^4 - 7*c^2*x^2 + 6)*sqrt(c*x + 1)*
sqrt(-c*x + 1) + 3)/((b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x^4*log(x) + ((b^
2*c^4*log(c) - a*b*c^4)*x^4 - 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c
) - a*b)*x^4 - (b^2*x^4*log(x) + (b^2*log(c) - a*b)*x^4)*(c*x + 1)*(c*x -
1) - 2*((b^2*c^2*x^2 - b^2)*x^4*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 -
 b^2*log(c) + a*b)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1) + ((c*x + 1)*(c*x - 1
)*b^2*x^4 + 2*(b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^4 - (b^2*
c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x^4)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1))
, x)
```

3.62.8 Giac [F]

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{(b\,\text{arsech}\,(cx)+a)^2x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*arcsech(c*x) + a)^2*x^4), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x^4\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

input
```
int(1/(x^4*(a + b*acosh(1/(c*x)))^2),x)
```

output
```
int(1/(x^4*(a + b*acosh(1/(c*x)))^2), x)
```

3.63 $\int \frac{x}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx$

3.63.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx = \mathrm{Int}\left(\frac{x}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3},x\right)$$

output
```
Unintegrable(x/(a+b*arcsech(c*x))^3,x)
```

3.63.2 Mathematica [N/A]

Not integrable

Time = 5.99 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx = \int \frac{x}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx$$

input
```
Integrate[x/(a + b*ArcSech[c*x])^3,x]
```

output
```
Integrate[x/(a + b*ArcSech[c*x])^3, x]
```

3.63.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx$$
$$\Big\downarrow \text{ 6865}$$
$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^3}dx$$

input
```
Int[x/(a + b*ArcSech[c*x])^3,x]
```

output
```
$Aborted
```

3.63.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.63.4 Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+b \,\operatorname{arcsech}\left(c x \right)\right)^{3}}d x$$

input
```
int(x/(a+b*arcsech(c*x))^3,x)
```

output
```
int(x/(a+b*arcsech(c*x))^3,x)
```

3.63.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.50

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{x}{\left(b\,\text{arsech}\,(cx)+a\right)^3}\,dx$$

input
```
integrate(x/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

output
```
integral(x/(b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(
c*x) + a^3), x)
```

3.63.6 Sympy [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{x}{\left(a+b\,\text{asech}\,(cx)\right)^3}\,dx$$

input
```
integrate(x/(a+b*asech(c*x))**3,x)
```

output
```
Integral(x/(a + b*asech(c*x))**3, x)
```

3.63.7 Maxima [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 2818, normalized size of antiderivative = 234.83

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{x}{\left(b\,\text{arsech}\,(cx)+a\right)^3}\,dx$$

input
```
integrate(x/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

output
```
-1/2*((2*(2*b*c^4*x^5 - 3*b*c^2*x^3 + b*x)*x*log(x) + (4*(b*c^4*log(c) - a
*c^4)*x^5 - (b*c^2*(6*log(c) + 1) - 6*a*c^2)*x^3 + (b*(2*log(c) + 1) - 2*a
)*x)*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + (3*(b*c^6*x^7 - 5*b*c^4*x^5 + 6
*b*c^2*x^3 - 2*b*x)*x*log(x) + (3*(b*c^6*log(c) - a*c^6)*x^7 - (b*c^4*(15*
log(c) + 2) - 15*a*c^4)*x^5 + (b*c^2*(18*log(c) + 5) - 18*a*c^2)*x^3 - 3*(
b*(2*log(c) + 1) - 2*a)*x)*x)*(c*x + 1)*(c*x - 1) - 2*(b*c^6*x^7 - 3*b*c^4
*x^5 + 3*b*c^2*x^3 - b*x)*x*log(x) - ((5*b*c^6*x^7 - 17*b*c^4*x^5 + 18*b*c
^2*x^3 - 6*b*x)*x*log(x) + ((b*c^6*(5*log(c) + 1) - 5*a*c^6)*x^7 - (b*c^4*
(17*log(c) + 5) - 17*a*c^4)*x^5 + (b*c^2*(18*log(c) + 7) - 18*a*c^2)*x^3 -
 3*(b*(2*log(c) + 1) - 2*a)*x)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((b*c^6*(
2*log(c) + 1) - 2*a*c^6)*x^7 - 3*(b*c^4*(2*log(c) + 1) - 2*a*c^4)*x^5 + 3*
(b*c^2*(2*log(c) + 1) - 2*a*c^2)*x^3 - (b*(2*log(c) + 1) - 2*a)*x)*x - (2*
(2*b*c^4*x^5 - 3*b*c^2*x^3 + b*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2)*x + 3*(
b*c^6*x^7 - 5*b*c^4*x^5 + 6*b*c^2*x^3 - 2*b*x)*(c*x + 1)*(c*x - 1)*x - (5*
b*c^6*x^7 - 17*b*c^4*x^5 + 18*b*c^2*x^3 - 6*b*x)*sqrt(c*x + 1)*sqrt(-c*x +
 1)*x - 2*(b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*x)*log(sqrt(c*x +
1)*sqrt(-c*x + 1) + 1))/((b^4*c^6*log(c)^2 - 2*a*b^3*c^6*log(c) + a^2*b^2*
c^6)*x^6 - b^4*log(c)^2 - 3*(b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a^2*b
^2*c^4)*x^4 + 2*a*b^3*log(c) - (b^4*log(c)^2 + b^4*log(x)^2 - 2*a*b^3*log(
c) + a^2*b^2 + 2*(b^4*log(c) - a*b^3)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1...
```

3.63.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{x}{\left(b\,\text{arsech}\,(cx)+a\right)^3}\,dx$$

input
```
integrate(x/(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

output
```
integrate(x/(b*arcsech(c*x) + a)^3, x)
```

3.63.9 Mupad [N/A]

Not integrable

Time = 4.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{x}{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

input
```
int(x/(a + b*acosh(1/(c*x)))^3,x)
```

output
```
int(x/(a + b*acosh(1/(c*x)))^3, x)
```

3.64 $\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3} dx$

3.64.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3} dx = \text{Int}\left(\frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3}, x\right)$$

output
```
Unintegrable(1/(a+b*arcsech(c*x))^3,x)
```

3.64.2 Mathematica [N/A]

Not integrable

Time = 93.53 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3} dx = \int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3} dx$$

input
```
Integrate[(a + b*ArcSech[c*x])^(-3),x]
```

output
```
Integrate[(a + b*ArcSech[c*x])^(-3), x]
```

3.64.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx$$
$$\Big\downarrow \text{ 6865}$$
$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3}dx$$

input
```
Int[(a + b*ArcSech[c*x])^(-3),x]
```

output
```
$Aborted
```

3.64.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.64.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\,\text{arcsech}\left(cx\right)\right)^{3}}dx$$

input
```
int(1/(a+b*arcsech(c*x))^3,x)
```

output
```
int(1/(a+b*arcsech(c*x))^3,x)
```

3.64.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 4.00

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\text{arsech}\left(cx\right)+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(
c*x) + a^3), x)
```

3.64.6 Sympy [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(a+b\,\text{asech}\left(cx\right)\right)^3}\,dx$$

input
```
integrate(1/(a+b*asech(c*x))**3,x)
```

output
```
Integral((a + b*asech(c*x))**(-3), x)
```

3.64.7 Maxima [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 2771, normalized size of antiderivative = 277.10

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\text{arsech}\left(cx\right)+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

output
```
1/2*((b*c^6*(log(c) + 1) - a*c^6)*x^7 - 3*(b*c^4*(log(c) + 1) - a*c^4)*x^5
 - (3*(b*c^4*log(c) - a*c^4)*x^5 - (b*c^2*(4*log(c) + 1) - 4*a*c^2)*x^3 +
(b*(log(c) + 1) - a)*x + (3*b*c^4*x^5 - 4*b*c^2*x^3 + b*x)*log(x))*(c*x +
1)^(3/2)*(-c*x + 1)^(3/2) + 3*(b*c^2*(log(c) + 1) - a*c^2)*x^3 - (2*(b*c^6
*log(c) - a*c^6)*x^7 - 2*(b*c^4*(5*log(c) + 1) - 5*a*c^4)*x^5 + (b*c^2*(11
*log(c) + 5) - 11*a*c^2)*x^3 - 3*(b*(log(c) + 1) - a)*x + (2*b*c^6*x^7 - 1
0*b*c^4*x^5 + 11*b*c^2*x^3 - 3*b*x)*log(x))*(c*x + 1)*(c*x - 1) + ((b*c^6*
(3*log(c) + 1) - 3*a*c^6)*x^7 - 5*(b*c^4*(2*log(c) + 1) - 2*a*c^4)*x^5 + (
b*c^2*(10*log(c) + 7) - 10*a*c^2)*x^3 - 3*(b*(log(c) + 1) - a)*x + (3*b*c^
6*x^7 - 10*b*c^4*x^5 + 10*b*c^2*x^3 - 3*b*x)*log(x))*sqrt(c*x + 1)*sqrt(-c
*x + 1) - (b*(log(c) + 1) - a)*x - (b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3
- (3*b*c^4*x^5 - 4*b*c^2*x^3 + b*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - (2*
b*c^6*x^7 - 10*b*c^4*x^5 + 11*b*c^2*x^3 - 3*b*x)*(c*x + 1)*(c*x - 1) + (3*
b*c^6*x^7 - 10*b*c^4*x^5 + 10*b*c^2*x^3 - 3*b*x)*sqrt(c*x + 1)*sqrt(-c*x +
 1) - b*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b*c^6*x^7 - 3*b*c^4*x^
5 + 3*b*c^2*x^3 - b*x)*log(x))/((b^4*c^6*log(c)^2 - 2*a*b^3*c^6*log(c) + a
^2*b^2*c^6)*x^6 - b^4*log(c)^2 - 3*(b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c)
+ a^2*b^2*c^4)*x^4 + 2*a*b^3*log(c) - (b^4*log(c)^2 + b^4*log(x)^2 - 2*a*b
^3*log(c) + a^2*b^2 + 2*(b^4*log(c) - a*b^3)*log(x))*(c*x + 1)^(3/2)*(-c*x
 + 1)^(3/2) - a^2*b^2 + 3*(b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2 - (b...
```

3.64.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^(-3), x)
```

3.64.9 Mupad [N/A]

Not integrable

Time = 4.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

input
```
int(1/(a + b*acosh(1/(c*x)))^3,x)
```

output
```
int(1/(a + b*acosh(1/(c*x)))^3, x)
```

3.65 $\int \frac{1}{x\left(a+b\mathbf{sech}^{-1}(cx)\right)^3}\,dx$

3.65.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx = \text{Int}\left(\frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^3},x\right)$$

output
```
Unintegrable(1/x/(a+b*arcsech(c*x))^3,x)
```

3.65.2 Mathematica [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx = \int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcSech[c*x])^3),x]
```

output
```
Integrate[1/(x*(a + b*ArcSech[c*x])^3), x]
```

3.65.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx$$
$$\Big\downarrow \text{ 6865}$$
$$\int \frac{1}{x \left(a+b\text{sech}^{-1}(cx)\right)^3}dx$$

input
```
Int[1/(x*(a + b*ArcSech[c*x])^3),x]
```

output
```
$Aborted
```

3.65.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.65.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \left(a+b \,\operatorname{arcsech}\left(c x\right)\right)^{3}}d x$$

input
```
int(1/x/(a+b*arcsech(c*x))^3,x)
```

output
```
int(1/x/(a+b*arcsech(c*x))^3,x)
```

3.65.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 3.21

$$\int \frac{1}{x\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arsech}\left(cx\right)+a\right)^3x}\,dx$$

input
```
integrate(1/x/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x*arcsech(c*x)^3 + 3*a*b^2*x*arcsech(c*x)^2 + 3*a^2*b*x*ar
csech(c*x) + a^3*x), x)
```

3.65.6 Sympy [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x\left(a+b\,\mathrm{asech}\left(cx\right)\right)^3}\,dx$$

input
```
integrate(1/x/(a+b*asech(c*x))**3,x)
```

output
```
Integral(1/(x*(a + b*asech(c*x))**3), x)
```

3.65.7 Maxima [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 2638, normalized size of antiderivative = 188.43

$$\int \frac{1}{x\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arsech}\left(cx\right)+a\right)^3x}\,dx$$

input
```
integrate(1/x/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

output
```
-1/2*(b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - (2*(b*c^4*log(c) - a*c^4)*x^
5 - (b*c^2*(2*log(c) + 1) - 2*a*c^2)*x^3 + b*x + 2*(b*c^4*x^5 - b*c^2*x^3)
*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - ((b*c^6*log(c) - a*c^6)*x^7 -
(b*c^4*(5*log(c) + 2) - 5*a*c^4)*x^5 + (b*c^2*(4*log(c) + 5) - 4*a*c^2)*x^
3 - 3*b*x + (b*c^6*x^7 - 5*b*c^4*x^5 + 4*b*c^2*x^3)*log(x))*(c*x + 1)*(c*x
 - 1) + ((b*c^6*(log(c) + 1) - a*c^6)*x^7 - (b*c^4*(3*log(c) + 5) - 3*a*c^
4)*x^5 + (b*c^2*(2*log(c) + 7) - 2*a*c^2)*x^3 - 3*b*x + (b*c^6*x^7 - 3*b*c
^4*x^5 + 2*b*c^2*x^3)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - b*x + (2*(b*c
^4*x^5 - b*c^2*x^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + (b*c^6*x^7 - 5*b*c^
4*x^5 + 4*b*c^2*x^3)*(c*x + 1)*(c*x - 1) - (b*c^6*x^7 - 3*b*c^4*x^5 + 2*b*
c^2*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x + 1)*sqrt(-c*x + 1) +
1))/((b^4*x*log(x)^2 + 2*(b^4*log(c) - a*b^3)*x*log(x) + (b^4*log(c)^2 - 2
*a*b^3*log(c) + a^2*b^2)*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - (b^4*c^6*x^
6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x*log(x)^2 + 3*((b^4*c^2*x^2 - b^
4)*x*log(x)^2 - 2*(b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*
x*log(x) - (b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2 - (b^4*c^2*log(c)^2 -
2*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x)*(c*x + 1)*(c*x - 1) + ((c*x + 1)
^(3/2)*(-c*x + 1)^(3/2)*b^4*x + 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1)*
x + 3*(b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x -
 (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x)*log(sqrt(c*x + ...
```

3.65.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx = \int \frac{1}{(b\,\text{arsech}\,(cx)+a)^3x}\,dx$$

input
```
integrate(1/x/(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*arcsech(c*x) + a)^3*x), x)
```

3.65.9 Mupad [N/A]

Not integrable

Time = 4.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

input
```
int(1/(x*(a + b*acosh(1/(c*x)))^3),x)
```

output
```
int(1/(x*(a + b*acosh(1/(c*x)))^3), x)
```

3.66 $\int \frac{1}{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}\,dx$

3.66.1 Optimal result

Integrand size = 14, antiderivative size = 114

$$\int \frac{1}{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}\,dx = \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{2bx\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}+\frac{1}{2b^2x\left(a+b\operatorname{sech}^{-1}(cx)\right)} + \frac{c\operatorname{Chi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{2b^3} - \frac{c\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)}{2b^3}$$

output
```
1/2/b^2/x/(a+b*arcsech(c*x))-1/2*c*cosh(a/b)*Shi(a/b+arcsech(c*x))/b^3+1/2
*c*Chi(a/b+arcsech(c*x))*sinh(a/b)/b^3+1/2*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2
)/b/x/(a+b*arcsech(c*x))^2
```

3.66.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}\,dx = \frac{\frac{b^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{x\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}+\frac{b}{ax+bx\operatorname{sech}^{-1}(cx)}+c\left(\operatorname{Chi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)\right)}{2b^3}$$

input
```
Integrate[1/(x^2*(a + b*ArcSech[c*x])^3),x]
```

output
```
((b^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x*(a + b*ArcSech[c*x])^2) + b/
(a*x + b*x*ArcSech[c*x]) + c*(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b] -
 Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]]))/(2*b^3)
```

3.66.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.24, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {6839, 3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx$$

$$\Big\downarrow \text{ 6839}$$

$$-c\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{cx\left(a+b\text{sech}^{-1}(cx)\right)^3}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{ 3042}$$

$$-c\int -\frac{i\sin\left(i\text{sech}^{-1}(cx)\right)}{\left(a+b\text{sech}^{-1}(cx)\right)^3}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{ 26}$$

$$ic\int \frac{\sin\left(i\text{sech}^{-1}(cx)\right)}{\left(a+b\text{sech}^{-1}(cx)\right)^3}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{ 3778}$$

$$ic\left(\frac{i\int \frac{1}{cx\left(a+b\text{sech}^{-1}(cx)\right)^2}d\text{sech}^{-1}(cx)}{2b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{2bcx\left(a+b\text{sech}^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$ic\left(\frac{i\int \frac{\sin\left(i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{\left(a+b\text{sech}^{-1}(cx)\right)^2}d\text{sech}^{-1}(cx)}{2b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{2bcx\left(a+b\text{sech}^{-1}(cx)\right)^2}\right)$$

$\downarrow$ 3778

$$ic\left(\frac{i\left(-\frac{1}{bcx\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{i\int -\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{cx\left(a+b\text{sech}^{-1}(cx)\right)}d\text{sech}^{-1}(cx)}{b}\right)}{2b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{2bcx\left(a+b\text{sech}^{-1}(cx)\right)^2}\right)$$

$\downarrow$ 26

$$ic\left(\frac{i\left(\frac{\int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{cx\left(a+b\text{sech}^{-1}(cx)\right)}d\text{sech}^{-1}(cx)}{b}-\frac{1}{bcx\left(a+b\text{sech}^{-1}(cx)\right)}\right)}{2b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{2bcx\left(a+b\text{sech}^{-1}(cx)\right)^2}\right)$$

$\downarrow$ 3042

$$ic\left(\frac{i\left(-\frac{1}{bcx\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{\int -\frac{i\sin\left(i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)}{b}\right)}{2b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{2bcx\left(a+b\text{sech}^{-1}(cx)\right)^2}\right)$$

$\downarrow$ 26

3.66. $\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx$

$$ic\left(\frac{i\left(-\frac{1}{bcx\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{i\int\frac{\sin\left(i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)}{b}\right)}{2b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{2bcx\left(a+b\text{sech}^{-1}(cx)\right)^2}\right)$$

$\big\downarrow$ 3784

$$ic\left(\frac{i\left(-\frac{1}{bcx\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{i\left(\cosh\left(\frac{a}{b}\right)\int\frac{i\sinh\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}\right)}{2b}-\right.$$

$\big\downarrow$ 26

$$ic\left(\frac{i\left(-\frac{1}{bcx\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{i\left(i\cosh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}\right)}{2b}-\right.$$

$\big\downarrow$ 3042

$$ic\left(\frac{i\left(-\frac{1}{bcx\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{i\left(i\cosh\left(\frac{a}{b}\right)\int-\frac{i\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right.}{b}\right.}{2b}\right.$$

$\big\downarrow$ 26

3.66. $\displaystyle\int\frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx$

$$ic\left(\frac{i\left(-\frac{1}{bcx\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{i\left(\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}\right)}{2b}\right.$$

$\downarrow$ 3779

$$ic\left(\frac{i\left(-\frac{1}{bcx\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{i\left(\frac{i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}-i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{ia}{b}+i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}\right)}{2b}-\frac{i\sqrt{\cdots}}{2bcx\,(a\cdots}\right.$$

$\downarrow$ 3782

$$ic\left(\frac{i\left(-\frac{1}{bcx\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{i\left(\frac{i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}-\frac{i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}\right)}{b}\right)}{2b}-\frac{i\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{2bcx\left(a+b\text{sech}^{-1}(cx)\right)\cdots}\right.$$

input
```
Int[1/(x^2*(a + b*ArcSech[c*x])^3),x]
```

output
```
I*c*(((-1/2*I)*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(b*c*x*(a + b*ArcSech[
c*x])^2) + ((I/2)*(-(1/(b*c*x*(a + b*ArcSech[c*x]))) - (I*(((-I)*CoshInteg
ral[a/b + ArcSech[c*x]]*Sinh[a/b])/b + (I*Cosh[a/b]*SinhIntegral[a/b + Arc
Sech[c*x]])/b))/b))/b)
```

3.66. $\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx$

3.66.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.66. $\int \frac{1}{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}\,dx$

3.66.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(104) = 208$.

Time = 0.61 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.14

method	result
derivativedivides	$c\left(-\frac{\left(\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}-1\right)(b\,\text{arcsech}(cx)+a-b)}{4cx\,b^2\left(b^2\,\text{arcsech}(cx)^2+2ab\,\text{arcsech}(cx)+a^2\right)}-\frac{\mathrm{e}^{\frac{a}{b}}\,\text{Ei}_1\left(\frac{a}{b}+\text{arcsech}(cx)\right)}{4b^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+1}{4bcx(a+b\,\text{arcsech}(cx))^2}+\frac{\sqrt{}}{4b^{}}\right.$
default	$c\left(-\frac{\left(\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}-1\right)(b\,\text{arcsech}(cx)+a-b)}{4cx\,b^2\left(b^2\,\text{arcsech}(cx)^2+2ab\,\text{arcsech}(cx)+a^2\right)}-\frac{\mathrm{e}^{\frac{a}{b}}\,\text{Ei}_1\left(\frac{a}{b}+\text{arcsech}(cx)\right)}{4b^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+1}{4bcx(a+b\,\text{arcsech}(cx))^2}+\frac{\sqrt{}}{4b^{}}\right.$

input
```
int(1/x^2/(a+b*arcsech(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
c*(-1/4*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)-1)*(b*arcsech(c*x)+a
-b)/c/x/b^2/(b^2*arcsech(c*x)^2+2*a*b*arcsech(c*x)+a^2)-1/4/b^3*exp(a/b)*E
i(1,a/b+arcsech(c*x))+1/4/b*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+
1)/c/x/(a+b*arcsech(c*x))^2+1/4/b^2*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x
)^(1/2)+1)/c/x/(a+b*arcsech(c*x))+1/4/b^3*exp(-a/b)*Ei(1,-arcsech(c*x)-a/b
))
```

3.66.5 Fricas [F]

$$\int \frac{1}{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{(b\,\text{arsech}\,(cx)+a)^3x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x^2*arcsech(c*x)^3 + 3*a*b^2*x^2*arcsech(c*x)^2 + 3*a^2*b*
x^2*arcsech(c*x) + a^3*x^2), x)
```

3.66.6 Sympy [F]

$$\int \frac{1}{x^2 \left(a + b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^2 \left(a + b \,\text{asech}\,(cx)\right)^3} \, dx$$

input
```
integrate(1/x**2/(a+b*asech(c*x))**3,x)
```

output
```
Integral(1/(x**2*(a + b*asech(c*x))**3), x)
```

3.66.7 Maxima [F]

$$\int \frac{1}{x^2 \left(a + b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{\left(b \,\text{arsech}\,(cx) + a\right)^3 x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

output
```
-1/2*((b*c^6*(log(c) - 1) - a*c^6)*x^7 - 3*(b*c^4*(log(c) - 1) - a*c^4)*x^
5 - (b*c^2*x^3 - (b*c^4*log(c) - a*c^4)*x^5 + (b*(log(c) - 1) - a)*x - (b*
c^4*x^5 - b*x)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 3*(b*c^2*(log(c)
 - 1) - a*c^2)*x^3 - (2*b*c^4*x^5 + (b*c^2*(3*log(c) - 5) - 3*a*c^2)*x^3 -
 3*(b*(log(c) - 1) - a)*x + 3*(b*c^2*x^3 - b*x)*log(x))*(c*x + 1)*(c*x - 1
) + ((b*c^6*(log(c) - 1) - a*c^6)*x^7 - (b*c^4*(4*log(c) - 5) - 4*a*c^4)*x
^5 + (b*c^2*(6*log(c) - 7) - 6*a*c^2)*x^3 - 3*(b*(log(c) - 1) - a)*x + (b*
c^6*x^7 - 4*b*c^4*x^5 + 6*b*c^2*x^3 - 3*b*x)*log(x))*sqrt(c*x + 1)*sqrt(-c
*x + 1) - (b*(log(c) - 1) - a)*x - (b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3
+ (b*c^4*x^5 - b*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - 3*(b*c^2*x^3 - b*x)
*(c*x + 1)*(c*x - 1) + (b*c^6*x^7 - 4*b*c^4*x^5 + 6*b*c^2*x^3 - 3*b*x)*sqr
t(c*x + 1)*sqrt(-c*x + 1) - b*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (
b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*log(x))/((b^4*c^6*x^6 - 3*b^4
*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^2*log(x)^2 - (b^4*x^2*log(x)^2 + 2*(b^4*
log(c) - a*b^3)*x^2*log(x) + (b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2)*x^2
)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 2*((b^4*c^6*log(c) - a*b^3*c^6)*x^6 -
 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*log(
c) - a*b^3*c^2)*x^2)*x^2*log(x) - 3*((b^4*c^2*x^2 - b^4)*x^2*log(x)^2 - 2*
(b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^2*log(x) - (b^4*
log(c)^2 - 2*a*b^3*log(c) + a^2*b^2 - (b^4*c^2*log(c)^2 - 2*a*b^3*c^2*l...
```

3.66.8 Giac [F]

$$\int \frac{1}{x^2 \left(a + b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{(b \operatorname{arsech}(cx) + a)^3 x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*arcsech(c*x) + a)^3*x^2), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a + b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^2 \left(a + b \operatorname{acosh}\left(\frac{1}{cx}\right)\right)^3} \, dx$$

input
```
int(1/(x^2*(a + b*acosh(1/(c*x)))^3),x)
```

output
```
int(1/(x^2*(a + b*acosh(1/(c*x)))^3), x)
```

3.67 $\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx$

3.67.1 Optimal result

Integrand size = 14, antiderivative size = 112

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx = \frac{c^2\cosh\left(2\text{sech}^{-1}(cx)\right)}{2b^2\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{c^2\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^3}+\frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{4b\left(a+b\text{sech}^{-1}(cx)\right)^2}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b^3}$$

output
```
1/2*c^2*cosh(2*arcsech(c*x))/b^2/(a+b*arcsech(c*x))-c^2*cosh(2*a/b)*Shi(2*
a/b+2*arcsech(c*x))/b^3+c^2*Chi(2*a/b+2*arcsech(c*x))*sinh(2*a/b)/b^3+1/4*
c^2*sinh(2*arcsech(c*x))/b/(a+b*arcsech(c*x))^2
```

3.67.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx = \frac{\frac{b^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}+\frac{b(2-c^2x^2)}{x^2\left(a+b\text{sech}^{-1}(cx)\right)}+2c^2\left(\text{Chi}\left(2\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{sec}\right.\right.\right.}{2b^3}$$

input
```
Integrate[1/(x^3*(a + b*ArcSech[c*x])^3),x]
```

output
```
((b^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x^2*(a + b*ArcSech[c*x])^2) +
(b*(2 - c^2*x^2))/(x^2*(a + b*ArcSech[c*x])) + 2*c^2*(CoshIntegral[2*(a/b
+ ArcSech[c*x])]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSe
ch[c*x])]))/(2*b^3)
```

3.67.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.16, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.214, Rules used = {6839, 5971, 27, 3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx$$

$$\Big\downarrow \text{6839}$$

$$-c^2 \int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{c^2x^2 \left(a+b\text{sech}^{-1}(cx)\right)^3}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{5971}$$

$$-c^2 \int \frac{\sinh\left(2\text{sech}^{-1}(cx)\right)}{2\left(a+b\text{sech}^{-1}(cx)\right)^3}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}c^2 \int \frac{\sinh\left(2\text{sech}^{-1}(cx)\right)}{\left(a+b\text{sech}^{-1}(cx)\right)^3}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}c^2 \int -\frac{i\sin\left(2i\text{sech}^{-1}(cx)\right)}{\left(a+b\text{sech}^{-1}(cx)\right)^3}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}ic^2 \int \frac{\sin\left(2i\text{sech}^{-1}(cx)\right)}{\left(a+b\text{sech}^{-1}(cx)\right)^3}d\text{sech}^{-1}(cx)$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}ic^2\left(\frac{i\int \frac{\cosh\left(2\text{sech}^{-1}(cx)\right)}{\left(a+b\text{sech}^{-1}(cx)\right)^2}d\text{sech}^{-1}(cx)}{b}-\frac{i\sinh\left(2\text{sech}^{-1}(cx)\right)}{2b\left(a+b\text{sech}^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}ic^2\left(\frac{i\int \frac{\sin\left(2i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{\left(a+b\text{sech}^{-1}(cx)\right)^2}d\text{sech}^{-1}(cx)}{b}-\frac{i\sinh\left(2\text{sech}^{-1}(cx)\right)}{2b\left(a+b\text{sech}^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}ic^2\left(\frac{i\left(-\frac{\cosh\left(2\text{sech}^{-1}(cx)\right)}{b\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{2i\int -\frac{i\sinh\left(2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)}{b}\right)}{b}-\frac{i\sinh\left(2\text{sech}^{-1}(cx)\right)}{2b\left(a+b\text{sech}^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}ic^2\left(\frac{i\left(\frac{2\int \frac{\sinh\left(2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)}{b}-\frac{\cosh\left(2\text{sech}^{-1}(cx)\right)}{b\left(a+b\text{sech}^{-1}(cx)\right)}\right)}{b}-\frac{i\sinh\left(2\text{sech}^{-1}(cx)\right)}{2b\left(a+b\text{sech}^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}ic^2\left(\frac{i\left(-\frac{\cosh\left(2\text{sech}^{-1}(cx)\right)}{b\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{2\int -\frac{i\sin\left(2i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)}{b}\right)}{b}-\frac{i\sinh\left(2\text{sech}^{-1}(cx)\right)}{2b\left(a+b\text{sech}^{-1}(cx)\right)^2}\right)$$

$$\Big\downarrow \text{26}$$

3.67. $\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx$

$$\frac{1}{2}ic^2\left(\frac{i\left(-\frac{\cosh\left(2\text{sech}^{-1}(cx)\right)}{b\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{2i\int\frac{\sin\left(2i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)}{b}\right)}{b}-\frac{i\sinh\left(2\text{sech}^{-1}(cx)\right)}{2b\left(a+b\text{sech}^{-1}(cx)\right)^2}\right)$$

$\downarrow$ 3784

$$\frac{1}{2}ic^2\left(\frac{i\left(-\frac{\cosh\left(2\text{sech}^{-1}(cx)\right)}{b\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{2i\left(\cosh\left(\frac{2a}{b}\right)\int\frac{i\sinh\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(c\right.}{b}\right.}{b}\right.$$

$\downarrow$ 26

$$\frac{1}{2}ic^2\left(\frac{i\left(-\frac{\cosh\left(2\text{sech}^{-1}(cx)\right)}{b\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{2i\left(i\cosh\left(\frac{2a}{b}\right)\int\frac{\sinh\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(c\right.}{b}\right.}{b}\right.$$

$\downarrow$ 3042

$$\frac{1}{2}ic^2\left(\frac{i\left(-\frac{\cosh\left(2\text{sech}^{-1}(cx)\right)}{b\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{2i\left(i\cosh\left(\frac{2a}{b}\right)\int-\frac{i\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sec}\right.}{b}\right.}{b}\right.$$

$\downarrow$ 26

3.67. $\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx$

$$\frac{1}{2}ic^2\left(\frac{i\left(-\frac{\cosh\left(2\text{sech}^{-1}(cx)\right)}{b\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{2i\left(\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)-i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}\right.}{b}\right.}{b}\right.$$

$\downarrow$ 3779

$$\frac{1}{2}ic^2\left(\frac{i\left(-\frac{\cosh\left(2\text{sech}^{-1}(cx)\right)}{b\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{2i\left(\frac{i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b}-i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2ia}{b}+2i\text{sech}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{sech}^{-1}(cx)}d\text{sech}^{-1}(cx)\right)}{b}\right)}{b}-\frac{}{2}\right.$$

$\downarrow$ 3782

$$\frac{1}{2}ic^2\left(\frac{i\left(-\frac{\cosh\left(2\text{sech}^{-1}(cx)\right)}{b\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{2i\left(\frac{i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b}-\frac{i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b}\right)}{b}\right)}{b}-\frac{i\sinh\left(2\text{sech}^{-1}\right.}{2b\left(a+b\text{sech}^{-1}\right.}\right.$$

input
```
Int[1/(x^3*(a + b*ArcSech[c*x])^3),x]
```

output
```
(I/2)*c^2*(((-1/2*I)*Sinh[2*ArcSech[c*x]])/(b*(a + b*ArcSech[c*x])^2) + (I
*(-(Cosh[2*ArcSech[c*x]]/(b*(a + b*ArcSech[c*x]))) - ((2*I)*(((-I)*CoshInt
egral[(2*a)/b + 2*ArcSech[c*x]]*Sinh[(2*a)/b])/b + (I*Cosh[(2*a)/b]*SinhIn
tegral[(2*a)/b + 2*ArcSech[c*x]])/b))/b))/b)
```

3.67. $\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx$

3.67.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.67. $\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx$

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.67.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(108) = 216$.

Time = 0.79 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.47

method	result
derivativedivides	$c^2\left(-\frac{\left(2\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+c^2x^2-2\right)(2b\,\mathrm{arcsech}(cx)+2a-b)}{8c^2x^2b^2\left(b^2\,\mathrm{arcsech}(cx)^2+2ab\,\mathrm{arcsech}(cx)+a^2\right)}-\frac{e^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(\frac{2a}{b}+2\,\mathrm{arcsech}(cx)\right)}{2b^3}-\frac{c^2x^2-2-2\sqrt{-\frac{c}{}}}{8b\,c^2x^2(a+b\,a}\right.$
default	$c^2\left(-\frac{\left(2\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+c^2x^2-2\right)(2b\,\mathrm{arcsech}(cx)+2a-b)}{8c^2x^2b^2\left(b^2\,\mathrm{arcsech}(cx)^2+2ab\,\mathrm{arcsech}(cx)+a^2\right)}-\frac{e^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(\frac{2a}{b}+2\,\mathrm{arcsech}(cx)\right)}{2b^3}-\frac{c^2x^2-2-2\sqrt{-\frac{c}{}}}{8b\,c^2x^2(a+b\,a}\right.$

input
```
int(1/x^3/(a+b*arcsech(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/8*(2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+c^2*x^2-2)*(2*b*
arcsech(c*x)+2*a-b)/c^2/x^2/b^2/(b^2*arcsech(c*x)^2+2*a*b*arcsech(c*x)+a^2
)-1/2/b^3*exp(2*a/b)*Ei(1,2*a/b+2*arcsech(c*x))-1/8/b*(c^2*x^2-2-2*(-(c*x-
1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2))/c^2/x^2/(a+b*arcsech(c*x))^2-1/4/b^
2*(c^2*x^2-2-2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2))/c^2/x^2/(a+b*
arcsech(c*x))+1/2/b^3*exp(-2*a/b)*Ei(1,-2*arcsech(c*x)-2*a/b))
```

3.67.5 Fricas [F]

$$\int \frac{1}{x^3\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arsech}\left(cx\right)+a\right)^3x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x^3*arcsech(c*x)^3 + 3*a*b^2*x^3*arcsech(c*x)^2 + 3*a^2*b*
x^3*arcsech(c*x) + a^3*x^3), x)
```

3.67.6 Sympy [F]

$$\int \frac{1}{x^3 \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^3 \left(a + b \operatorname{asech}{\left(cx \right)}\right)^3} \, dx$$

input
```
integrate(1/x**3/(a+b*asech(c*x))**3,x)
```

output
```
Integral(1/(x**3*(a + b*asech(c*x))**3), x)
```

3.67.7 Maxima [F]

$$\int \frac{1}{x^3 \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{\left(b \operatorname{arsech}\left(cx\right) + a\right)^3 x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

output
```
-1/2*((b*c^6*(2*log(c) - 1) - 2*a*c^6)*x^7 - 3*(b*c^4*(2*log(c) - 1) - 2*a
*c^4)*x^5 + ((b*c^2*(2*log(c) - 1) - 2*a*c^2)*x^3 - (b*(2*log(c) - 1) - 2*
a)*x + 2*(b*c^2*x^3 - b*x)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 3*(b
*c^2*(2*log(c) - 1) - 2*a*c^2)*x^3 - ((b*c^6*log(c) - a*c^6)*x^7 - (b*c^4*
(5*log(c) - 2) - 5*a*c^4)*x^5 + 5*(b*c^2*(2*log(c) - 1) - 2*a*c^2)*x^3 - 3
*(b*(2*log(c) - 1) - 2*a)*x + (b*c^6*x^7 - 5*b*c^4*x^5 + 10*b*c^2*x^3 - 6*
b*x)*log(x))*(c*x + 1)*(c*x - 1) + ((b*c^6*(3*log(c) - 1) - 3*a*c^6)*x^7 -
 (b*c^4*(11*log(c) - 5) - 11*a*c^4)*x^5 + 7*(b*c^2*(2*log(c) - 1) - 2*a*c^
2)*x^3 - 3*(b*(2*log(c) - 1) - 2*a)*x + (3*b*c^6*x^7 - 11*b*c^4*x^5 + 14*b
*c^2*x^3 - 6*b*x)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b*(2*log(c) - 1)
 - 2*a)*x - (2*b*c^6*x^7 - 6*b*c^4*x^5 + 6*b*c^2*x^3 + 2*(b*c^2*x^3 - b*x)
*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - (b*c^6*x^7 - 5*b*c^4*x^5 + 10*b*c^2*x^
3 - 6*b*x)*(c*x + 1)*(c*x - 1) + (3*b*c^6*x^7 - 11*b*c^4*x^5 + 14*b*c^2*x^
3 - 6*b*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - 2*b*x)*log(sqrt(c*x + 1)*sqrt(-c
*x + 1) + 1) + 2*(b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*log(x))/((b
^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^3*log(x)^2 + 2*((b^4*c
^6*log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(
c) + a*b^3 + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^3*log(x) - (b^4*x^3*log
(x)^2 + 2*(b^4*log(c) - a*b^3)*x^3*log(x) + (b^4*log(c)^2 - 2*a*b^3*log(c)
 + a^2*b^2)*x^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + ((b^4*c^6*log(c)^2 ...
```

3.67.8 Giac [F]

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^3x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*arcsech(c*x) + a)^3*x^3), x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x^3\left(a+b\,\text{acosh}\left(\frac{1}{cx}\right)\right)^3}\,dx$$

input
```
int(1/(x^3*(a + b*acosh(1/(c*x)))^3),x)
```

output
```
int(1/(x^3*(a + b*acosh(1/(c*x)))^3), x)
```

3.68 $\int \frac{1}{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)^3}\,dx$

3.68.1 Optimal result

Integrand size = 14, antiderivative size = 240

$$\begin{aligned}\int \frac{1}{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)^3}\,dx &= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{8bx\left(a+b\,\text{sech}^{-1}(cx)\right)^2}+\frac{c^2}{8b^2x\left(a+b\,\text{sech}^{-1}(cx)\right)}\\
&\quad+\frac{3c^3\cosh\left(3\,\text{sech}^{-1}(cx)\right)}{8b^2\left(a+b\,\text{sech}^{-1}(cx)\right)}+\frac{c^3\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8b^3}\\
&\quad+\frac{9c^3\text{Chi}\left(\frac{3a}{b}+3\,\text{sech}^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{8b^3}\\
&\quad+\frac{c^3\sinh\left(3\,\text{sech}^{-1}(cx)\right)}{8b\left(a+b\,\text{sech}^{-1}(cx)\right)^2}-\frac{c^3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{8b^3}\\
&\quad-\frac{9c^3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\,\text{sech}^{-1}(cx)\right)}{8b^3}\end{aligned}$$

output
```
1/8*c^2/b^2/x/(a+b*arcsech(c*x))+3/8*c^3*cosh(3*arcsech(c*x))/b^2/(a+b*arc
sech(c*x))-1/8*c^3*cosh(a/b)*Shi(a/b+arcsech(c*x))/b^3-9/8*c^3*cosh(3*a/b)
*Shi(3*a/b+3*arcsech(c*x))/b^3+1/8*c^3*Chi(a/b+arcsech(c*x))*sinh(a/b)/b^3
+9/8*c^3*Chi(3*a/b+3*arcsech(c*x))*sinh(3*a/b)/b^3+1/8*c^3*sinh(3*arcsech(
c*x))/b/(a+b*arcsech(c*x))^2+1/8*c^2*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/b/x/
(a+b*arcsech(c*x))^2
```

3.68.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx$$
$$= \frac{\frac{4b^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}+\frac{4b(3-2c^2x^2)}{x^3\left(a+b\text{sech}^{-1}(cx)\right)}-8c^3\left(\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right.}{}$$

input
```
Integrate[1/(x^4*(a + b*ArcSech[c*x])^3),x]
```

output
```
((4*b^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x^3*(a + b*ArcSech[c*x])^2)
+ (4*b*(3 - 2*c^2*x^2))/(x^3*(a + b*ArcSech[c*x])) - 8*c^3*(CoshIntegral[a
/b + ArcSech[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])
 + 9*c^3*(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b] + CoshIntegral[3*(a/b
 + ArcSech[c*x])]*Sinh[(3*a)/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x
]] - Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSech[c*x])]))/(8*b^3)
```

3.68.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6839, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx$$
$$\Big\downarrow \text{6839}$$
$$-c^3 \int \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{c^3x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}d\text{sech}^{-1}(cx)$$
$$\Big\downarrow \text{5971}$$
$$-c^3 \int \left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{4cx\left(a+b\text{sech}^{-1}(cx)\right)^3}+\frac{\sinh\left(3\text{sech}^{-1}(cx)\right)}{4\left(a+b\text{sech}^{-1}(cx)\right)^3}\right)d\text{sech}^{-1}(cx)$$

$$\downarrow \text{2009}$$

$$-c^3\left(-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{8b^3}-\frac{9\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)}{8b^3}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{8b^3}+\frac{9}{}\right.$$

input
```
Int[1/(x^4*(a + b*ArcSech[c*x])^3),x]
```

output
```
-(c^3*(-1/8*(Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(b*c*x*(a + b*ArcSech[c*
x])^2) - 1/(8*b^2*c*x*(a + b*ArcSech[c*x])) - (3*Cosh[3*ArcSech[c*x]])/(8*
b^2*(a + b*ArcSech[c*x])) - (CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/(
8*b^3) - (9*CoshIntegral[(3*a)/b + 3*ArcSech[c*x]]*Sinh[(3*a)/b])/(8*b^3)
- Sinh[3*ArcSech[c*x]]/(8*b*(a + b*ArcSech[c*x])^2) + (Cosh[a/b]*SinhInteg
ral[a/b + ArcSech[c*x]])/(8*b^3) + (9*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b +
 3*ArcSech[c*x]])/(8*b^3)))
```

3.68.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6839
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, A
rcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.68.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(222) = 444$.

Time = 1.00 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.62

method	result
derivativedivides	$c^3\left(\frac{\left(\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}\,c^3x^3-4\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}-3c^2x^2+4\right)(3b\,\mathrm{arcsech}(cx)+3a-b)}{16c^3x^3b^2\left(b^2\,\mathrm{arcsech}(cx)^2+2ab\,\mathrm{arcsech}(cx)+a^2\right)}-\frac{9\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(\frac{3a}{b}+3\,\mathrm{arcsech}(\right.}{16b^3}\right.$
default	$c^3\left(\frac{\left(\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}\,c^3x^3-4\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}-3c^2x^2+4\right)(3b\,\mathrm{arcsech}(cx)+3a-b)}{16c^3x^3b^2\left(b^2\,\mathrm{arcsech}(cx)^2+2ab\,\mathrm{arcsech}(cx)+a^2\right)}-\frac{9\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(\frac{3a}{b}+3\,\mathrm{arcsech}(\right.}{16b^3}\right.$

input
```
int(1/x^4/(a+b*arcsech(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
c^3*(1/16*(((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x^3-4*(-(c*x-1)/c/
x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)-3*c^2*x^2+4)*(3*b*arcsech(c*x)+3*a-b)/c^3
/x^3/b^2/(b^2*arcsech(c*x)^2+2*a*b*arcsech(c*x)+a^2)-9/16/b^3*exp(3*a/b)*E
i(1,3*a/b+3*arcsech(c*x))-1/16*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/
2)-1)*(b*arcsech(c*x)+a-b)/c/x/b^2/(b^2*arcsech(c*x)^2+2*a*b*arcsech(c*x)+
a^2)-1/16/b^3*exp(a/b)*Ei(1,a/b+arcsech(c*x))+1/16/b*((-(c*x-1)/c/x)^(1/2)
*c*x*((c*x+1)/c/x)^(1/2)+1)/c/x/(a+b*arcsech(c*x))^2+1/16/b^2*((-(c*x-1)/c
/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+1)/c/x/(a+b*arcsech(c*x))+1/16/b^3*exp(-
a/b)*Ei(1,-arcsech(c*x)-a/b)-1/16/b*(((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1
/2)*c^3*x^3-4*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+3*c^2*x^2-4)/c^
3/x^3/(a+b*arcsech(c*x))^2-3/16/b^2*(((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1
/2)*c^3*x^3-4*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+3*c^2*x^2-4)/c^
3/x^3/(a+b*arcsech(c*x))+9/16/b^3*exp(-3*a/b)*Ei(1,-3*arcsech(c*x)-3*a/b))
```

3.68.5 Fricas [F]

$$\int \frac{1}{x^4\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arsech}\,(cx)+a\right)^3x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x^4*arcsech(c*x)^3 + 3*a*b^2*x^4*arcsech(c*x)^2 + 3*a^2*b*
x^4*arcsech(c*x) + a^3*x^4), x)
```

3.68.6 Sympy [F]

$$\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x^4\left(a+b\,\text{asech}\,(cx)\right)^3}\,dx$$

input
```
integrate(1/x**4/(a+b*asech(c*x))**3,x)
```

output
```
Integral(1/(x**4*(a + b*asech(c*x))**3), x)
```

3.68.7 Maxima [F]

$$\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^3x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

output
```
-1/2*((b*c^6*(3*log(c) - 1) - 3*a*c^6)*x^7 - 3*(b*c^4*(3*log(c) - 1) - 3*a
*c^4)*x^5 - ((b*c^4*log(c) - a*c^4)*x^5 - (b*c^2*(4*log(c) - 1) - 4*a*c^2)
*x^3 + (b*(3*log(c) - 1) - 3*a)*x + (b*c^4*x^5 - 4*b*c^2*x^3 + 3*b*x)*log(
x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 3*(b*c^2*(3*log(c) - 1) - 3*a*c^2)*
x^3 - (2*(b*c^6*log(c) - a*c^6)*x^7 - 2*(b*c^4*(5*log(c) - 1) - 5*a*c^4)*x
^5 + (b*c^2*(17*log(c) - 5) - 17*a*c^2)*x^3 - 3*(b*(3*log(c) - 1) - 3*a)*x
 + (2*b*c^6*x^7 - 10*b*c^4*x^5 + 17*b*c^2*x^3 - 9*b*x)*log(x))*(c*x + 1)*(
c*x - 1) + ((b*c^6*(5*log(c) - 1) - 5*a*c^6)*x^7 - (b*c^4*(18*log(c) - 5)
- 18*a*c^4)*x^5 + (b*c^2*(22*log(c) - 7) - 22*a*c^2)*x^3 - 3*(b*(3*log(c)
- 1) - 3*a)*x + (5*b*c^6*x^7 - 18*b*c^4*x^5 + 22*b*c^2*x^3 - 9*b*x)*log(x)
)*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b*(3*log(c) - 1) - 3*a)*x - (3*b*c^6*x^7
 - 9*b*c^4*x^5 + 9*b*c^2*x^3 - (b*c^4*x^5 - 4*b*c^2*x^3 + 3*b*x)*(c*x + 1)
^(3/2)*(-c*x + 1)^(3/2) - (2*b*c^6*x^7 - 10*b*c^4*x^5 + 17*b*c^2*x^3 - 9*b
*x)*(c*x + 1)*(c*x - 1) + (5*b*c^6*x^7 - 18*b*c^4*x^5 + 22*b*c^2*x^3 - 9*b
*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - 3*b*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1)
 + 1) + 3*(b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*log(x))/((b^4*c^6*
x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^4*log(x)^2 + 2*((b^4*c^6*log(
c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*
b^3 + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^4*log(x) + ((b^4*c^6*log(c)^2
- 2*a*b^3*c^6*log(c) + a^2*b^2*c^6)*x^6 - b^4*log(c)^2 - 3*(b^4*c^4*log...
```

3.68.8 Giac [F]

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{\left(b \operatorname{arsech}\left(cx\right)+a\right)^3 x^4} \, dx$$

input
```
integrate(1/x^4/(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*arcsech(c*x) + a)^3*x^4), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^4 \left(a+b \operatorname{acosh}\left(\frac{1}{cx}\right)\right)^3} \, dx$$

input
```
int(1/(x^4*(a + b*acosh(1/(c*x)))^3),x)
```

output
```
int(1/(x^4*(a + b*acosh(1/(c*x)))^3), x)
```

3.69 $\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx$

3.69.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx=\text{Int}\left((dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3,x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arcsech(c*x))^3,x)
```

3.69.2 Mathematica [N/A]

Not integrable

Time = 5.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx=\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcSech[c*x])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcSech[c*x])^3, x]
```

3.69.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx$$

$$\Big\downarrow \text{6865}$$

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 dx$$

input
```
Int[(d*x)^m*(a + b*ArcSech[c*x])^3,x]
```

output
```
$Aborted
```

3.69.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.69.4 Maple [N/A] (verified)

Not integrable

Time = 0.65 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b \,\operatorname{arcsech}\left(cx\right)\right)^3 dx$$

input
```
int((d*x)^m*(a+b*arcsech(c*x))^3,x)
```

output
```
int((d*x)^m*(a+b*arcsech(c*x))^3,x)
```

3.69.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (b\,\text{arsech}\,(cx)+a)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c*
x) + a^3)*(d*x)^m, x)
```

3.69.6 Sympy [N/A]

Not integrable

Time = 5.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a+b\,\text{asech}\,(cx)\right)^3 \, dx$$

input
```
integrate((d*x)**m*(a+b*asech(c*x))**3,x)
```

output
```
Integral((d*x)**m*(a + b*asech(c*x))**3, x)
```

3.69.7 Maxima [N/A]

Not integrable

Time = 10.45 (sec) , antiderivative size = 1450, normalized size of antiderivative = 90.62

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (b\,\text{arsech}\,(cx)+a)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

output
```
b^3*d^m*x*x^m*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^3/(m + 1) + (d*x)^(m +
 1)*a^3/(d*(m + 1)) - integrate(((b^3*c^2*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1
))*x^m*log(x)^3 - 3*(b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) - (b^3*c^2
*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*x^m*log(x)^2 + 3*((b^3*c
^2*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(x) + ((b^3*c^2*d^m*(m + 1)*x
^2 - b^3*d^m*(m + 1))*x^m*log(x) - (b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m
+ 1) + (a*b^2*c^2*d^m*(m + 1) - (d^m*(m + 1)*log(c) + d^m)*b^3*c^2)*x^2)*x
^m)*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m
+ 1) - (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*x^m)*log(
sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^2 - 3*(b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^
2*d^m*(m + 1)*log(c) + a^2*b*d^m*(m + 1) - (b^3*c^2*d^m*(m + 1)*log(c)^2 -
 2*a*b^2*c^2*d^m*(m + 1)*log(c) + a^2*b*c^2*d^m*(m + 1))*x^2)*x^m*log(x) +
 ((b^3*c^2*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(x)^3 - 3*(b^3*d^m*(m
 + 1)*log(c) - a*b^2*d^m*(m + 1) - (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2
*d^m*(m + 1))*x^2)*x^m*log(x)^2 - 3*(b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^
m*(m + 1)*log(c) + a^2*b*d^m*(m + 1) - (b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a
*b^2*c^2*d^m*(m + 1)*log(c) + a^2*b*c^2*d^m*(m + 1))*x^2)*x^m*log(x) - (b^
3*d^m*(m + 1)*log(c)^3 - 3*a*b^2*d^m*(m + 1)*log(c)^2 + 3*a^2*b*d^m*(m + 1
)*log(c) - (b^3*c^2*d^m*(m + 1)*log(c)^3 - 3*a*b^2*c^2*d^m*(m + 1)*log(c)^
2 + 3*a^2*b*c^2*d^m*(m + 1)*log(c))*x^2)*x^m)*sqrt(c*x + 1)*sqrt(-c*x +...
```

3.69.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (b\,\text{arsech}\,(cx)+a)^3(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^3*(d*x)^m, x)
```

3.69.9 Mupad [N/A]

Not integrable

Time = 4.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (d\,x)^m \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

input
```
int((d*x)^m*(a + b*acosh(1/(c*x)))^3,x)
```

output
```
int((d*x)^m*(a + b*acosh(1/(c*x)))^3, x)
```

3.70 $\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 dx$

3.70.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 dx = \text{Int}\left((dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arcsech(c*x))^2,x)
```

3.70.2 Mathematica [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 dx = \int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcSech[c*x])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcSech[c*x])^2, x]
```

3.70.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx$$

$$\Big\downarrow \ 6865$$

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 dx$$

input
```
Int[(d*x)^m*(a + b*ArcSech[c*x])^2,x]
```

output
```
$Aborted
```

3.70.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.70.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\ \operatorname{arcsech}\left(cx\right)\right)^2 dx$$

input
```
int((d*x)^m*(a+b*arcsech(c*x))^2,x)
```

output
```
int((d*x)^m*(a+b*arcsech(c*x))^2,x)
```

3.70.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx = \int (b\,\text{arsech}\,(cx)+a)^2(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2)*(d*x)^m, x)
```

3.70.6 Sympy [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx = \int (dx)^m\,(a+b\,\text{asech}\,(cx))^2 \, dx$$

input
```
integrate((d*x)**m*(a+b*asech(c*x))**2,x)
```

output
```
Integral((d*x)**m*(a + b*asech(c*x))**2, x)
```

3.70.7 Maxima [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 704, normalized size of antiderivative = 44.00

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx = \int (b\,\text{arsech}\,(cx)+a)^2(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

output
```
b^2*d^m*x*x^m*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^2/(m + 1) + (d*x)^(m +
 1)*a^2/(d*(m + 1)) - integrate(-((b^2*c^2*d^m*(m + 1)*x^2 - b^2*d^m*(m +
1))*x^m*log(x)^2 - 2*(b^2*d^m*(m + 1)*log(c) - a*b*d^m*(m + 1) - (b^2*c^2*
d^m*(m + 1)*log(c) - a*b*c^2*d^m*(m + 1))*x^2)*x^m*log(x) + ((b^2*c^2*d^m*
(m + 1)*x^2 - b^2*d^m*(m + 1))*x^m*log(x)^2 - 2*(b^2*d^m*(m + 1)*log(c) -
a*b*d^m*(m + 1) - (b^2*c^2*d^m*(m + 1)*log(c) - a*b*c^2*d^m*(m + 1))*x^2)*
x^m*log(x) - (b^2*d^m*(m + 1)*log(c)^2 - 2*a*b*d^m*(m + 1)*log(c) - (b^2*c
^2*d^m*(m + 1)*log(c)^2 - 2*a*b*c^2*d^m*(m + 1)*log(c))*x^2)*x^m)*sqrt(c*x
 + 1)*sqrt(-c*x + 1) - (b^2*d^m*(m + 1)*log(c)^2 - 2*a*b*d^m*(m + 1)*log(c
) - (b^2*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b*c^2*d^m*(m + 1)*log(c))*x^2)*x^m
 - 2*((b^2*c^2*d^m*(m + 1)*x^2 - b^2*d^m*(m + 1))*x^m*log(x) + ((b^2*c^2*d
^m*(m + 1)*x^2 - b^2*d^m*(m + 1))*x^m*log(x) - (b^2*d^m*(m + 1)*log(c) - a
*b*d^m*(m + 1) + (a*b*c^2*d^m*(m + 1) - (d^m*(m + 1)*log(c) + d^m)*b^2*c^2
)*x^2)*x^m)*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^2*d^m*(m + 1)*log(c) - a*b*d
^m*(m + 1) - (b^2*c^2*d^m*(m + 1)*log(c) - a*b*c^2*d^m*(m + 1))*x^2)*x^m)*
log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1))/(c^2*(m + 1)*x^2 + (c^2*(m + 1)*x^2
 - m - 1)*sqrt(c*x + 1)*sqrt(-c*x + 1) - m - 1), x)
```

3.70.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx = \int (b\,\text{arsech}\,(cx)+a)^2(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)^2*(d*x)^m, x)
```

3.70.9 Mupad [N/A]

Not integrable

Time = 4.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx = \int (d\,x)^m \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

input
```
int((d*x)^m*(a + b*acosh(1/(c*x)))^2,x)
```

output
```
int((d*x)^m*(a + b*acosh(1/(c*x)))^2, x)
```

3.70. $\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx$

3.71 $\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

3.71.1 Optimal result

Integrand size = 14, antiderivative size = 87

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \frac{(dx)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{d(1+m)} + \frac{b(dx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{d(1+m)^2}$$

output
```
(d*x)^(1+m)*(a+b*arcsech(c*x))/d/(1+m)+b*(d*x)^(1+m)*hypergeom([1/2, 1/2+1
/2*m],[3/2+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d/(1+m)^2
```

3.71.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \frac{x(dx)^m\left((1+m)(-1+cx)\left(a+b\text{sech}^{-1}(cx)\right)-b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x\right.}{(1+m)^2(-1+cx)}$$

input
```
Integrate[(d*x)^m*(a + b*ArcSech[c*x]),x]
```

output
```
(x*(d*x)^m*((1 + m)*(-1 + c*x)*(a + b*ArcSech[c*x]) - b*Sqrt[(1 - c*x)/(1
+ c*x)]*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2
*x^2]))/((1 + m)^2*(-1 + c*x))
```

3.71.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6837, 135, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{ 6837}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(dx)^m}{\sqrt{1-cx}\sqrt{cx+1}}dx}{m+1}+\frac{(dx)^{m+1} \left(a+b\text{sech}^{-1}(cx)\right)}{d(m+1)}$$

$$\Big\downarrow \text{ 135}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(dx)^m}{\sqrt{1-c^2x^2}}dx}{m+1}+\frac{(dx)^{m+1} \left(a+b\text{sech}^{-1}(cx)\right)}{d(m+1)}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(dx)^{m+1} \left(a+b\text{sech}^{-1}(cx)\right)}{d(m+1)}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(dx)^{m+1} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)}{d(m+1)^2}$$

input
```
Int[(d*x)^m*(a + b*ArcSech[c*x]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcSech[c*x]))/(d*(1 + m)) + (b*(d*x)^(1 + m)*Sqrt[(
1 + c*x)^(-1)]*Sqrt[1 + c*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
c^2*x^2])/(d*(1 + m)^2)
```

3.71.3.1 Defintions of rubi rules used

rule 135
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f, m, n, p},
 x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && GtQ[a, 0] && GtQ[c, 0]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6837
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Simp[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)]   Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x])
, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

3.71.4 Maple [F]

$$\int (dx)^m (a + b \operatorname{arcsech}(cx)) \, dx$$

input
```
int((d*x)^m*(a+b*arcsech(c*x)),x)
```

output
```
int((d*x)^m*(a+b*arcsech(c*x)),x)
```

3.71.5 Fricas [F]

$$\int (dx)^m \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int (b \operatorname{arsech}(cx) + a)(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)*(d*x)^m, x)
```

3.71.6 Sympy [F]

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right) dx = \int (dx)^m (a+b \operatorname{asech}(cx))\, dx$$

input
```
integrate((d*x)**m*(a+b*asech(c*x)),x)
```

output
```
Integral((d*x)**m*(a + b*asech(c*x)), x)
```

3.71.7 Maxima [F]

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right) dx = \int (b \operatorname{arsech}(cx)+a)(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
(c^2*d^m*integrate(x^2*x^m/(c^2*(m + 1)*x^2 + (c^2*(m + 1)*x^2 - m - 1)*sq
rt(c*x + 1)*sqrt(-c*x + 1) - m - 1), x) + (d^m*x*x^m*log(sqrt(c*x + 1)*sqr
t(-c*x + 1) + 1) - d^m*x*x^m*log(x))/(m + 1) - integrate((c^2*d^m*(m + 1)*
x^2*log(c) - d^m*(m + 1)*log(c) + d^m)*x^m/(c^2*(m + 1)*x^2 - m - 1), x))*
b + (d*x)^(m + 1)*a/(d*(m + 1))
```

3.71.8 Giac [F]

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right) dx = \int (b \operatorname{arsech}(cx)+a)(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*(d*x)^m, x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (d\,x)^m\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d*x)^m*(a + b*acosh(1/(c*x))),x)
```

output
```
int((d*x)^m*(a + b*acosh(1/(c*x))), x)
```

3.72 $\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)} dx$

3.72.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)} dx = \text{Int}\left(\frac{(dx)^m}{a+b\text{sech}^{-1}(cx)},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arcsech(c*x)),x)
```

3.72.2 Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)} dx = \int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)} dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcSech[c*x]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcSech[c*x]), x]
```

3.72.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)} \, dx$$
$$\Big\downarrow \; 6865$$
$$\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcSech[c*x]),x]
```

output
```
$Aborted
```

3.72.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.72.4 Maple [N/A] (verified)

Not integrable

Time = 0.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b \operatorname{arcsech}(cx)}dx$$

input
```
int((d*x)^m/(a+b*arcsech(c*x)),x)
```

output
```
int((d*x)^m/(a+b*arcsech(c*x)),x)
```

3.72.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{(dx)^m}{b\,\text{arsech}\,(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arcsech(c*x) + a), x)
```

3.72.6 Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\,\text{asech}\,(cx)}\,dx$$

input
```
integrate((d*x)**m/(a+b*asech(c*x)),x)
```

output
```
Integral((d*x)**m/(a + b*asech(c*x)), x)
```

3.72.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{(dx)^m}{b\,\text{arsech}\,(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arcsech(c*x) + a), x)
```

3.72.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a + b\text{sech}^{-1}(cx)}\, dx = \int \frac{(dx)^m}{b\,\text{arsech}\,(cx) + a}\, dx$$

input
```
integrate((d*x)^m/(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arcsech(c*x) + a), x)
```

3.72.9 Mupad [N/A]

Not integrable

Time = 3.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{a + b\text{sech}^{-1}(cx)}\, dx = \int \frac{(d\,x)^m}{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}\, dx$$

input
```
int((d*x)^m/(a + b*acosh(1/(c*x))),x)
```

output
```
int((d*x)^m/(a + b*acosh(1/(c*x))), x)
```

3.73 $\int \frac{(dx)^m}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$

3.73.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx=\mathrm{Int}\left(\frac{(dx)^m}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arcsech(c*x))^2,x)
```

3.73.2 Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcSech[c*x])^2,x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcSech[c*x])^2, x]
```

3.73.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2} \, dx$$
$$\Big\downarrow \ 6865$$
$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2}dx$$

input
```
Int[(d*x)^m/(a + b*ArcSech[c*x])^2,x]
```

output
```
$Aborted
```

3.73.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.73.4 Maple [N/A] (verified)

Not integrable

Time = 0.72 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{\left(a+b \,\operatorname{arcsech}\left(cx\right)\right)^2}dx$$

input
```
int((d*x)^m/(a+b*arcsech(c*x))^2,x)
```

output
```
int((d*x)^m/(a+b*arcsech(c*x))^2,x)
```

3.73.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(b\,\text{arsech}\,(cx)+a\right)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2), x)
```

3.73.6 Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\,\text{asech}\,(cx)\right)^2}\,dx$$

input
```
integrate((d*x)**m/(a+b*asech(c*x))**2,x)
```

output
```
Integral((d*x)**m/(a + b*asech(c*x))**2, x)
```

3.73.7 Maxima [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 616, normalized size of antiderivative = 38.50

$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(b\,\text{arsech}\,(cx)+a\right)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*d^m*x^3 - d^m*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m + (c^2*d^m*x^3 -
d^m*x)*x^m)/((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) - (b^2*log(c) + b
^2*log(x) - a*b)*sqrt(c*x + 1)*sqrt(-c*x + 1) + a*b - (b^2*c^2*x^2 - sqrt(
c*x + 1)*sqrt(-c*x + 1)*b^2 - b^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) +
 (b^2*c^2*x^2 - b^2)*log(x)) + integrate(((c^2*d^m*(m + 3)*x^2 - d^m*(m +
1))*(c*x + 1)*(c*x - 1)*x^m + (c^4*d^m*(m + 2)*x^4 - c^2*d^m*(3*m + 5)*x^2
 + 2*d^m*(m + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m + (c^4*d^m*(m + 1)*x^4
- 2*c^2*d^m*(m + 1)*x^2 + d^m*(m + 1))*x^m)/((b^2*c^4*log(c) - a*b*c^4)*x^
4 - (b^2*log(c) + b^2*log(x) - a*b)*(c*x + 1)*(c*x - 1) - 2*(b^2*c^2*log(c
) - a*b*c^2)*x^2 + b^2*log(c) - 2*((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*lo
g(c) + a*b + (b^2*c^2*x^2 - b^2)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - a*
b - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 - (c*x + 1)*(c*x - 1)*b^2 - 2*(b^2*c^2*x^
2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) + b^2)*log(sqrt(c*x + 1)*sqrt(-c*x +
 1) + 1) + (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(x)), x)
```

3.73.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(b\,\text{arsech}\,(cx)+a\right)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arcsech(c*x) + a)^2, x)
```

3.73.9 Mupad [N/A]

Not integrable

Time = 3.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{(d\,x)^m}{\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

input
```
int((d*x)^m/(a + b*acosh(1/(c*x)))^2,x)
```

output
```
int((d*x)^m/(a + b*acosh(1/(c*x)))^2, x)
```

3.74 $\int (d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

3.74.1 Optimal result

Integrand size = 16, antiderivative size = 264

$$\begin{aligned}\int (d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = &-\frac{be(9c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^4} \\ &-\frac{bde^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2} \\ &-\frac{be^3x^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^2} \\ &+\frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e} \\ &+\frac{bd(2c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{2c^3} \\ &-\frac{bd^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{4e}\end{aligned}$$

output
```
1/4*(e*x+d)^4*(a+b*arcsech(c*x))/e+1/2*b*d*(2*c^2*d^2+e^2)*arcsin(c*x)*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^3-1/4*b*d^4*arctanh((-c^2*x^2+1)^(1/2))*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)/e-1/6*b*e*(9*c^2*d^2+e^2)*(1/(c*x+1))^(1/2)*(
c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/2*b*d*e^2*x*(1/(c*x+1))^(1/2)*(c*x+1
)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2-1/12*b*e^3*x^2*(1/(c*x+1))^(1/2)*(c*x+1)^(1
/2)*(-c^2*x^2+1)^(1/2)/c^2
```

3.74.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.72

$$\int (d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \frac{1}{4}\left(4ad^3x+6ad^2ex^2+4ade^2x^3+ae^3x^4 - \frac{be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(2e^2+c^2(18d^2+6dex+e^2x^2)\right)}{3c^4} + bx\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right)\text{sech}^{-1}(cx) + \frac{2ibd(2c^2d^2+e^2)\log\left(-2icx+2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)}{c^3}\right)$$

input
```
Integrate[(d + e*x)^3*(a + b*ArcSech[c*x]),x]
```

output
```
(4*a*d^3*x + 6*a*d^2*e*x^2 + 4*a*d*e^2*x^3 + a*e^3*x^4 - (b*e*Sqrt[(1 - c*
x)/(1 + c*x)]*(1 + c*x)*(2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)))/(3*c^4
) + b*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcSech[c*x] + ((2*I)*
b*d*(2*c^2*d^2 + e^2)*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*
x)])/c^3)/4
```

3.74.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.74, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6842, 541, 25, 2340, 27, 2340, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{6842}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(d+ex)^4}{x\sqrt{1-c^2x^2}}dx}{4e}+\frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{541}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{3c^2d^4+12c^2exd^3+12c^2e^3x^3d+2e^2\left(9c^2d^2+e^2\right)x^2}{x\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{e^4x^2\sqrt{1-c^2x^2}}{3c^2}\right)}{4e}+\frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{3c^2d^4+12c^2exd^3+12c^2e^3x^3d+2e^2\left(9c^2d^2+e^2\right)x^2}{x\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{e^4x^2\sqrt{1-c^2x^2}}{3c^2}\right)}{4e}+\frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{2340}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{-\frac{\int -\frac{2\left(3c^4d^4+6c^2e\left(2c^2d^2+e^2\right)xd+2c^2e^2\left(9c^2d^2+e^2\right)x^2\right)}{x\sqrt{1-c^2x^2}}dx}{2c^2}-6de^3x\sqrt{1-c^2x^2}}{3c^2}-\frac{e^4x^2\sqrt{1-c^2x^2}}{3c^2}\right)}{4e}+\frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\int \frac{3c^4d^4+6c^2e\left(2c^2d^2+e^2\right)xd+2c^2e^2\left(9c^2d^2+e^2\right)x^2}{x\sqrt{1-c^2x^2}}dx}{c^2}-6de^3x\sqrt{1-c^2x^2}}{3c^2}-\frac{e^4x^2\sqrt{1-c^2x^2}}{3c^2}\right)}{4e}+\frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{2340}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{-\frac{\int -\frac{3c^4d\left(c^2d^3+2e\left(2c^2d^2+e^2\right)x\right)}{x\sqrt{1-c^2x^2}}dx}{c^2}-2e^2\sqrt{1-c^2x^2}\left(9c^2d^2+e^2\right)}{c^2}-6de^3x\sqrt{1-c^2x^2}}{3c^2}-\frac{e^4x^2\sqrt{1-c^2x^2}}{3c^2}\right)}{4e}+\frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{27}$$

3.74. $\int (d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{3c^2d\int\frac{c^2d^3+2e\left(2c^2d^2+e^2\right)x}{x\sqrt{1-c^2x^2}}dx-2e^2\sqrt{1-c^2x^2}\left(9c^2d^2+e^2\right)}{c^2}-6de^3x\sqrt{1-c^2x^2}}{3c^2}-\frac{e^4x^2\sqrt{1-c^2x^2}}{3c^2}\right)}{4e}+\frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \; 538$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{3c^2d\left(c^2d^3\int\frac{1}{x\sqrt{1-c^2x^2}}dx+2e\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{1-c^2x^2}}dx\right)-2e^2\sqrt{1-c^2x^2}\left(9c^2d^2+e^2\right)}{c^2}-6de^3x\sqrt{1-c^2x^2}}{3c^2}-\frac{e^4x^2\sqrt{1-c^2x^2}}{3c^2}\right)}{4e}+\frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \; 223$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{3c^2d\left(c^2d^3\int\frac{1}{x\sqrt{1-c^2x^2}}dx+\frac{2e\arcsin(cx)\left(2c^2d^2+e^2\right)}{c}\right)-2e^2\sqrt{1-c^2x^2}\left(9c^2d^2+e^2\right)}{c^2}-6de^3x\sqrt{1-c^2x^2}}{3c^2}-\frac{e^4x^2\sqrt{1-c^2x^2}}{3c^2}\right)}{4e}+\frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \; 243$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{3c^2d\left(\frac{1}{2}c^2d^3\int\frac{1}{x^2\sqrt{1-c^2x^2}}dx^2+\frac{2e\arcsin(cx)\left(2c^2d^2+e^2\right)}{c}\right)-2e^2\sqrt{1-c^2x^2}\left(9c^2d^2+e^2\right)}{c^2}-6de^3x\sqrt{1-c^2x^2}}{3c^2}-\frac{e^4x^2\sqrt{1-c^2x^2}}{3c^2}\right)}{4e}+\frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \; 73$$

3.74. $\int (d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{3c^2d\left(\frac{2e\arcsin(cx)\left(2c^2d^2+e^2\right)}{c}-d^3\int\frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}\right)-2e^2\sqrt{1-c^2x^2}\left(9c^2d^2+e^2\right)}{c^2}-6de^3x\sqrt{1-c^2x^2}}{3c^2}-\frac{e^4x^2\sqrt{1-c^2x^2}}{3c^2}\right)}{4e}+\frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{221}$$

$$\frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{3c^2d\left(\frac{2e\arcsin(cx)\left(2c^2d^2+e^2\right)}{c}-c^2d^3\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-2e^2\sqrt{1-c^2x^2}\left(9c^2d^2+e^2\right)}{c^2}-6de^3x\sqrt{1-c^2x^2}}{3c^2}-\frac{e^4x^2\sqrt{1-c^2x^2}}{3c^2}\right)}{4e}$$

input
```
Int[(d + e*x)^3*(a + b*ArcSech[c*x]),x]
```

output
```
((d + e*x)^4*(a + b*ArcSech[c*x]))/(4*e) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
+ c*x]*(-1/3*(e^4*x^2*Sqrt[1 - c^2*x^2])/c^2 + (-6*d*e^3*x*Sqrt[1 - c^2*x^
2] + (-2*e^2*(9*c^2*d^2 + e^2)*Sqrt[1 - c^2*x^2] + 3*c^2*d*((2*e*(2*c^2*d^
2 + e^2)*ArcSin[c*x])/c - c^2*d^3*ArcTanh[Sqrt[1 - c^2*x^2]]))/c^2)/(3*c^2
)))/(4*e)
```

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6842
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Simp[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x)^(m + 1)/(x*
Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.74. $\int (d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

3.74.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00

method	result
parts	$\frac{a(ex+d)^4}{4e} + \frac{b\left(\frac{c\,e^3 \operatorname{arcsech}(cx)x^4}{4} + c\,e^2 \operatorname{arcsech}(cx)x^3 d + \frac{3ce \operatorname{arcsech}(cx)x^2 d^2}{2} + \operatorname{arcsech}(cx)xc\,d^3 + \frac{c \operatorname{arcsech}(cx)d^4}{4e} + \frac{\sqrt{-\frac{cx}{c}}}{}\right.}{}$
derivativedivides	$\frac{\frac{a(cex+cd)^4}{4c^3e} + \frac{b\left(\frac{\operatorname{arcsech}(cx)c^4d^4}{4e} + \operatorname{arcsech}(cx)c^4d^3x + \frac{3e \operatorname{arcsech}(cx)c^4d^2x^2}{2} + e^2 \operatorname{arcsech}(cx)c^4d\,x^3 + \frac{e^3 \operatorname{arcsech}(cx)c^4x^4}{4} - \frac{\sqrt{-\frac{cx-1}{cx}}}{}\right.}{}}{}$
default	$\frac{\frac{a(cex+cd)^4}{4c^3e} + \frac{b\left(\frac{\operatorname{arcsech}(cx)c^4d^4}{4e} + \operatorname{arcsech}(cx)c^4d^3x + \frac{3e \operatorname{arcsech}(cx)c^4d^2x^2}{2} + e^2 \operatorname{arcsech}(cx)c^4d\,x^3 + \frac{e^3 \operatorname{arcsech}(cx)c^4x^4}{4} - \frac{\sqrt{-\frac{cx-1}{cx}}}{}\right.}{}}{}$

input
```
int((e*x+d)^3*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x+d)^4/e+b/c*(1/4*c*e^3*arcsech(c*x)*x^4+c*e^2*arcsech(c*x)*x^3*d
+3/2*c*e*arcsech(c*x)*x^2*d^2+arcsech(c*x)*x*c*d^3+1/4*c/e*arcsech(c*x)*d^
4+1/12/c^2/e*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(-3*c^4*d^4*arctan
h(1/(-c^2*x^2+1)^(1/2))+12*c^3*d^3*e*arcsin(c*x)-e^4*(-c^2*x^2+1)^(1/2)*c^
2*x^2-6*c^2*d*e^3*x*(-c^2*x^2+1)^(1/2)-18*c^2*d^2*e^2*(-c^2*x^2+1)^(1/2)+6
*c*d*e^3*arcsin(c*x)-2*e^4*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2))
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(144) = 288$.

Time = 0.36 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.36

$$\int (d+ex)^3 \left(a + b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$= \frac{3\,ac^3e^3x^4 + 12\,ac^3de^2x^3 + 18\,ac^3d^2ex^2 + 12\,ac^3d^3x - 12\left(2\,bc^2d^3 + bde^2\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 1}{cx}\right) - 3\,(4\,bc}{}$$

input
```
integrate((e*x+d)^3*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*a*c^3*e^3*x^4 + 12*a*c^3*d*e^2*x^3 + 18*a*c^3*d^2*e*x^2 + 12*a*c^3
*d^3*x - 12*(2*b*c^2*d^3 + b*d*e^2)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2)) - 1)/(c*x)) - 3*(4*b*c^3*d^3 + 6*b*c^3*d^2*e + 4*b*c^3*d*e^2 + b*c^3*
e^3)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 3*(b*c^3*e^3*x^4 +
4*b*c^3*d*e^2*x^3 + 6*b*c^3*d^2*e*x^2 + 4*b*c^3*d^3*x - 4*b*c^3*d^3 - 6*b*
c^3*d^2*e - 4*b*c^3*d*e^2 - b*c^3*e^3)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2)) + 1)/(c*x)) - (b*c^2*e^3*x^3 + 6*b*c^2*d*e^2*x^2 + 2*(9*b*c^2*d^2*e +
 b*e^3)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^3
```

3.74.6 Sympy [F]

$$\int (d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (a + b \operatorname{asech}(cx)) (d+ex)^3 \, dx$$

input
```
integrate((e*x+d)**3*(a+b*asech(c*x)),x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x)**3, x)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.84

$$\begin{aligned}
&\int (d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx \\
&= \frac{1}{4} ae^3x^4 + ade^2x^3 + \frac{3}{2} ad^2ex^2 + \frac{3}{2} \left(x^2 \operatorname{arsech}(cx) - \frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c} \right) bd^2e \\
&+ \frac{1}{2} \left(2x^3 \operatorname{arsech}(cx) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c} \right) bde^2 \\
&+ \frac{1}{12} \left(3x^4 \operatorname{arsech}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} - 3x\sqrt{\frac{1}{c^2x^2}-1}}{c^3} \right) be^3 \\
&+ ad^3x + \frac{\left(cx \operatorname{arsech}(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)bd^3}{c}
\end{aligned}$$

input
```
integrate((e*x+d)^3*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*arcsech(c*x) - x*
sqrt(1/(c^2*x^2) - 1)/c)*b*d^2*e + 1/2*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*
x^2) - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^
2)/c)*b*d*e^2 + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2
) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*e^3 + a*d^3*x + (c*x*arcsech(c*x) -
arctan(sqrt(1/(c^2*x^2) - 1)))*b*d^3/c
```

3.74.8 Giac [F]

$$\int (d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (ex+d)^3 (b\,\text{arsech}\,(cx)+a) \, dx$$

input
```
integrate((e*x+d)^3*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(b*arcsech(c*x) + a), x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right) (d+e\,x)^3 \, dx$$

input
```
int((a + b*acosh(1/(c*x)))*(d + e*x)^3,x)
```

output
```
int((a + b*acosh(1/(c*x)))*(d + e*x)^3, x)
```

3.75 $\int (d+ex)^2 \left(a+b\text{sech}^{-1}(cx)\right) dx$

3.75.1 Optimal result

Integrand size = 16, antiderivative size = 201

$$\int (d+ex)^2 \left(a+b\text{sech}^{-1}(cx)\right) dx = -\frac{bde\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + \frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e} + \frac{b(6c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{6c^3} - \frac{bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{3e}$$

output
```
1/3*(e*x+d)^3*(a+b*arcsech(c*x))/e+1/6*b*(6*c^2*d^2+e^2)*arcsin(c*x)*(1/(c
*x+1))^(1/2)*(c*x+1)^(1/2)/c^3-1/3*b*d^3*arctanh((-c^2*x^2+1)^(1/2))*(1/(c
*x+1))^(1/2)*(c*x+1)^(1/2)/e-b*d*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x
^2+1)^(1/2)/c^2-1/6*b*e^2*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(
1/2)/c^2
```

3.75.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.73

$$\int (d+ex)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$
$$= \frac{-bce\sqrt{\frac{1-cx}{1+cx}}(1+cx)(6d+ex)+2ac^3x(3d^2+3dex+e^2x^2)+2bc^3x(3d^2+3dex+e^2x^2)\,\text{sech}^{-1}(cx)+ib(6}{6c^3}$$

input
```
Integrate[(d + e*x)^2*(a + b*ArcSech[c*x]),x]
```

output
```
(-(b*c*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(6*d + e*x)) + 2*a*c^3*x*(3*d
^2 + 3*d*e*x + e^2*x^2) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcSech[c*
x] + I*b*(6*c^2*d^2 + e^2)*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1
 + c*x)])/(6*c^3)
```

3.75.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.74, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {6842, 541, 25, 2340, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$
$$\Big\downarrow \; 6842$$
$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(d+ex)^3}{x\sqrt{1-c^2x^2}}dx}{3e}+\frac{(d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$
$$\Big\downarrow \; 541$$
$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{2c^2d^3+6c^2e^2x^2d+e\left(6c^2d^2+e^2\right)x}{x\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{e^3x\sqrt{1-c^2x^2}}{2c^2}\right)}{3e}+\frac{(d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$
$$\Big\downarrow \; 25$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{2c^2d^3+6c^2e^2x^2d+e\left(6c^2d^2+e^2\right)x}{x\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{e^3x\sqrt{1-c^2x^2}}{2c^2}\right)}{3e}+\frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{2340}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{-\frac{\int -\frac{c^2\left(2c^2d^3+e\left(6c^2d^2+e^2\right)x\right)}{x\sqrt{1-c^2x^2}}dx}{c^2}-6de^2\sqrt{1-c^2x^2}}{2c^2}-\frac{e^3x\sqrt{1-c^2x^2}}{2c^2}\right)}{3e}+\frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\int \frac{c^2\left(2c^2d^3+e\left(6c^2d^2+e^2\right)x\right)}{x\sqrt{1-c^2x^2}}dx}{c^2}-6de^2\sqrt{1-c^2x^2}}{2c^2}-\frac{e^3x\sqrt{1-c^2x^2}}{2c^2}\right)}{3e}+\frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{2c^2d^3+e\left(6c^2d^2+e^2\right)x}{x\sqrt{1-c^2x^2}}dx-6de^2\sqrt{1-c^2x^2}}{2c^2}-\frac{e^3x\sqrt{1-c^2x^2}}{2c^2}\right)}{3e}+\frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{538}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2c^2d^3\int \frac{1}{x\sqrt{1-c^2x^2}}dx+e\left(6c^2d^2+e^2\right)\int \frac{1}{\sqrt{1-c^2x^2}}dx-6de^2\sqrt{1-c^2x^2}}{2c^2}-\frac{e^3x\sqrt{1-c^2x^2}}{2c^2}\right)}{3e}+\frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{223}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2c^2d^3\int \frac{1}{x\sqrt{1-c^2x^2}}dx+\frac{e\arcsin(cx)\left(6c^2d^2+e^2\right)}{c}-6de^2\sqrt{1-c^2x^2}}{2c^2}-\frac{e^3x\sqrt{1-c^2x^2}}{2c^2}\right)}{3e}+\frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{243}$$

3.75. $\int (d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)dx$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{c^2d^3\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2+\frac{e\arcsin(cx)\left(6c^2d^2+e^2\right)}{c}-6de^2\sqrt{1-c^2x^2}}{2c^2}-\frac{e^3x\sqrt{1-c^2x^2}}{2c^2}\right)}{3e}+\frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{73}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{-2d^3\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}+\frac{e\arcsin(cx)\left(6c^2d^2+e^2\right)}{c}-6de^2\sqrt{1-c^2x^2}}{2c^2}-\frac{e^3x\sqrt{1-c^2x^2}}{2c^2}\right)}{3e}+\frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{221}$$

$$\frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{e\arcsin(cx)\left(6c^2d^2+e^2\right)}{c}-2c^2d^3\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-6de^2\sqrt{1-c^2x^2}}{2c^2}-\frac{e^3x\sqrt{1-c^2x^2}}{2c^2}\right)}{3e}$$

input
```
Int[(d + e*x)^2*(a + b*ArcSech[c*x]),x]
```

output
```
((d + e*x)^3*(a + b*ArcSech[c*x]))/(3*e) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
+ c*x]*(-1/2*(e^3*x*Sqrt[1 - c^2*x^2])/c^2 + (-6*d*e^2*Sqrt[1 - c^2*x^2] +
 (e*(6*c^2*d^2 + e^2)*ArcSin[c*x])/c - 2*c^2*d^3*ArcTanh[Sqrt[1 - c^2*x^2]
])/(2*c^2)))/(3*e)
```

3.75.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.75. $\int (d+ex)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

rule 6842
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Simp[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x)^(m + 1)/(x*
Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.75.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.99

method	result
parts	$\frac{a(ex+d)^3}{3e} + \frac{b\Bigg(\frac{c\,e^2 \operatorname{arcsech}(cx)x^3}{3} + ce \operatorname{arcsech}(cx)x^2 d + \operatorname{arcsech}(cx)xc\,d^2 + \frac{c \operatorname{arcsech}(cx)d^3}{3e} + \frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\Big(-2c^3d^3 \mathrm{a}}{}}{c}$
derivativedivides	$\frac{\frac{a(cex+cd)^3}{3c^2e} + \frac{b\Bigg(\frac{\operatorname{arcsech}(cx)c^3d^3}{3e} + \operatorname{arcsech}(cx)c^3d^2x + e \operatorname{arcsech}(cx)c^3d\,x^2 + \frac{e^2 \operatorname{arcsech}(cx)c^3x^3}{3} + \frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\Big(-2c^3d^3 \mathrm{arct}}{}}{c^2}}{c}$
default	$\frac{\frac{a(cex+cd)^3}{3c^2e} + \frac{b\Bigg(\frac{\operatorname{arcsech}(cx)c^3d^3}{3e} + \operatorname{arcsech}(cx)c^3d^2x + e \operatorname{arcsech}(cx)c^3d\,x^2 + \frac{e^2 \operatorname{arcsech}(cx)c^3x^3}{3} + \frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\Big(-2c^3d^3 \mathrm{arct}}{}}{c^2}}{c}$

input
```
int((e*x+d)^2*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*(e*x+d)^3/e+b/c*(1/3*c*e^2*arcsech(c*x)*x^3+c*e*arcsech(c*x)*x^2*d+a
rcsech(c*x)*x*c*d^2+1/3*c/e*arcsech(c*x)*d^3+1/6/c/e*(-(c*x-1)/c/x)^(1/2)*
x*((c*x+1)/c/x)^(1/2)*(-2*c^3*d^3*arctanh(1/(-c^2*x^2+1)^(1/2))+6*c^2*d^2*
e*arcsin(c*x)-6*c*d*e^2*(-c^2*x^2+1)^(1/2)-e^3*c*x*(-c^2*x^2+1)^(1/2)+e^3*
arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

3.75.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(107) = 214$.

Time = 0.33 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.39

$$\int (d+ex)^2 \left(a + b\operatorname{sech}^{-1}(cx)\right)\,dx$$

$$= \frac{2\,ac^3e^2x^3 + 6\,ac^3dex^2 + 6\,ac^3d^2x - 2\left(6\,bc^2d^2 + be^2\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) - 2\left(3\,bc^3d^2 + 3\,bc^3de + bc^3e^2\right.}{}$$

input
```
integrate((e*x+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e^2*x^3 + 6*a*c^3*d*e*x^2 + 6*a*c^3*d^2*x - 2*(6*b*c^2*d^2 +
b*e^2)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - 2*(3*b*c^3
*d^2 + 3*b*c^3*d*e + b*c^3*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) -
1)/x) + 2*(b*c^3*e^2*x^3 + 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x - 3*b*c^3*d^2 -
 3*b*c^3*d*e - b*c^3*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*
x)) - (b*c^2*e^2*x^2 + 6*b*c^2*d*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^3
```

3.75.6 Sympy [F]

$$\int (d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int\left(a+b\operatorname{asech}(cx)\right)(d+ex)^2\,dx$$

input
```
integrate((e*x+d)**2*(a+b*asech(c*x)),x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x)**2, x)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.76

$$\begin{aligned}\int (d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx &= \frac{1}{3}\,ae^2x^3+adex^2+\left(x^2\operatorname{arsech}(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)bde\\ &\quad+\frac{1}{6}\left(2\,x^3\operatorname{arsech}(cx)-\frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)be^2\\ &\quad+ad^2x+\frac{\left(cx\operatorname{arsech}(cx)-\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)bd^2}{c}\end{aligned}$$

input
```
integrate((e*x+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)/c)
*b*d*e + 1/6*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*x^2
) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b*e^2 + a*d^2*x + (c
*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d^2/c
```

3.75.8 Giac [F]

$$\int (d+ex)^2 \left(a+b\text{sech}^{-1}(cx)\right)\,dx = \int (ex+d)^2 (b\,\text{arsech}\,(cx)+a)\,dx$$

input
```
integrate((e*x+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(b*arcsech(c*x) + a), x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(a+b\text{sech}^{-1}(cx)\right)\,dx = \int \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right) (d+e\,x)^2\,dx$$

input
```
int((a + b*acosh(1/(c*x)))*(d + e*x)^2,x)
```

output
```
int((a + b*acosh(1/(c*x)))*(d + e*x)^2, x)
```

3.76 $\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.76.1 Optimal result

Integrand size = 14, antiderivative size = 142

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=-\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}+\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c}-\frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{2e}$$

output
```
1/2*(e*x+d)^2*(a+b*arcsech(c*x))/e+b*d*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+
1)^(1/2)/c-1/2*b*d^2*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)
^(1/2)/e-1/2*b*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

3.76.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx = adx+\frac{1}{2}aex^2+be\left(-\frac{1}{2c^2}-\frac{x}{2c}\right)\sqrt{\frac{1-cx}{1+cx}} + bdx\text{sech}^{-1}(cx)+\frac{1}{2}bex^2\text{sech}^{-1}(cx) + \frac{2bd\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\arctan\left(\frac{\sqrt{1-c^2x^2}}{1-cx}\right)}{c-c^2x}$$

input
```
Integrate[(d + e*x)*(a + b*ArcSech[c*x]),x]
```

output
```
a*d*x + (a*e*x^2)/2 + b*e*(-1/2*1/c^2 - x/(2*c))*Sqrt[(1 - c*x)/(1 + c*x)]
 + b*d*x*ArcSech[c*x] + (b*e*x^2*ArcSech[c*x])/2 + (2*b*d*Sqrt[(1 - c*x)/(
1 + c*x)]*Sqrt[1 - c^2*x^2]*ArcTan[Sqrt[1 - c^2*x^2]/(1 - c*x)])/(c - c^2*
x)
```

3.76.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.74, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6842, 541, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6842}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{(d+ex)^2}{x\sqrt{1-c^2x^2}}dx}{2e}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}$$

$$\Big\downarrow \text{541}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{c^2d(d+2ex)}{x\sqrt{1-c^2x^2}}dx}{c^2}-\frac{e^2\sqrt{1-c^2x^2}}{c^2}\right)}{2e}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{c^2d(d+2ex)}{x\sqrt{1-c^2x^2}}dx}{c^2}-\frac{e^2\sqrt{1-c^2x^2}}{c^2}\right)}{2e}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d\int \frac{d+2ex}{x\sqrt{1-c^2x^2}}dx-\frac{e^2\sqrt{1-c^2x^2}}{c^2}\right)}{2e}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}$$

$$\Big\downarrow \text{538}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d\left(d\int \frac{1}{x\sqrt{1-c^2x^2}}dx+2e\int \frac{1}{\sqrt{1-c^2x^2}}dx\right)-\frac{e^2\sqrt{1-c^2x^2}}{c^2}\right)}{2e}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}$$

$$\Big\downarrow \text{223}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d\left(d\int \frac{1}{x\sqrt{1-c^2x^2}}dx+\frac{2e\arcsin(cx)}{c}\right)-\frac{e^2\sqrt{1-c^2x^2}}{c^2}\right)}{2e}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}$$

$$\Big\downarrow \text{243}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d\left(\frac{1}{2}d\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2+\frac{2e\arcsin(cx)}{c}\right)-\frac{e^2\sqrt{1-c^2x^2}}{c^2}\right)}{2e}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}$$

$$\Big\downarrow \text{73}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d\left(\frac{2e\arcsin(cx)}{c}-\frac{d\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2}\right)-\frac{e^2\sqrt{1-c^2x^2}}{c^2}\right)}{2e}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}$$

$$\Big\downarrow \text{221}$$

$$\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d\left(\frac{2e\arcsin(cx)}{c}-d\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{e^2\sqrt{1-c^2x^2}}{c^2}\right)}{2e}$$

input
```
Int[(d + e*x)*(a + b*ArcSech[c*x]),x]
```

3.76. $\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

output
```
((d + e*x)^2*(a + b*ArcSech[c*x]))/(2*e) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
+ c*x]*(-((e^2*Sqrt[1 - c^2*x^2])/c^2) + d*((2*e*ArcSin[c*x])/c - d*ArcTan
h[Sqrt[1 - c^2*x^2]])))/(2*e)
```

3.76.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6842
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Simp[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x)^(m + 1)/(x*
Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.76.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

method	result	size
parts	$a\left(\frac{1}{2}e\,x^2+dx\right)+\frac{b\left(\frac{c\ \operatorname{arcsech}(cx)e\,x^2}{2}+\operatorname{arcsech}(cx)dxc+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,\left(2dc\arcsin(cx)-e\sqrt{-c^2x^2+1}\right)}{2\sqrt{-c^2x^2+1}}\right)}{c}$	107
derivativedivides	$\frac{\frac{a\left(d\,c^2x+\frac{1}{2}e\,c^2x^2\right)}{c}+\frac{b\left(\operatorname{arcsech}(cx)d\,c^2x+\frac{\operatorname{arcsech}(cx)e\,c^2x^2}{2}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(2dc\arcsin(cx)-e\sqrt{-c^2x^2+1}\right)}{2\sqrt{-c^2x^2+1}}\right)}{c}}{c}$	125
default	$\frac{\frac{a\left(d\,c^2x+\frac{1}{2}e\,c^2x^2\right)}{c}+\frac{b\left(\operatorname{arcsech}(cx)d\,c^2x+\frac{\operatorname{arcsech}(cx)e\,c^2x^2}{2}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(2dc\arcsin(cx)-e\sqrt{-c^2x^2+1}\right)}{2\sqrt{-c^2x^2+1}}\right)}{c}}{c}$	125

input
```
int((e*x+d)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e*x^2+d*x)+b/c*(1/2*c*arcsech(c*x)*e*x^2+arcsech(c*x)*d*x*c+1/2*(-(
c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(2*d*c*arcsin(c*x)-e*(-c^2*x^2+1)^
(1/2))/(-c^2*x^2+1)^(1/2))
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(72) = 144.

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.25

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{acex^2+2\,acdx-bex\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-4\,bd\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-(2\,bcd+bce)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)+(bce}{2\,c}$$

input
```
integrate((e*x+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/2*(a*c*e*x^2 + 2*a*c*d*x - b*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 4*b*d*
arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - (2*b*c*d + b*c*e)
*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c*e*x^2 + 2*b*c*d*x
- 2*b*c*d - b*c*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c
```

3.76.6 Sympy [F]

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (a+b\operatorname{asech}(cx))\,(d+ex)\,dx$$

input
```
integrate((e*x+d)*(a+b*asech(c*x)),x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x), x)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\frac{1}{2}\,aex^2+\frac{1}{2}\left(x^2\operatorname{arsech}(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)be$$
$$+adx+\frac{\left(cx\operatorname{arsech}(cx)-\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)bd}{c}$$

input
```
integrate((e*x+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + 1/2*(x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)/c)*b*e + a*d
*x + (c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d/c
```

3.76.8 Giac [F]

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (ex+d)(b\,\text{arsech}\,(cx)+a)\,dx$$

input
```
integrate((e*x+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)*(b*arcsech(c*x) + a), x)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\frac{a\,x\,(2\,d+e\,x)}{2}+\frac{b\,d\,\mathrm{atan}\left(\frac{1}{\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}\right)}{c}+\frac{b\,e\,x^2\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{2}+b\,d\,x\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)-\frac{b\,e\,x\,\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}{2\,c}$$

input
```
int((a + b*acosh(1/(c*x)))*(d + e*x),x)
```

output
```
(a*x*(2*d + e*x))/2 + (b*d*atan(1/((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2)
)))/c + (b*e*x^2*acosh(1/(c*x)))/2 + b*d*x*acosh(1/(c*x)) - (b*e*x*(1/(c*x
) - 1)^(1/2)*(1/(c*x) + 1)^(1/2))/(2*c)
```

3.77 $\int \left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.77.1 Optimal result

Integrand size = 8, antiderivative size = 40

$$\int \left(a+b\text{sech}^{-1}(cx)\right)\,dx=ax+bx\text{sech}^{-1}(cx)+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c}$$

output
```
a*x+b*x*arcsech(c*x)+b*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c
```

3.77.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

$$\int \left(a+b\text{sech}^{-1}(cx)\right)\,dx=ax+bx\text{sech}^{-1}(cx)+\frac{2b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\arctan\left(\frac{\sqrt{1-c^2x^2}}{1-cx}\right)}{c-c^2x}$$

input
```
Integrate[a + b*ArcSech[c*x],x]
```

output
```
a*x + b*x*ArcSech[c*x] + (2*b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*
ArcTan[Sqrt[1 - c^2*x^2]/(1 - c*x)])/(c - c^2*x)
```

3.77.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)}{c}+bx\text{sech}^{-1}(cx)$$

input
```
Int[a + b*ArcSech[c*x],x]
```

output
```
a*x + b*x*ArcSech[c*x] + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x]
)/c
```

3.77.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.77.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

method	result	size
default	$ax+bx\,\text{arcsech}\,(cx)-\frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)}{c}$	42
parts	$ax+bx\,\text{arcsech}\,(cx)-\frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)}{c}$	42
derivativedivides	$\frac{acx+b\left(cx\,\text{arcsech}(cx)-\arctan\left(\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\right)}{c}$	46

input
```
int(a+b*arcsech(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arcsech(c*x)-b/c*arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
```

3.77.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(20) = 40$.

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.98

$$\int \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{acx - bc\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 2\,b\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + (bcx-bc)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{c}$$

input
```
integrate(a+b*arcsech(c*x),x, algorithm="fricas")
```

output
```
(a*c*x - b*c*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2*b*arctan(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + (b*c*x - b*c)*log((c*x*s
qrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c
```

3.77.6 Sympy [F]

$$\int \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \int \left(a+b\operatorname{asech}(cx)\right)\,dx$$

input
```
integrate(a+b*asech(c*x),x)
```

output
```
Integral(a + b*asech(c*x), x)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \left(a+b\operatorname{sech}^{-1}(cx)\right) dx = ax + \frac{\left(cx\operatorname{arsech}(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)b}{c}$$

input
```
integrate(a+b*arcsech(c*x),x, algorithm="maxima")
```

output
```
a*x + (c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b/c
```

3.77.8 Giac [F]

$$\int \left(a+b\operatorname{sech}^{-1}(cx)\right) dx = \int b\operatorname{arsech}(cx) + a\, dx$$

input
```
integrate(a+b*arcsech(c*x),x, algorithm="giac")
```

output
```
integrate(b*arcsech(c*x) + a, x)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(a+b\operatorname{sech}^{-1}(cx)\right) dx = a\,x + b\,x\,\operatorname{acosh}\left(\frac{1}{c\,x}\right) + \frac{b\,\operatorname{atan}\left(\frac{1}{\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}\right)}{c}$$

input
```
int(a + b*acosh(1/(c*x)),x)
```

output
```
a*x + b*x*acosh(1/(c*x)) + (b*atan(1/((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1
/2))))/c
```

3.78 $\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex}\,dx$

3.78.1 Optimal result

Integrand size = 16, antiderivative size = 229

$$\begin{aligned}\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex}\,dx =& -\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{e}\\
&+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e}\\
&+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e}\\
&+\frac{b\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)}{2e}\\
&-\frac{b\,\text{PolyLog}\left(2,-\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e}\\
&-\frac{b\,\text{PolyLog}\left(2,-\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e}\end{aligned}$$

output
```
-(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/e+(
a+b*arcsech(c*x))*ln(1+(e-(-c^2*d^2+e^2)^(1/2))/c/d/(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2)))/e+(a+b*arcsech(c*x))*ln(1+(e+(-c^2*d^2+e^2)^(1/2))/c/d
/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/e+1/2*b*polylog(2,-1/(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/e-b*polylog(2,(-e+(-c^2*d^2+e^2)^(1/2))/
c/d/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/e-b*polylog(2,(-e-(-c^2*d^2+
e^2)^(1/2))/c/d/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/e
```

3.78.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.72

$$\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex}\,dx=\frac{a\log(d+ex)}{e} + \frac{b\left(\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)-2\left(-4i\arcsin\left(\frac{\sqrt{1+\frac{e}{cd}}}{\sqrt{2}}\right)\text{arctanh}\left(\frac{(-cd+e)\tanh\left(\frac{1}{2}\text{sech}^{-1}(cx)\right)}{\sqrt{-c^2d^2+e^2}}\right)+\text{sech}^{-1}(c\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(d + e*x),x]
```

output
```
(a*Log[d + e*x])/e + (b*(PolyLog[2, -E^(-2*ArcSech[c*x])] - 2*((-4*I)*ArcS
in[Sqrt[1 + e/(c*d)]/Sqrt[2]]*ArcTanh[((-(c*d) + e)*Tanh[ArcSech[c*x]/2])/
Sqrt[-(c^2*d^2) + e^2]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcS
ech[c*x]*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])] + (2*I
)*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]]*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(
c*d*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (e + Sqrt[-(c^2*d^2) + e^2])/(
c*d*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]]*Log[1 + (e
+ Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])] + PolyLog[2, (-e + Sqrt[-(
c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])] + PolyLog[2, -((e + Sqrt[-(c^2*d^2)
 + e^2])/(c*d*E^ArcSech[c*x]))])))/(2*e)
```

3.78.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6841, 2998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex}\,dx$$

$$\Big\downarrow \text{ 6841}$$

$$\frac{b\int \frac{\sqrt{\frac{1-cx}{cx+1}}\log\left(\frac{e^{-\text{sech}^{-1}(cx)}\left(e-\sqrt{e^2-c^2d^2}\right)}{cd}+1\right)}{x(1-cx)}dx}{e}+\frac{b\int \frac{\sqrt{\frac{1-cx}{cx+1}}\log\left(\frac{e^{-\text{sech}^{-1}(cx)}\left(e+\sqrt{e^2-c^2d^2}\right)}{cd}+1\right)}{x(1-cx)}dx}{e}-\frac{b\int \frac{\sqrt{\frac{1-cx}{cx+1}}\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{x(1-cx)}dx}{e}+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}+1\right)}{e}+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{\left(\sqrt{e^2-c^2d^2}+e\right)e^{-\text{sech}^{-1}(cx)}}{cd}+1\right)}{e}-\frac{\log\left(e^{-2\text{sech}^{-1}(cx)}+1\right)\left(a+b\text{sech}^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{ 2998}$$

$$\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}+1\right)}{e}+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{\left(\sqrt{e^2-c^2d^2}+e\right)e^{-\text{sech}^{-1}(cx)}}{cd}+1\right)}{e}-\frac{\log\left(e^{-2\text{sech}^{-1}(cx)}+1\right)\left(a+b\text{sech}^{-1}(cx)\right)}{e}-\frac{b\,\text{PolyLog}\left(2,-\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e}-\frac{b\,\text{PolyLog}\left(2,-\frac{\left(e+\sqrt{e^2-c^2d^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e}+\frac{b\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)}{2e}$$

input
```
Int[(a + b*ArcSech[c*x])/(d + e*x),x]
```

output
```
-(((a + b*ArcSech[c*x])*Log[1 + E^(-2*ArcSech[c*x])])/e) + ((a + b*ArcSech
[c*x])*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])])/e + ((a
 + b*ArcSech[c*x])*Log[1 + (e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x
])])/e + (b*PolyLog[2, -E^(-2*ArcSech[c*x])])/(2*e) - (b*PolyLog[2, -((e -
 Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x]))])/e - (b*PolyLog[2, -((e +
Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x]))])/e
```

3.78.3.1 Defintions of rubi rules used

rule 2998
```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

rule 6841
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :>
Simp[(a + b*ArcSech[c*x])*(Log[1 + (e - Sqrt[(-c^2)*d^2 + e^2])/(c*d*E^ArcS
ech[c*x])]/e), x] + (Simp[(a + b*ArcSech[c*x])*(Log[1 + (e + Sqrt[(-c^2)*d^
2 + e^2])/(c*d*E^ArcSech[c*x])]/e), x] - Simp[(a + b*ArcSech[c*x])*(Log[1 +
 1/E^(2*ArcSech[c*x])]/e), x] + Simp[b/e   Int[(Sqrt[(1 - c*x)/(1 + c*x)]*L
og[1 + (e - Sqrt[(-c^2)*d^2 + e^2])/(c*d*E^ArcSech[c*x])])/(x*(1 - c*x)), x
], x] + Simp[b/e   Int[(Sqrt[(1 - c*x)/(1 + c*x)]*Log[1 + (e + Sqrt[(-c^2)*
d^2 + e^2])/(c*d*E^ArcSech[c*x])])/(x*(1 - c*x)), x], x] - Simp[b/e   Int[(
Sqrt[(1 - c*x)/(1 + c*x)]*Log[1 + 1/E^(2*ArcSech[c*x])])/(x*(1 - c*x)), x],
 x]) /; FreeQ[{a, b, c, d, e}, x]
```

3.78.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 511, normalized size of antiderivative = 2.23

method	result
parts	$\frac{a\ln(ex+d)}{e}+\frac{b\,\mathrm{arcsech}(cx)\ln\left(\frac{-cd\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)+\sqrt{-c^2d^2+e^2}-e}{-e+\sqrt{-c^2d^2+e^2}}\right)}{e}+\frac{b\,\mathrm{arcsech}(cx)\ln\left(\frac{cd\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\right.}{e+\sqrt{}}\right.}{e}$
derivativedivides	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(\frac{\mathrm{arcsech}(cx)\ln\left(\frac{-cd\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)+\sqrt{-c^2d^2+e^2}-e}{-e+\sqrt{-c^2d^2+e^2}}\right)}{e}+\frac{\mathrm{arcsech}(cx)\ln\left(\frac{cd\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right.}{e+\sqrt{-c^2d^2}}\right.}{e}\right.}{}$
default	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(\frac{\mathrm{arcsech}(cx)\ln\left(\frac{-cd\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)+\sqrt{-c^2d^2+e^2}-e}{-e+\sqrt{-c^2d^2+e^2}}\right)}{e}+\frac{\mathrm{arcsech}(cx)\ln\left(\frac{cd\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right.}{e+\sqrt{-c^2d^2}}\right.}{e}\right.}{}$

input
```
int((a+b*arcsech(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*ln(e*x+d)/e+b/e*arcsech(c*x)*ln((-c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2))+(-c^2*d^2+e^2)^(1/2)-e)/(-e+(-c^2*d^2+e^2)^(1/2)))+b/e*arcsech(c*x)
*ln((c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))+(-c^2*d^2+e^2)^(1/2)+e)/
(e+(-c^2*d^2+e^2)^(1/2)))+b/e*dilog((-c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))+(-c^2*d^2+e^2)^(1/2)-e)/(-e+(-c^2*d^2+e^2)^(1/2)))+b/e*dilog((c*d
*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))+(-c^2*d^2+e^2)^(1/2)+e)/(e+(-c^2
*d^2+e^2)^(1/2)))-b/e*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2)))-b/e*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
)-b/e*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e*dilog(1-I*(1
/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))
```

3.78.5 Fricas [F]

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{d+ex}\,dx=\int \frac{b\,\mathrm{arsech}\,(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)/(e*x + d), x)
```

3.78.6 Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{d + ex}\,dx = \int \frac{a + b\,\text{asech}\,(cx)}{d + ex}\,dx$$

input
```
integrate((a+b*asech(c*x))/(e*x+d),x)
```

output
```
Integral((a + b*asech(c*x))/(d + e*x), x)
```

3.78.7 Maxima [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{d + ex}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{ex + d}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x + d),
x) + a*log(e*x + d)/e
```

3.78.8 Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{d + ex}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{ex + d}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(e*x + d), x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex}\,dx=\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{d+e\,x}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(d + e*x),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(d + e*x), x)
```

3.79 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2}\,dx$

3.79.1 Optimal result

Integrand size = 16, antiderivative size = 147

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{a+b\text{sech}^{-1}(cx)}{e(d+ex)}+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{d\sqrt{c^2d^2-e^2}}+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{de}$$

output
```
(-a-b*arcsech(c*x))/e/(e*x+d)+b*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1
/2)*(c*x+1)^(1/2)/d/e+b*arctan((c^2*d*x+e)/(c^2*d^2-e^2)^(1/2)/(-c^2*x^2+1
)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d/(c^2*d^2-e^2)^(1/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.51

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{a}{e(d+ex)}-\frac{b\text{sech}^{-1}(cx)}{e(d+ex)}-\frac{b\log(x)}{de}+\frac{b\log(d+ex)}{d\sqrt{-c^2d^2+e^2}}+\frac{b\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{de}-\frac{b\log\left(e+c^2dx+\sqrt{-c^2d^2+e^2}\sqrt{\frac{1-cx}{1+cx}}+c\sqrt{-c^2d^2+e^2}\,x\sqrt{\frac{1-cx}{1+cx}}\right)}{d\sqrt{-c^2d^2+e^2}}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(d + e*x)^2,x]
```

output
```
-(a/(e*(d + e*x))) - (b*ArcSech[c*x])/(e*(d + e*x)) - (b*Log[x])/(d*e) + (
b*Log[d + e*x])/(d*Sqrt[-(c^2*d^2) + e^2]) + (b*Log[1 + Sqrt[(1 - c*x)/(1
+ c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/(d*e) - (b*Log[e + c^2*d*x + Sqr
t[-(c^2*d^2) + e^2]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[-(c^2*d^2) + e^2]*x
*Sqrt[(1 - c*x)/(1 + c*x)]])/(d*Sqrt[-(c^2*d^2) + e^2])
```

3.79.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6842, 617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2}\,dx$$

$$\Big\downarrow \ 6842$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x(d+ex)\sqrt{1-c^2x^2}}dx}{e}-\frac{a+b\text{sech}^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \ 617$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \left(\frac{1}{dx\sqrt{1-c^2x^2}}-\frac{e}{d(d+ex)\sqrt{1-c^2x^2}}\right)dx}{e}-\frac{a+b\text{sech}^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \ 2009$$

$$-\frac{a+b\text{sech}^{-1}(cx)}{e(d+ex)}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{e\arctan\left(\frac{c^2dx+e}{\sqrt{1-c^2x^2}\sqrt{c^2d^2-e^2}}\right)}{d\sqrt{c^2d^2-e^2}}-\frac{\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d}\right)}{e}$$

input
```
Int[(a + b*ArcSech[c*x])/(d + e*x)^2,x]
```

output
```
-((a + b*ArcSech[c*x])/(e*(d + e*x))) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c
*x]*(-((e*ArcTan[(e + c^2*d*x)/(Sqrt[c^2*d^2 - e^2]*Sqrt[1 - c^2*x^2])])/(
d*Sqrt[c^2*d^2 - e^2])) - ArcTanh[Sqrt[1 - c^2*x^2]]/d))/e
```

3.79.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6842
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Simp[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x)^(m + 1)/(x*
Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.79.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.41

method	result
parts	$-\frac{a}{(ex+d)e}+\frac{b\left(-\frac{c^2\operatorname{arcsech}(cx)}{(cex+cd)e}+\frac{c^2\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,\left(\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)-\ln\left(\frac{2\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,\sqrt{-c^2x^2+\cdots}}{cex+cd}\cdots\right.\right.}{e\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,d\sqrt{-c^2x^2+1}}\right.}{c}$
derivativedivides	$\frac{-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\operatorname{arcsech}(cx)}{(cex+cd)e}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,\left(\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)-\ln\left(\frac{2\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,\sqrt{-c^2x^2+1}\,e+\cdots}{cex+cd}\cdots\right.\right.}{e\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,d\sqrt{-c^2x^2+1}}\right.}{c}$
default	$\frac{-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\operatorname{arcsech}(cx)}{(cex+cd)e}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,\left(\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)-\ln\left(\frac{2\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,\sqrt{-c^2x^2+1}\,e+\cdots}{cex+cd}\cdots\right.\right.}{e\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,d\sqrt{-c^2x^2+1}}\right.}{c}$

input
```
int((a+b*arcsech(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/(e*x+d)/e+b/c*(-c^2/(c*e*x+c*d)/e*arcsech(c*x)+c^2/e*(-(c*x-1)/c/x)^(1/
2)*x*((c*x+1)/c/x)^(1/2)*((-(c^2*d^2-e^2)/e^2)^(1/2)*arctanh(1/(-c^2*x^2+1
)^(1/2))-ln(2*((-(c^2*d^2-e^2)/e^2)^(1/2)*(-c^2*x^2+1)^(1/2)*e+d*c^2*x+e)/
(c*e*x+c*d)))/(-(c^2*d^2-e^2)/e^2)^(1/2)/d/(-c^2*x^2+1)^(1/2))
```

3.79.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(99) = 198$.

Time = 0.30 (sec) , antiderivative size = 578, normalized size of antiderivative = 3.93

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^2}\, dx$$
$$= \left[-\frac{ac^2d^3 - ade^2 + \sqrt{-c^2d^2 + e^2}(be^2x + bde)\log\left(\frac{c^2dex-(c^3d^2-ce^2)x\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+e^2-\sqrt{-c^2d^2+e^2}\left(c^2dx+cex\sqrt{-\frac{c^2x^2-1}{c^2x^2}}\right.}{ex+d}\right.}{c^2d^4e - d^2e^3 + (c}\right.$$
$$-\frac{ac^2d^3 - ade^2 - 2\sqrt{c^2d^2 - e^2}(be^2x + bde)\arctan\left(-\frac{\sqrt{c^2d^2-e^2}cdx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-\sqrt{c^2d^2-e^2}(ex+d)}{(c^2d^2-e^2)x}\right) + (bc^2d^3 - bde}{c^2d^4e - d^2e^3 + (c^2d^3e^2 - de}$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[-(a*c^2*d^3 - a*d*e^2 + sqrt(-c^2*d^2 + e^2)*(b*e^2*x + b*d*e)*log((c^2*d
*e*x - (c^3*d^2 - c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2 - sqrt(-c^
2*d^2 + e^2)*(c^2*d*x + c*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e))/(e*x +
d)) + (b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*log((c*x*sqrt(-(c^2*
x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c^2*d^3 - b*d*e^2)*log((c*x*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4
)*x), -(a*c^2*d^3 - a*d*e^2 - 2*sqrt(c^2*d^2 - e^2)*(b*e^2*x + b*d*e)*arct
an(-(sqrt(c^2*d^2 - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(c^2*d
^2 - e^2)*(e*x + d))/((c^2*d^2 - e^2)*x)) + (b*c^2*d^3 - b*d*e^2 + (b*c^2*
d^2*e - b*e^3)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c^2
*d^3 - b*d*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*
d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x)]
```

3.79.6 Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{a+b\,\text{asech}\,(cx)}{(d+ex)^2}\,dx$$

input
```
integrate((a+b*asech(c*x))/(e*x+d)**2,x)
```

output
```
Integral((a + b*asech(c*x))/(d + e*x)**2, x)
```

3.79.7 Maxima [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{b\,\text{arsech}\,(cx)+a}{(ex+d)^2}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

output
```
(c^2*integrate(x^2/(c^2*d^2*x^2 + (c^2*d^2*x^2 - d^2 + (c^2*d*e*x^2 - d*e)
*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - d^2 + (c^2*d*e*x^2 - d*e)*x), x) + (x*l
og(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - x*log(c) - x*log(x))/(d*e*x + d^2)
- integrate(1/(c^2*d^2*x^2 - d^2 + (c^2*d*e*x^2 - d*e)*x), x))*b - a/(e^2*
x + d*e)
```

3.79.8 Giac [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{b\,\text{arsech}\,(cx)+a}{(ex+d)^2}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(e*x + d)^2, x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex)^2}\, dx = \int \frac{a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{\left(d + e\,x\right)^2}\, dx$$

input
```
int((a + b*acosh(1/(c*x)))/(d + e*x)^2,x)
```

output
```
int((a + b*acosh(1/(c*x)))/(d + e*x)^2, x)
```

3.80 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^3} dx$

3.80.1 Optimal result

Integrand size = 16, antiderivative size = 306

$$\begin{aligned}\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^3}\,dx &= \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2d\left(c^2d^2-e^2\right)(d+ex)}-\frac{a+b\text{sech}^{-1}(cx)}{2e(d+ex)^2}\\&\quad+\frac{bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{2\left(c^2d^2-e^2\right)^{3/2}}\\&\quad+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{2d^2\sqrt{c^2d^2-e^2}}\\&\quad+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{2d^2e}\end{aligned}$$

output
```
1/2*(-a-b*arcsech(c*x))/e/(e*x+d)^2+1/2*b*c^2*arctan((c^2*d*x+e)/(c^2*d^2-
e^2)^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/(c^2*d^2-e^
2)^(3/2)+1/2*b*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)
/d^2/e+1/2*b*arctan((c^2*d*x+e)/(c^2*d^2-e^2)^(1/2)/(-c^2*x^2+1)^(1/2))*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*d^2-e^2)^(1/2)+1/2*b*e*(1/(c*x+1))^
(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d^2-e^2)/(e*x+d)
```

3.80.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^3}\,dx$$
$$=\frac{1}{2}\left(-\frac{a}{e(d+ex)^2}+\frac{b\sqrt{\frac{1-cx}{1+cx}}(e+cex)}{d(cd-e)(cd+e)(d+ex)}-\frac{b\text{sech}^{-1}(cx)}{e(d+ex)^2}-\frac{b\log(x)}{d^2e}\right.$$
$$+\frac{b\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{d^2e}$$
$$\left.-\frac{ib(2c^2d^2-e^2)\log\left(\frac{4d^2e\sqrt{c^2d^2-e^2}\left(ie+ic^2dx+\sqrt{c^2d^2-e^2}\sqrt{\frac{1-cx}{1+cx}}+c\sqrt{c^2d^2-e^2}x\sqrt{\frac{1-cx}{1+cx}}\right)}{b(2c^2d^2-e^2)(d+ex)}\right)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}}\right)$$

input
```
Integrate[(a + b*ArcSech[c*x])/(d + e*x)^3,x]
```

output
```
(-(a/(e*(d + e*x)^2)) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(e + c*e*x))/(d*(c*d
 - e)*(c*d + e)*(d + e*x)) - (b*ArcSech[c*x])/(e*(d + e*x)^2) - (b*Log[x])/
(d^2*e) + (b*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c
*x)]])/(d^2*e) - (I*b*(2*c^2*d^2 - e^2)*Log[(4*d^2*e*Sqrt[c^2*d^2 - e^2]*(
I*e + I*c^2*d*x + Sqrt[c^2*d^2 - e^2]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c
^2*d^2 - e^2]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/(b*(2*c^2*d^2 - e^2)*(d + e*x)
)])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d^2 - e^2]))/2
```

3.80.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6842, 617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^3}\,dx$$

$\Big\downarrow$ 6842

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x(d+ex)^2\sqrt{1-c^2x^2}}dx}{2e}-\frac{a+b\text{sech}^{-1}(cx)}{2e(d+ex)^2}$$

$\Big\downarrow$ 617

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \left(-\frac{e}{d^2(d+ex)\sqrt{1-c^2x^2}}-\frac{e}{d(d+ex)^2\sqrt{1-c^2x^2}}+\frac{1}{d^2x\sqrt{1-c^2x^2}}\right)dx}{2e}-\frac{a+b\text{sech}^{-1}(cx)}{2e(d+ex)^2}$$

$\Big\downarrow$ 2009

$$-\frac{a+b\text{sech}^{-1}(cx)}{2e(d+ex)^2}-$$
$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{c^2e\arctan\left(\frac{c^2dx+e}{\sqrt{1-c^2x^2}\sqrt{c^2d^2-e^2}}\right)}{(c^2d^2-e^2)^{3/2}}-\frac{e\arctan\left(\frac{c^2dx+e}{\sqrt{1-c^2x^2}\sqrt{c^2d^2-e^2}}\right)}{d^2\sqrt{c^2d^2-e^2}}-\frac{\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d^2}-\frac{e^2\sqrt{1-c^2x^2}}{d(c^2d^2-e^2)(d+ex)}\right)}{2e}$$

input
```
Int[(a + b*ArcSech[c*x])/(d + e*x)^3,x]
```

output
```
-1/2*(a + b*ArcSech[c*x])/(e*(d + e*x)^2) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
 + c*x]*(-((e^2*Sqrt[1 - c^2*x^2])/(d*(c^2*d^2 - e^2)*(d + e*x))) - (c^2*e
*ArcTan[(e + c^2*d*x)/(Sqrt[c^2*d^2 - e^2]*Sqrt[1 - c^2*x^2])])/(c^2*d^2 -
 e^2)^(3/2) - (e*ArcTan[(e + c^2*d*x)/(Sqrt[c^2*d^2 - e^2]*Sqrt[1 - c^2*x^
2])])/(d^2*Sqrt[c^2*d^2 - e^2]) - ArcTanh[Sqrt[1 - c^2*x^2]]/d^2))/(2*e)
```

3.80.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6842
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Simp[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x)^(m + 1)/(x*
Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.80.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(267) = 534$.

Time = 1.88 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.94

method	result
parts	$-\frac{a}{2(ex+d)^2 e}+\frac{b\left(-\frac{c^3\,\mathrm{arcsech}(cx)}{2(cex+cd)^2 e}+\frac{c^2\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^3d^3\sqrt{-\frac{c^2d^2-e^2}{e^2}}+\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{a\,c^3}{2(cex+cd)^2 e}+b\,c^3\left(-\frac{\mathrm{arcsech}(cx)}{2(cex+cd)^2 e}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^3d^3\sqrt{-\frac{c^2d^2-e^2}{e^2}}+\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^3\right.}{}\right.}{}$
default	$\frac{-\frac{a\,c^3}{2(cex+cd)^2 e}+b\,c^3\left(-\frac{\mathrm{arcsech}(cx)}{2(cex+cd)^2 e}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^3d^3\sqrt{-\frac{c^2d^2-e^2}{e^2}}+\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^3\right.}{}\right.}{}$

input
```
int((a+b*arcsech(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(e*x+d)^2/e+b/c*(-1/2*c^3/(c*e*x+c*d)^2/e*arcsech(c*x)+1/2*c^2/e*(-
(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(arctanh(1/(-c^2*x^2+1)^(1/2))*c^
3*d^3*(-(c^2*d^2-e^2)/e^2)^(1/2)+arctanh(1/(-c^2*x^2+1)^(1/2))*c^3*d^2*e*(
-(c^2*d^2-e^2)/e^2)^(1/2)*x-2*ln(2*((-(c^2*d^2-e^2)/e^2)^(1/2)*(-c^2*x^2+1
)^(1/2)*e+d*c^2*x+e)/(c*e*x+c*d))*c^3*d^3-2*ln(2*((-(c^2*d^2-e^2)/e^2)^(1/
2)*(-c^2*x^2+1)^(1/2)*e+d*c^2*x+e)/(c*e*x+c*d))*c^3*d^2*e*x-arctanh(1/(-c^
2*x^2+1)^(1/2))*c*d*e^2*(-(c^2*d^2-e^2)/e^2)^(1/2)-arctanh(1/(-c^2*x^2+1)^
(1/2))*e^3*(-(c^2*d^2-e^2)/e^2)^(1/2)*c*x+c*d*e^2*(-c^2*x^2+1)^(1/2)*(-(c^
2*d^2-e^2)/e^2)^(1/2)+ln(2*((-(c^2*d^2-e^2)/e^2)^(1/2)*(-c^2*x^2+1)^(1/2)*
e+d*c^2*x+e)/(c*e*x+c*d))*c*d*e^2+ln(2*((-(c^2*d^2-e^2)/e^2)^(1/2)*(-c^2*x
^2+1)^(1/2)*e+d*c^2*x+e)/(c*e*x+c*d))*e^3*c*x)/(-c^2*x^2+1)^(1/2)/d^2/(c*d
+e)/(c*d-e)/(c*e*x+c*d)/(-(c^2*d^2-e^2)/e^2)^(1/2))
```

3.80.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(200) = 400$.

Time = 0.37 (sec) , antiderivative size = 1212, normalized size of antiderivative = 3.96

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

output
```
[-1/2*(a*c^4*d^6 - (2*a + b)*c^2*d^4*e^2 + (a + b)*d^2*e^4 - (b*c^2*d^2*e^
4 - b*e^6)*x^2 + (2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2*d^2*e^3 - b*e^5)*x^
2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(-c^2*d^2 + e^2)*log((c^2*d*e*x -
 (c^3*d^2 - c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2 - sqrt(-c^2*d^2
+ e^2)*(c^2*d*x + c*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e))/(e*x + d)) -
2*(b*c^2*d^3*e^3 - b*d*e^5)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 +
 (b*c^4*d^4*e^2 - 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*
d^3*e^3 + b*d*e^5)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b
*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*
x^2)) + 1)/(c*x)) - ((b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 + (b*c^3*d^4*e^2 - b*
c*d^2*e^4)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^4*d^8*e - 2*c^2*d^6*e^3 +
 d^4*e^5 + (c^4*d^6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 -
2*c^2*d^5*e^4 + d^3*e^6)*x), -1/2*(a*c^4*d^6 - (2*a + b)*c^2*d^4*e^2 + (a
+ b)*d^2*e^4 - (b*c^2*d^2*e^4 - b*e^6)*x^2 - 2*(2*b*c^2*d^4*e - b*d^2*e^3
+ (2*b*c^2*d^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(c^
2*d^2 - e^2)*arctan(-(sqrt(c^2*d^2 - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2)) - sqrt(c^2*d^2 - e^2)*(e*x + d))/((c^2*d^2 - e^2)*x)) - 2*(b*c^2*d^3*
e^3 - b*d*e^5)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e
^2 - 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d
*e^5)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c^4*d^6 -...
```

3.80.6 Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^3} \, dx = \int \frac{a + b \operatorname{asech}(cx)}{(d + ex)^3} \, dx$$

input
```
integrate((a+b*asech(c*x))/(e*x+d)**3,x)
```

output
```
Integral((a + b*asech(c*x))/(d + e*x)**3, x)
```

3.80.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^3} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.80.8 Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^3} \, dx = \int \frac{b \,\text{arsech}\,(cx) + a}{(ex + d)^3} \, dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(e*x + d)^3, x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^3} \, dx = \int \frac{a + b \,\text{acosh}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^3} \, dx$$

input
```
int((a + b*acosh(1/(c*x)))/(d + e*x)^3,x)
```

output
```
int((a + b*acosh(1/(c*x)))/(d + e*x)^3, x)
```

3.81 $\int (d+ex)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

3.81.1 Optimal result

Integrand size = 18, antiderivative size = 343

$$\int (d+ex)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2}+\frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e} - \frac{28bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4b(2c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^3\sqrt{d+ex}} - \frac{4bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5e\sqrt{d+ex}}$$

output
```
2/5*(e*x+d)^(5/2)*(a+b*arcsech(c*x))/e-28/15*b*d*EllipticE(1/2*(-c*x+1)^(1
/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*
x+d)^(1/2)/c/(c*(e*x+d)/(c*d+e))^(1/2)-4/15*b*(2*c^2*d^2+e^2)*EllipticF(1/
2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x
+1)^(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/c^3/(e*x+d)^(1/2)-4/5*b*d^3*EllipticPi
(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)
*(c*x+1)^(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/e/(e*x+d)^(1/2)-4/15*b*e*(1/(c*x+
1))^(1/2)*(c*x+1)^(1/2)*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

3.81.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 21.84 (sec) , antiderivative size = 2653, normalized size of antiderivative = 7.73

$$\int (d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \text{Result too large to show}$$

input
```
Integrate[(d + e*x)^(3/2)*(a + b*ArcSech[c*x]),x]
```

output
```
Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[d + e*x]*((-4*b*e)/(15*c^2) - (4*b*e*x)/(15
*c)) + Sqrt[d + e*x]*((2*a*d^2)/(5*e) + (4*a*d*x)/5 + (2*a*e*x^2)/5) + (2*
b*(d + e*x)^(5/2)*ArcSech[c*x])/(5*e) - (4*b*(7*c*d*e*Sqrt[(1 - c*x)/(1 +
c*x)]*(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x))) + ((7*
I)*c^2*d^2*e*(c*d + e)*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x
))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))/(c*d + e)]*(EllipticE[I*ArcS
inh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] - EllipticF[I*ArcSinh
[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)]))/(c*d - e) - ((7*I)*c*d
*e^2*(c*d + e)*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 +
c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))/(c*d + e)]*(EllipticE[I*ArcSinh[Sqrt
[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] - EllipticF[I*ArcSinh[Sqrt[(1
 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)]))/(c*d - e) + (3*I)*c^3*d^3*Sqrt
[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1
- c*x)/(1 + c*x)))/(c*d + e)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)
]], (c*d - e)/(c*d + e)] - (2*I)*c^2*d^2*e*Sqrt[1 + (1 - c*x)/(1 + c*x)]*S
qrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))/(c*d + e
)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] -
I*e^3*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*
d*(1 + (1 - c*x)/(1 + c*x)))/(c*d + e)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)
/(1 + c*x)]], (c*d - e)/(c*d + e)] + ((3 + 3*I)*c^3*d^3*(-I + Sqrt[(1 -...
```

3.81.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6842, 634, 632, 186, 413, 412, 2185, 27, 600, 508, 327, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{6842}$$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(d+ex)^{5/2}}{x\sqrt{1-c^2x^2}}dx}{5e}+\frac{2(d+ex)^{5/2} \left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{634}$$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d^3\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\int \frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx\right)}{5e}+\frac{2(d+ex)^{5/2} \left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{632}$$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d^3\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx-\int \frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx\right)}{5e}+\frac{2(d+ex)^{5/2} \left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{186}$$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\int \frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-2d^3\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}\right)}{5e}+\frac{2(d+ex)^{5/2} \left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{413}$$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\int \frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5e}+\frac{2(d+ex)^{5/2} \left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{412}$$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\int \frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5e}+\frac{2(d+ex)^{5/2} \left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{2185}$$

3.81. $\int (d+ex)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2\int \frac{e^3\left(9d^2c^2+7dexc^2+e^2\right)}{2\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3c^2e^2}-\frac{2d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2e^2\sqrt{1-c^2x^2}\sqrt{d+ex}}{3c^2}\right)}{5e}+\frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{e\int \frac{9d^2c^2+7dexc^2+e^2}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2e^2\sqrt{1-c^2x^2}\sqrt{d+ex}}{3c^2}\right)}{5e}+\frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 600$$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{e\left(\left(2c^2d^2+e^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+7c^2d\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx\right)}{3c^2}-\frac{2d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\right.}{5e}+\frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 508$$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{e\left(\left(2c^2d^2+e^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{14cd\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-}}{\sqrt{2}}\right.\right.}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right.}{5e}+\frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 327$$

3.81. $\int (d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{e\left(\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{14cd\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\sqrt{\right.\right.}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right.}{5e}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 511$$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{e\left(-\frac{2\left(2c^2d^2+e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{d+ex}}-\frac{14cd\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-\frac{e(1-}{cd}}}{}\right.}{5e}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 321$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}+$$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{e\left(-\frac{2\left(2c^2d^2+e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{14cd\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}}{}\right.}{5e}$$

input
```
Int[(d + e*x)^(3/2)*(a + b*ArcSech[c*x]),x]
```

output
```
(2*(d + e*x)^(5/2)*(a + b*ArcSech[c*x]))/(5*e) + (2*b*Sqrt[(1 + c*x)^(-1)]
*Sqrt[1 + c*x]*((-2*e^2*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2])/(3*c^2) + (e*((-1
4*c*d*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
e)])/Sqrt[(c*(d + e*x))/(c*d + e)] - (2*(2*c^2*d^2 + e^2)*Sqrt[(c*(d + e*x
))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(
c*Sqrt[d + e*x])))/(3*c^2) - (2*d^3*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*Elli
pticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/Sqrt[d + e/c -
(e*(1 - c*x))/c]))/(5*e)
```

3.81. $\int (d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.81.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 634
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] - Int[(
1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(c^(n + 1/2) - (c + d*x)^(n
+ 1/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n - 1/2, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

rule 6842
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Simp[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x)^(m + 1)/(x*
Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.81. $\int (d+ex)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

3.81.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(302) = 604$.

Time = 12.72 (sec) , antiderivative size = 818, normalized size of antiderivative = 2.38

method	result
derivativedivides	$\frac{\frac{2a(ex+d)^{\frac{5}{2}}}{5}+2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsech}(cx)}{5}-\frac{2e^2\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(\sqrt{\frac{c}{cd+e}}\,c^2(ex+d)^{\frac{5}{2}}+9\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{2a(ex+d)^{\frac{5}{2}}}{5}+2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsech}(cx)}{5}-\frac{2e^2\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(\sqrt{\frac{c}{cd+e}}\,c^2(ex+d)^{\frac{5}{2}}+9\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\cdots\right.}{\cdots}\right.}{\cdots}$
parts	$\frac{2a(ex+d)^{\frac{5}{2}}}{5e}+\frac{2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsech}(cx)}{5}-\frac{2e^2\sqrt{-\frac{c(ex+d)-cd-e}{cex}}\,x\sqrt{\frac{c(ex+d)-cd+e}{cex}}\left(\sqrt{\frac{c}{cd+e}}\,c^2(ex+d)^{\frac{5}{2}}+9\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int((e*x+d)^(3/2)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
2/e*(1/5*a*(e*x+d)^(5/2)+b*(1/5*(e*x+d)^(5/2)*arcsech(c*x)-2/15/c*e^2*((-c
*(e*x+d)+c*d+e)/c/e/x)^(1/2)*x*(-(-c*(e*x+d)+c*d-e)/c/e/x)^(1/2)*((c/(c*d+
e))^(1/2)*c^2*(e*x+d)^(5/2)+9*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticF
((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*((-c*(e*x+d)+c*d
+e)/(c*d+e))^(1/2)*c^2*d^2-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticE(
(e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*((-c*(e*x+d)+c*d+
e)/(c*d+e))^(1/2)*c^2*d^2-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticPi(
(e*x+d)^(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e)
)^(1/2))*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*c^2*d^2-2*(c/(c*d+e))^(1/2)*c^
2*d*(e*x+d)^(3/2)-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticF((e*x+d)^(
1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*((-c*(e*x+d)+c*d+e)/(c*d+e
))^(1/2)*c*d*e+7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1/2
)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*((-c*(e*x+d)+c*d+e)/(c*d+e))^
(1/2)*c*d*e+(c/(c*d+e))^(1/2)*c^2*d^2*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c
*d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(
1/2))*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*e^2-(c/(c*d+e))^(1/2)*e^2*(e*x+d)
^(1/2))/(c/(c*d+e))^(1/2)/(c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)))
```

3.81.5 Fricas [F(-1)]

Timed out.

$$\int (d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
Timed out
```

3.81.6 Sympy [F]

$$\int (d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (a+b\,\text{asech}\,(cx))\,(d+ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(a+b*asech(c*x)),x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x)**(3/2), x)
```

3.81.7 Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.81.8 Giac [F]

$$\int (d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (ex+d)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)\,dx$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(3/2)*(b*arcsech(c*x) + a), x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,(d+e\,x)^{3/2}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))*(d + e*x)^(3/2),x)
```

output
```
int((a + b*acosh(1/(c*x)))*(d + e*x)^(3/2), x)
```

3.82 $\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.82.1 Optimal result

Integrand size = 18, antiderivative size = 279

$$\begin{aligned}
&\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)\,dx \\
&= \frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&\quad - \frac{4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c\sqrt{d+ex}} \\
&\quad - \frac{4bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3e\sqrt{d+ex}}
\end{aligned}$$

output
```
2/3*(e*x+d)^(3/2)*(a+b*arcsech(c*x))/e-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*
2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x+d)
^(1/2)/c/(c*(e*x+d)/(c*d+e))^(1/2)-4/3*b*d*EllipticF(1/2*(-c*x+1)^(1/2)*2^
(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d
)/(c*d+e))^(1/2)/c/(e*x+d)^(1/2)-4/3*b*d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2
^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*
x+d)/(c*d+e))^(1/2)/e/(e*x+d)^(1/2)
```

3.82.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 26.04 (sec) , antiderivative size = 2938, normalized size of antiderivative = 10.53

$$\int \sqrt{d+ex}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\text{Result too large to show}$$

input
```
Integrate[Sqrt[d + e*x]*(a + b*ArcSech[c*x]),x]
```

output
```
((2*a*d)/(3*e) + (2*a*x)/3)*Sqrt[d + e*x] + (2*b*(d + e*x)^(3/2)*ArcSech[c
*x])/(3*e) + (4*b*(-((e*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[c*(1 + (1 - c*x)/(1
 + c*x))]*Sqrt[c + (c*(1 - c*x))/(1 + c*x)]*Sqrt[(c*d + e + (c*d*(1 - c*x)
)/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/(1 + c*x))])/(c*
(1 + (1 - c*x)/(1 + c*x)))) + (Sqrt[c*(1 + (1 - c*x)/(1 + c*x))]*Sqrt[c +
(c*(1 - c*x))/(1 + c*x)]*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + (c*d*
(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))]*Sqrt[(c*d + e + (c*d*(1 -
 c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/(1 + c*x))]
*((I*c*d*(-(c*d) - e)*e*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*
(1 - c*x))/((-(c*d) - e)*(1 + c*x))]*(EllipticE[I*ArcSinh[Sqrt[(1 - c*x)/(
1 + c*x)]], -((c*d - e)/(-(c*d) - e))] - EllipticF[I*ArcSinh[Sqrt[(1 - c*x
)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))]))/((c*d - e)*Sqrt[c*(1 + (1 - c*
x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) - (I*(-(c*d) -
 e)*e^2*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*(1 - c*x))/((-(c
*d) - e)*(1 + c*x))]*(EllipticE[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c
*d - e)/(-(c*d) - e))] - EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -
((c*d - e)/(-(c*d) - e))]))/((c*d - e)*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c
*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) - (I*c^2*d^2*Sqrt[1 + (1 - c*x
)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*(1 - c*x))/((-(c*d) - e)*(1 + c*x))]*Elli
pticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))])...
```

3.82.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {6842, 634, 600, 508, 327, 511, 321, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

$\downarrow$ 6842

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}dx}{3e}+\frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$\downarrow$ 634

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\int \frac{-xe^2-2de}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx\right)}{3e}+\frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$\downarrow$ 600

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+de\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+e\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx\right)}{3e}+\frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$\downarrow$ 508

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+de\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2e\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3e}+\frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$\downarrow$ 327

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+de\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3e}+\frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$\downarrow$ 511

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\int \frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}}{c\sqrt{\frac{c(d+ex)}{cd+e}}}}{3e}$$
$$\frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$\downarrow$ 321

3.82. $\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right.}{3e}$$
$$\frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$\Big\downarrow$ 632

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d^2\int\frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2}{cd}\right.}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right.}{3e}$$
$$\frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$\Big\downarrow$ 186

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-2d^2\int\frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}E\left(\text{arc}\right.}{c\sqrt{}}\right.}{3e}$$
$$\frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$\Big\downarrow$ 413

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{2d^2\sqrt{1-\frac{e(1-cx)}{cd+e}}\int\frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}E\left(\right.}{}\right.}{3e}$$
$$\frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$\Big\downarrow$ 412

$$\frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}+$$
$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{2d^2\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+e}}{}\right.}{3e}$$

input
```
Int[Sqrt[d + e*x]*(a + b*ArcSech[c*x]),x]
```

3.82. $\int\sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

output
```
(2*(d + e*x)^(3/2)*(a + b*ArcSech[c*x]))/(3*e) + (2*b*Sqrt[(1 + c*x)^(-1)]
*Sqrt[1 + c*x]*((-2*e*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]
], (2*e)/(c*d + e)])/(c*Sqrt[(c*(d + e*x))/(c*d + e)]) - (2*d*e*Sqrt[(c*(d
 + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
e)])/(c*Sqrt[d + e*x]) - (2*d^2*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*Elliptic
Pi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/Sqrt[d + e/c - (e*(
1 - c*x))/c]))/(3*e)
```

3.82.3.1 Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 634
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] - Int[(
1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(c^(n + 1/2) - (c + d*x)^(n
+ 1/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n - 1/2, 0]
```

rule 6842
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Simp[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x)^(m + 1)/(x*
Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.82. $\int \sqrt{d+ex}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$

3.82.4 Maple [A] (verified)

Time = 11.26 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsech}(cx)}{3}-\frac{2e^2\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(2\,\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)cd-\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsech}(cx)}{3}-\frac{2e^2\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(2\,\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)cd-\cdots\right.}{\cdots}\right.}{\cdots}$
parts	$\frac{2a(ex+d)^{\frac{3}{2}}}{3e}+\frac{2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsech}(cx)}{3}-\frac{2e^2\sqrt{-\frac{c(ex+d)-cd-e}{cex}}\,x\sqrt{\frac{c(ex+d)-cd+e}{cex}}\left(2\,\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)cd-\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
2/e*(1/3*(e*x+d)^(3/2)*a+b*(1/3*(e*x+d)^(3/2)*arcsech(c*x)-2/3*e^2*((-c*(e
*x+d)+c*d+e)/c/e/x)^(1/2)*x*(-(-c*(e*x+d)+c*d-e)/c/e/x)^(1/2)*(2*EllipticF
((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c*d-EllipticE((e
*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c*d-EllipticPi((e*x
+d)^(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1
/2))*c*d-EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2)
)*e+EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*e)*
((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)/(c/(
c*d+e))^(1/2)/(c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)))
```

3.82.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
Timed out
```

3.82.6 Sympy [F]

$$\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int(a+b\,\text{asech}\,(cx))\,\sqrt{d+ex}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(a+b*asech(c*x)),x)
```

output
```
Integral((a + b*asech(c*x))*sqrt(d + e*x), x)
```

3.82.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.82.8 Giac [F]

$$\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\,\text{arsech}\,(cx)+a)\,dx$$

input
```
integrate((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(b*arcsech(c*x) + a), x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))*(d + e*x)^(1/2),x)
```

output
```
int((a + b*acosh(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.83 $\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex}}\,dx$

3.83.1 Optimal result

Integrand size = 18, antiderivative size = 187

$$\begin{aligned}\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex}}\,dx &= \frac{2\sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)}{e}\\ &- \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}\\ &- \frac{4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{e\sqrt{d+ex}}\end{aligned}$$

output
```
2*(a+b*arcsech(c*x))*(e*x+d)^(1/2)/e-4*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1
/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/
(c*d+e))^(1/2)/c/(e*x+d)^(1/2)-4*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2)
,2,2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(
c*d+e))^(1/2)/e/(e*x+d)^(1/2)
```

3.83.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 18.89 (sec) , antiderivative size = 1707, normalized size of antiderivative = 9.13

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2a\sqrt{d+ex}}{e}+\frac{2b\sqrt{d+ex}\operatorname{sech}^{-1}(cx)}{e}$$

$$-\frac{4ib\sqrt{\frac{cd+e+\frac{cd(1-cx)}{1+cx}-\frac{e(1-cx)}{1+cx}}{c+\frac{c(1-cx)}{1+cx}}}\left(2cd\sqrt{-\frac{i\left(\sqrt{-cd-e}\sqrt{cd-e}+cd\sqrt{\frac{1-cx}{1+cx}}-e\sqrt{\frac{1-cx}{1+cx}}\right)}{\left(-icd+\sqrt{-cd-e}\sqrt{cd-e}+ie\right)\left(-i+\sqrt{\frac{1-cx}{1+cx}}\right)}}\sqrt{-\frac{i\left(\sqrt{-cd-e}\sqrt{cd-e}-cd\sqrt{\frac{1-cx}{1+cx}}+e\sqrt{\frac{1-cx}{1+cx}}\right)}{\left(icd+\sqrt{-cd-e}\sqrt{cd-e}-ie\right)\left(-i+\sqrt{\frac{1-cx}{1+cx}}\right)}}\left(\cdots\right.\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcSech[c*x])/Sqrt[d + e*x],x]
```

output
```
(2*a*Sqrt[d + e*x])/e + (2*b*Sqrt[d + e*x]*ArcSech[c*x])/e - ((4*I)*b*Sqrt
[(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(
1 - c*x))/(1 + c*x))]*(2*c*d*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] +
c*d*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)]))/(((-I)*c*d +
 Sqrt[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*
Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)]
 + e*Sqrt[(1 - c*x)/(1 + c*x)]))/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e]
- I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*(1 + (1 - c*x)/(1 + c*x))*Ellipt
icF[ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/
(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1
 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I
*Sqrt[c*d - e])^2] + (c*d - e)*Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*
(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I
 + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e
*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))/(c*d + e)]*Elliptic
F[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] + (2*I)*c*d*S
qrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d*Sqrt[(1 - c*x)/(1 + c*x)]
- e*Sqrt[(1 - c*x)/(1 + c*x)]))/(((-I)*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e
] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[((-I)*(Sqrt[-(c*d) - e]*S
qrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e*Sqrt[(1 - c*x)/(1 + c*...
```

3.83.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6842, 637, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 6842}$$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}dx}{e}+\frac{2\sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{ 637}$$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \left(\frac{d}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx}{e}+\frac{2\sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)}{e}+\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{2e\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2d\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{d+ex}}\right)}{e}$$

input
```
Int[(a + b*ArcSech[c*x])/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(a + b*ArcSech[c*x]))/e + (2*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[
1 + c*x]*((-2*e*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]) - (2*d*Sqrt[(c*(d + e*x))
/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])
/Sqrt[d + e*x]))/e
```

3.83.3.1 Defintions of rubi rules used

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6842
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Simp[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x)^(m + 1)/(x*
Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.83.4 Maple [A] (verified)

Time = 11.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\operatorname{arcsech}(cx)-\frac{2c\,e^2\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)-\mathrm{Ell}\cdots\right.}{\left(c^2(ex+d)^2-2c^2d(ex+d)\cdots\right.}\right.}{e}$
default	$\frac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\operatorname{arcsech}(cx)-\frac{2c\,e^2\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)-\mathrm{Ell}\cdots\right.}{\left(c^2(ex+d)^2-2c^2d(ex+d)\cdots\right.}\right.}{e}$
parts	$\frac{2a\sqrt{ex+d}}{e}+\frac{2b\left(\sqrt{ex+d}\,\operatorname{arcsech}(cx)-\frac{2c\,e^2\sqrt{-\frac{c(ex+d)-cd-e}{cex}}\,x\sqrt{\frac{c(ex+d)-cd+e}{cex}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)-\mathrm{Elli}\cdots\right.}{\sqrt{\frac{c}{cd+e}}\left(c^2(ex+d)^2-2c^2\cdots\right.}\right.}{e}$

input
```
int((a+b*arcsech(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*((e*x+d)^(1/2)*a+b*((e*x+d)^(1/2)*arcsech(c*x)-2*c*e^2*((-c*(e*x+d)+c*
d+e)/c/e/x)^(1/2)*x*(-(-c*(e*x+d)+c*d-e)/c/e/x)^(1/2)*(EllipticF((e*x+d)^(
1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))-EllipticPi((e*x+d)^(1/2)*(
c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2)))*((-c*
(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)/(c^2*(e*x
+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/(c/(c*d+e))^(1/2)))
```

3.83.5 Fricas [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{\sqrt{d + ex}}\, dx = \text{Timed out}$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.83.6 Sympy [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{\sqrt{d + ex}}\, dx = \int \frac{a + b\operatorname{asech}(cx)}{\sqrt{d + ex}}\, dx$$

input
```
integrate((a+b*asech(c*x))/(e*x+d)**(1/2),x)
```

output
```
Integral((a + b*asech(c*x))/sqrt(d + e*x), x)
```

3.83.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{\sqrt{d + ex}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for mor
e details)
```

3.83.8 Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{\sqrt{d + ex}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{ex + d}}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/sqrt(e*x + d), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{\sqrt{d + ex}}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{\sqrt{d + e\,x}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(d + e*x)^(1/2),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(d + e*x)^(1/2), x)
```

3.84 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

3.84.1 Optimal result

Integrand size = 18, antiderivative size = 105

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}} + \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{e\sqrt{d+ex}}$$

output
```
-2*(a+b*arcsech(c*x))/e/(e*x+d)^(1/2)+4*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^
(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x
+d)/(c*d+e))^(1/2)/e/(e*x+d)^(1/2)
```

3.84.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 18.02 (sec) , antiderivative size = 1675, normalized size of antiderivative = 15.95

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2a}{e\sqrt{d+ex}} - \frac{2b\text{sech}^{-1}(cx)}{e\sqrt{d+ex}} + \frac{4ib\left(2\sqrt{-\frac{i\left(\sqrt{-cd-e}\sqrt{cd-e}+cd\sqrt{\frac{1-cx}{1+cx}}-e\sqrt{\frac{1-cx}{1+cx}}\right)}{\left(-icd+\sqrt{-cd-e}\sqrt{cd-e}+ie\right)\left(-i+\sqrt{\frac{1-cx}{1+cx}}\right)}}\sqrt{-\frac{i\left(\sqrt{-cd-e}\sqrt{cd-e}-cd\sqrt{\frac{1-cx}{1+cx}}+e\sqrt{\frac{1-cx}{1+cx}}\right)}{\left(icd+\sqrt{-cd-e}\sqrt{cd-e}-ie\right)\left(-i+\sqrt{\frac{1-cx}{1+cx}}\right)}}\left(1+\frac{1-cx}{1+cx}\right)\text{EllipticF}\left(\text{arcsin}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(d + e*x)^(3/2),x]
```

output
```
(-2*a)/(e*Sqrt[d + e*x]) - (2*b*ArcSech[c*x])/(e*Sqrt[d + e*x]) + ((4*I)*b
*(2*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d*Sqrt[(1 - c*x)/(1 + c
*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)]))/(((-I)*c*d + Sqrt[-(c*d) - e]*Sqrt[c*
d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[((-I)*(Sqrt[-(c*d) -
 e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e*Sqrt[(1 - c*x)/(1 +
c*x)]))/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e)*(-I + Sqrt[(1 - c*
x)/(1 + c*x)]))]*(1 + (1 - c*x)/(1 + c*x))*EllipticF[ArcSin[Sqrt[((Sqrt[-(
c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d
) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d
) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] + Sqrt
[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((
Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqr
t[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1
 - c*x)/(1 + c*x)))/(c*d + e)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x
)]], (c*d - e)/(c*d + e)] + (2*I)*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d -
e] + c*d*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)]))/(((-I)*
c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)
]))]*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 +
c*x)] + e*Sqrt[(1 - c*x)/(1 + c*x)]))/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d
- e] - I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*(1 + (1 - c*x)/(1 + c*x)...
```

3.84.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6842, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6842}$$

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{e}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 632}$$

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{e}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{186}$$

$$\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{e}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{413}$$

$$\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int\frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{e\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{412}$$

$$\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{e\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

input
```
Int[(a + b*ArcSech[c*x])/(d + e*x)^(3/2),x]
```

output
```
(-2*(a + b*ArcSech[c*x]))/(e*Sqrt[d + e*x]) + (4*b*Sqrt[(1 + c*x)^(-1)]*Sq
rt[1 + c*x]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1
- c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(e*Sqrt[d + e/c - (e*(1 - c*x))/c])
```

3.84.3.1 Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

3.84. $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 6842
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Simp[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x)^(m + 1)/(x*
Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.84.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(96) = 192$.

Time = 11.35 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.39

method	result
derivativedivides	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arcsech}(cx)}{\sqrt{ex+d}}-\frac{2c\,e^2\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\frac{cd+e}{cd},\frac{\sqrt{\frac{c}{cd-e}}}{\sqrt{\frac{c}{cd+e}}}\right)\sqrt{-c(e\cdots}}{d\sqrt{\frac{c}{cd+e}}\left(c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2\right)}\cdots\right)}{e}$
default	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arcsech}(cx)}{\sqrt{ex+d}}-\frac{2c\,e^2\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\frac{cd+e}{cd},\frac{\sqrt{\frac{c}{cd-e}}}{\sqrt{\frac{c}{cd+e}}}\right)\sqrt{-c(e\cdots}}{d\sqrt{\frac{c}{cd+e}}\left(c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2\right)}\cdots\right)}{e}$
parts	$-\frac{2a}{\sqrt{ex+d}\,e}+\frac{2b\left(-\frac{\operatorname{arcsech}(cx)}{\sqrt{ex+d}}-\frac{2c\,e^2\sqrt{-\frac{c(ex+d)-cd-e}{cex}}\,x\sqrt{\frac{c(ex+d)-cd+e}{cex}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\frac{cd+e}{cd},\frac{\sqrt{\frac{c}{cd-e}}}{\sqrt{\frac{c}{cd+e}}}\right)\sqrt{-\cdots}}{d\sqrt{\frac{c}{cd+e}}\left(c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2\right)}\cdots\right)}{e}$

input
```
int((a+b*arcsech(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arcsech(c*x)-2*c*e^2*((-c*(e*x+d
)+c*d+e)/c/e/x)^(1/2)*x*(-(-c*(e*x+d)+c*d-e)/c/e/x)^(1/2)*EllipticPi((e*x+
d)^(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/
2))*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)/
d/(c/(c*d+e))^(1/2)/(c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)))
```

3.84.5 Fricas [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{3/2}} \, dx = \int \frac{b \,\text{arsech}\,(cx) + a}{(ex + d)^{\frac{3}{2}}} \, dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arcsech(c*x) + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.84.6 Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{3/2}} \, dx = \int \frac{a + b \,\text{asech}\,(cx)}{(d + ex)^{\frac{3}{2}}} \, dx$$

input
```
integrate((a+b*asech(c*x))/(e*x+d)**(3/2),x)
```

output
```
Integral((a + b*asech(c*x))/(d + e*x)**(3/2), x)
```

3.84.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{3/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.84.8 Giac [F]

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\int \frac{b\,\operatorname{arsech}(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(e*x + d)^(3/2), x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\int \frac{a+b\,\operatorname{acosh}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^{3/2}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(d + e*x)^(3/2),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(d + e*x)^(3/2), x)
```

3.85 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

3.85.1 Optimal result

Integrand size = 18, antiderivative size = 278

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d^2-e^2\right)\sqrt{d+ex}} - \frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{4bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3d\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3de\sqrt{d+ex}}$$

output
```
-2/3*(a+b*arcsech(c*x))/e/(e*x+d)^(3/2)-4/3*b*c*EllipticE(1/2*(-c*x+1)^(1/
2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x
+d)^(1/2)/d/(c^2*d^2-e^2)/(c*(e*x+d)/(c*d+e))^(1/2)+4/3*b*EllipticPi(1/2*(
-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+
1)^(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/d/e/(e*x+d)^(1/2)+4/3*b*e*(1/(c*x+1))^(
1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d^2-e^2)/(e*x+d)^(1/2)
```

3.85.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 25.13 (sec) , antiderivative size = 4527, normalized size of antiderivative = 16.28

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(d + e*x)^(5/2),x]
```

output
```
(-2*a)/(3*e*(d + e*x)^(3/2)) + Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[d + e*x]*((4
*b*c)/(3*d*(c^2*d^2 - e^2)) - (4*b)/(3*d*(c*d + e)*(d + e*x))) - (2*b*ArcS
ech[c*x])/(3*e*(d + e*x)^(3/2)) - (4*b*((e*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[
c*(1 + (1 - c*x)/(1 + c*x))]*(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1
- c*x))/(1 + c*x)))/((1 + (1 - c*x)/(1 + c*x))*Sqrt[c + (c*(1 - c*x))/(1 +
 c*x)]*Sqrt[(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x)
)/(c + (c*(1 - c*x))/(1 + c*x))]) - ((c*d - e)*Sqrt[c*(1 + (1 - c*x)/(1 +
c*x))]*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + (c*d*(1 - c*x))/(1 + c*
x) - (e*(1 - c*x))/(1 + c*x))]*((I*(-(c*d) - e)*e*Sqrt[1 + (1 - c*x)/(1 +
c*x)]*Sqrt[1 - ((c*d - e)*(1 - c*x))/((-(c*d) - e)*(1 + c*x))]*(EllipticE[
I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))] - Ellipti
cF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))]))/((c*
d - e)*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(
1 + c*x))]) + (I*c*d*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*(1
- c*x))/((-(c*d) - e)*(1 + c*x))]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 +
c*x)]], -((c*d - e)/(-(c*d) - e))])/Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d
+ e + ((c*d - e)*(1 - c*x))/(1 + c*x))] + (I*e*Sqrt[1 + (1 - c*x)/(1 + c*x
)]*Sqrt[1 - ((c*d - e)*(1 - c*x))/((-(c*d) - e)*(1 + c*x))]*EllipticF[I*Ar
cSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))])/Sqrt[c*(1 +
(1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))] - (I*...
```

3.85.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.91, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6842, 635, 25, 27, 498, 27, 508, 327, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$$

$\Big\downarrow$ 6842

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx}{3e}-\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\Big\downarrow$ 635

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\int -\frac{e}{d(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx+\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}\right)}{3e}-\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\int \frac{e}{d(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3e}-\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\int \frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx}{d}\right)}{3e}-\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\Big\downarrow$ 498

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c^2\int -\frac{\sqrt{d+ex}}{2\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}\right)}{d}\right)}{3e}-\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{c^2\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}+\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{d}\right)}{3e}-\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\Big\downarrow$ 508

3.85. $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)}{3e}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 327

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)}{3e}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 632

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)}{3e}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 186

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{2\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)}{3e}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 413

3.85. $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{d\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)}{3e}-\frac{2(a+b\text{sech}^{-1}(cx))}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \; 412$$

$$-\frac{2(a+b\text{sech}^{-1}(cx))}{3e(d+ex)^{3/2}}-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{3e}$$

input
```
Int[(a + b*ArcSech[c*x])/(d + e*x)^(5/2),x]
```

output
```
(-2*(a + b*ArcSech[c*x]))/(3*e*(d + e*x)^(3/2)) - (2*b*Sqrt[(1 + c*x)^(-1)
]*Sqrt[1 + c*x]*(-((e*((2*e*Sqrt[1 - c^2*x^2])/((c^2*d^2 - e^2)*Sqrt[d + e
*x]) - (2*c*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(
c*d + e)])/((c^2*d^2 - e^2)*Sqrt[(c*(d + e*x))/(c*d + e)])))/d) - (2*Sqrt[
1 - (e*(1 - c*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(d*Sqrt[d + e/c - (e*(1 - c*x))/c])))/(3*e)
```

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

3.85. $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 6842
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Simp[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x)^(m + 1)/(x*
Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.85.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(246) = 492$.

Time = 13.39 (sec) , antiderivative size = 890, normalized size of antiderivative = 3.20

method	result
derivativedivides	$\frac{-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsech}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2c\,e^{2}\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(\sqrt{\frac{c}{cd+e}}\,c^{2}d(ex+d)^{2}-\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsech}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2c\,e^{2}\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(\sqrt{\frac{c}{cd+e}}\,c^{2}d(ex+d)^{2}-\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$
parts	$-\frac{2a}{3(ex+d)^{\frac{3}{2}}e}+\frac{2b\left(-\frac{\operatorname{arcsech}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2c\,e^{2}\sqrt{-\frac{c(ex+d)-cd-e}{cex}}\,x\sqrt{\frac{c(ex+d)-cd+e}{cex}}\left(\sqrt{\frac{c}{cd+e}}\,c^{2}d(ex+d)^{2}-\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int((a+b*arcsech(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(-1/3*a/(e*x+d)^(3/2)+b*(-1/3/(e*x+d)^(3/2)*arcsech(c*x)+2/3*c*e^2*((-
c*(e*x+d)+c*d+e)/c/e/x)^(1/2)*x*(-(-c*(e*x+d)+c*d-e)/c/e/x)^(1/2)*((c/(c*d
+e))^(1/2)*c^2*d*(e*x+d)^2-((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)
+c*d-e)/(c*d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/
(c*d-e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)+c^2*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/
2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^
(1/2),((c*d+e)/(c*d-e))^(1/2))*d^2*(e*x+d)^(1/2)-((-c*(e*x+d)+c*d+e)/(c*d+
e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(
c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*c^2*d^2*(
e*x+d)^(1/2)-2*(c/(c*d+e))^(1/2)*c^2*d^2*(e*x+d)+((-c*(e*x+d)+c*d+e)/(c*d+
e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c
*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c*d*e*(e*x+d)^(1/2)-((-c*(e*x+d)+c*d
+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1
/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c*d*e*(e*x+d)^(1/2)+(c/(c*d
+e))^(1/2)*c^2*d^3+((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c
/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*e^2*(e*x+d)^(1/2)-(c/(c*d+e))^(1/2)*d*e
^2)/(c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/d^2/(c/(c*d+e))^(1/2)/(c*d
+e)/(c*d-e)/(e*x+d)^(1/2)))
```

3.85.5 Fricas [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{5/2}}\, dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex + d)^{\frac{5}{2}}}\, dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arcsech(c*x) + a)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2
*e*x + d^3), x)
```

3.85.6 Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{5/2}} \, dx = \int \frac{a + b \operatorname{asech}(cx)}{(d + ex)^{\frac{5}{2}}} \, dx$$

input
```
integrate((a+b*asech(c*x))/(e*x+d)**(5/2),x)
```

output
```
Integral((a + b*asech(c*x))/(d + e*x)**(5/2), x)
```

3.85.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{5/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.85.8 Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{5/2}} \, dx = \int \frac{b \operatorname{arsech}(cx) + a}{(ex + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(e*x + d)^(5/2), x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^{5/2}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(d + e*x)^(5/2),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(d + e*x)^(5/2), x)
```

3.86 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{7/2}} \, dx$

3.86.1 Optimal result

Integrand size = 18, antiderivative size = 609

$$\begin{aligned}\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{7/2}} \, dx &= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d\left(c^2d^2-e^2\right)(d+ex)^{3/2}} \\ &+ \frac{16bc^2e\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15\left(c^2d^2-e^2\right)^2\sqrt{d+ex}} + \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{5d^2\left(c^2d^2-e^2\right)\sqrt{d+ex}} \\ &- \frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{16bc^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15\left(c^2d^2-e^2\right)^2\sqrt{\frac{c(d+ex)}{cd+e}}} \\ &- \frac{4bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5d^2\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}} \\ &+ \frac{4bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15d\left(c^2d^2-e^2\right)\sqrt{d+ex}} \\ &+ \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5d^2e\sqrt{d+ex}}\end{aligned}$$

output
```
-2/5*(a+b*arcsech(c*x))/e/(e*x+d)^(5/2)-16/15*b*c^3*EllipticE(1/2*(-c*x+1)
^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*
(e*x+d)^(1/2)/(c^2*d^2-e^2)^2/(c*(e*x+d)/(c*d+e))^(1/2)-4/5*b*c*EllipticE(
1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c
*x+1)^(1/2)*(e*x+d)^(1/2)/d^2/(c^2*d^2-e^2)/(c*(e*x+d)/(c*d+e))^(1/2)+4/15
*b*c*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c
*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/d/(c^2*d^2-e^2)/(e*x+
d)^(1/2)+4/5*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))
^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/d^2/e/(e
*x+d)^(1/2)+4/15*b*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/
(c^2*d^2-e^2)/(e*x+d)^(3/2)+16/15*b*c^2*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*
(-c^2*x^2+1)^(1/2)/(c^2*d^2-e^2)^2/(e*x+d)^(1/2)+4/5*b*e*(1/(c*x+1))^(1/2)
*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/(c^2*d^2-e^2)/(e*x+d)^(1/2)
```

3.86.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 26.78 (sec) , antiderivative size = 8675, normalized size of antiderivative = 14.24

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{7/2}}\, dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(d + e*x)^(7/2),x]
```

output
```
Result too large to show
```

3.86.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.71, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6842, 635, 632, 186, 413, 412, 688, 27, 688, 27, 600, 508, 327, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{7/2}}\, dx$$

$$\Big\downarrow \; 6842$$

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{1}{x(d+ex)^{5/2}\sqrt{1-c^2x^2}}dx}{5e}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{635}$$

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\int\frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{1-c^2x^2}}dx+\frac{\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d^2}\right)}{5e}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{632}$$

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\int\frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{1-c^2x^2}}dx+\frac{\int\frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{d^2}\right)}{5e}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{186}$$

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\int\frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{1-c^2x^2}}dx-\frac{2\int\frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{d^2}\right)}{5e}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{413}$$

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\int\frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{1-c^2x^2}}dx-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\int\frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{d^2\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5e}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\int\frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{1-c^2x^2}}dx-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d^2\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5e}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{688}$$

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2\int-\frac{e\left(3d\left(2c^2-\frac{e^2}{d^2}\right)-c^2ex\right)}{2d(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx}{3(c^2d^2-e^2)}-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d^2\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2e^2\sqrt{1-c^2x^2}}{3d(c^2d^2-e^2)(d+ex)^{3/2}}\right)}{5e}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

3.86. $\int\frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$\downarrow$ 27

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{e\int\frac{3\left(2c^2d-\frac{e^2}{d}\right)-c^2ex}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx}{3d(c^2d^2-e^2)}-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d^2\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2e^2\sqrt{1-c^2x^2}}{3d(c^2d^2-e^2)(d+ex)^{3/2}}\right)}{5e}-\frac{2(a+b\text{sech}^{-1}(cx))}{5e(d+ex)^{5/2}}$$

$\downarrow$ 688

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{e\left(\frac{2\int\frac{c^2\left(2d\left(3c^2d^2-e^2\right)+e\left(7c^2d^2-3e^2\right)x\right)}{2d\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}+\frac{2e\sqrt{1-c^2x^2}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d(c^2d^2-e^2)}-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-}}{\sqrt{2}}\right.\right.}{d^2\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right.}{5e}$$

$$\frac{2(a+b\text{sech}^{-1}(cx))}{5e(d+ex)^{5/2}}$$

$\downarrow$ 27

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{e\left(\frac{c^2\int\frac{2d\left(3c^2d^2-e^2\right)+e\left(7c^2d^2-3e^2\right)x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d\left(c^2d^2-e^2\right)}+\frac{2e\sqrt{1-c^2x^2}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d(c^2d^2-e^2)}-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right.}{d^2\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right.}{5e}$$

$$\frac{2(a+b\text{sech}^{-1}(cx))}{5e(d+ex)^{5/2}}$$

$\downarrow$ 600

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{e\left(\frac{c^2\left(\left(7c^2d^2-3e^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx-d(cd-e)(cd+e)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx\right)}{d\left(c^2d^2-e^2\right)}+\frac{2e\sqrt{1-c^2x^2}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d(c^2d^2-e^2)}-\frac{2\sqrt{1-\frac{e(1-cx}{cd+e}}}{}\right.}{5e}$$

$$\frac{2(a+b\text{sech}^{-1}(cx))}{5e(d+ex)^{5/2}}$$

$\downarrow$ 508

3.86. $\int\frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{e\left(\frac{c^2\left(-\frac{2\left(7c^2d^2-3e^2\right)\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{\frac{c(d+ex)}{cd+e}}}-d(cd-e)(cd+e)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx\right)}{d\left(c^2d^2-e^2\right)}+\frac{2e\sqrt{1-c^2x^2}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d(c^2d^2-e^2)}\right)}{5e}$$

$$\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 327

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{e\left(\frac{c^2\left(-d(cd-e)(cd+e)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2\left(7c^2d^2-3e^2\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d\left(c^2d^2-e^2\right)}+\frac{2e\sqrt{1-c^2x^2}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d(c^2d^2-e^2)}\right)}{5e}$$

$$\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 511

3.86. $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$$-\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{e\left(\frac{c^2\left(\frac{2d(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{d+ex}}-\frac{2\left(7c^2d^2-3e^2\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d\left(c^2d^2-e^2\right)}}{3d(c^2d^2-e^2)}\right.}{5e}$$

$$\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 321

$$-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}-$$

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{e\left(\frac{c^2\left(\frac{2d(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2\left(7c^2d^2-3e^2\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d\left(c^2d^2-e^2\right)}+\frac{2e}{}}{3d(c^2d^2-e^2)}\right.}{5e}$$

input
```
Int[(a + b*ArcSech[c*x])/(d + e*x)^(7/2),x]
```

3.86. $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

output
```
(-2*(a + b*ArcSech[c*x]))/(5*e*(d + e*x)^(5/2)) - (2*b*Sqrt[(1 + c*x)^(-1)
]*Sqrt[1 + c*x]*((-2*e^2*Sqrt[1 - c^2*x^2])/(3*d*(c^2*d^2 - e^2)*(d + e*x)
^(3/2)) - (e*((2*e*(7*c^2*d^2 - 3*e^2)*Sqrt[1 - c^2*x^2])/(d*(c^2*d^2 - e^
2)*Sqrt[d + e*x]) + (c^2*((-2*(7*c^2*d^2 - 3*e^2)*Sqrt[d + e*x]*EllipticE[
ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[(c*(d + e*x))/(c*
d + e)]) + (2*d*(c*d - e)*(c*d + e)*Sqrt[(c*(d + e*x))/(c*d + e)]*Elliptic
F[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x])))/(d*
(c^2*d^2 - e^2))))/(3*d*(c^2*d^2 - e^2)) - (2*Sqrt[1 - (e*(1 - c*x))/(c*d
+ e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(d^2*
Sqrt[d + e/c - (e*(1 - c*x))/c])))/(5*e)
```

3.86.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

3.86. $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 6842
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Simp[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x)^(m + 1)/(x*
Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.86.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1611 vs. $2(539) = 1078$.

Time = 14.25 (sec) , antiderivative size = 1612, normalized size of antiderivative = 2.65

method	result	size
derivativedivides	Expression too large to display	1612
default	Expression too large to display	1612
parts	Expression too large to display	1634

input
```
int((a+b*arcsech(c*x))/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arcsech(c*x)-2/15*c*e^2*((
-c*(e*x+d)+c*d+e)/c/e/x)^(1/2)*x*(-(-c*(e*x+d)+c*d-e)/c/e/x)^(1/2)*(6*((-c
*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*Elliptic
F((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c^4*d^4*(e*x+d)
^(3/2)-7*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(
1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c^
4*d^4*(e*x+d)^(3/2)+3*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-
e)/(c*d-e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d
,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*c^4*d^4*(e*x+d)^(3/2)-7*(c/(c*d+e))^
(1/2)*c^4*d^3*(e*x+d)^3-7*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+
c*d-e)/(c*d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(
c*d-e))^(1/2))*c^3*d^3*e*(e*x+d)^(3/2)+7*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2
)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^(
1/2),((c*d+e)/(c*d-e))^(1/2))*c^3*d^3*e*(e*x+d)^(3/2)+13*(c/(c*d+e))^(1/2)
*c^4*d^4*(e*x+d)^2-2*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e
)/(c*d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e
))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3/2)+3*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*(
(-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2
),((c*d+e)/(c*d-e))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3/2)-6*((-c*(e*x+d)+c*d+e)
/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticPi((e*x+d)^(...
```

3.86.5 Fricas [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arcsech(c*x) + a)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2
*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

3.86.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{7/2}}\, dx = \text{Timed out}$$

input
```
integrate((a+b*asech(c*x))/(e*x+d)**(7/2),x)
```

output
```
Timed out
```

3.86.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{7/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.86.8 Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{7/2}}\, dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex + d)^{\frac{7}{2}}}\, dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(e*x + d)^(7/2), x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{a+b\,\operatorname{acosh}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^{7/2}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(d + e*x)^(7/2),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(d + e*x)^(7/2), x)
```

3.87 $\int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right) dx$

3.87.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right) dx = \frac{(d+ex)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{e(1+m)} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{Int}\left(\frac{(d+ex)^{1+m}}{x\sqrt{1-c^2x^2}},x\right)}{e(1+m)}$$

output
```
(e*x+d)^(1+m)*(a+b*arcsech(c*x))/e/(1+m)+b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)
*Unintegrable((e*x+d)^(1+m)/x/(-c^2*x^2+1)^(1/2),x)/e/(1+m)
```

3.87.2 Mathematica [N/A]

Not integrable

Time = 21.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right) dx = \int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right) dx$$

input
```
Integrate[(d + e*x)^m*(a + b*ArcSech[c*x]),x]
```

output
```
Integrate[(d + e*x)^m*(a + b*ArcSech[c*x]), x]
```

3.87.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6842, 638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{6842}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(d+ex)^{m+1}}{x\sqrt{1-c^2x^2}}dx}{e(m+1)}+\frac{(d+ex)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{e(m+1)}$$

$$\Big\downarrow \text{638}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(d+ex)^{m+1}}{x\sqrt{1-c^2x^2}}dx}{e(m+1)}+\frac{(d+ex)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{e(m+1)}$$

input
```
Int[(d + e*x)^m*(a + b*ArcSech[c*x]),x]
```

output
```
$Aborted
```

3.87.3.1 Defintions of rubi rules used

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 6842
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Simp[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x)^(m + 1)/(x*
Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.87.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (ex+d)^m (a+b \operatorname{arcsech}(cx))\,dx$$

input
```
int((e*x+d)^m*(a+b*arcsech(c*x)),x)
```

output
```
int((e*x+d)^m*(a+b*arcsech(c*x)),x)
```

3.87.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (b\,\text{arsech}\,(cx)+a)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)*(e*x + d)^m, x)
```

3.87.6 Sympy [N/A]

Not integrable

Time = 13.56 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (a+b\,\text{asech}\,(cx))\,(d+ex)^m\,dx$$

input
```
integrate((e*x+d)**m*(a+b*asech(c*x)),x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x)**m, x)
```

3.87.7 Maxima [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 222, normalized size of antiderivative = 13.88

$$\int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right) dx = \int (b\,\text{arsech}\,(cx)+a)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
b*(((e*x + d)*(e*x + d)^m*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - (e*x + d
)*(e*x + d)^m*log(x))/(e*(m + 1)) - integrate((c^2*e*(m + 1)*x^3*log(c) -
(e*(m + 1)*log(c) - e)*x + d)*(e*x + d)^m/(c^2*e*(m + 1)*x^3 - e*(m + 1)*x
), x) + integrate((c^2*e*x^2 + c^2*d*x)*(e*x + d)^m/(c^2*e*(m + 1)*x^2 + (
c^2*e*(m + 1)*x^2 - e*(m + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1) - e*(m + 1)),
x)) + (e*x + d)^(m + 1)*a/(e*(m + 1))
```

3.87.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right) dx = \int (b\,\text{arsech}\,(cx)+a)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*(e*x + d)^m, x)
```

3.87.9 Mupad [N/A]

Not integrable

Time = 4.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right) dx = \int \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)(d+e\,x)^m\,dx$$

input
```
int((a + b*acosh(1/(c*x)))*(d + e*x)^m,x)
```

output
```
int((a + b*acosh(1/(c*x)))*(d + e*x)^m, x)
```

3.87. $\int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

3.88 $\int x^4(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.88.1 Optimal result

Integrand size = 19, antiderivative size = 229

$$\int x^4(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx = -\frac{b(42c^2d+25e)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{560c^6} - \frac{b(42c^2d+25e)\,x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{840c^4} - \frac{bex^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2} + \frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right) + \frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right) + \frac{b(42c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{560c^7}$$

output
```
1/5*d*x^5*(a+b*arcsech(c*x))+1/7*e*x^7*(a+b*arcsech(c*x))+1/560*b*(42*c^2*
d+25*e)*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^7-1/560*b*(42*c^2*d+
25*e)*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6-1/840*b*(42
*c^2*d+25*e)*x^3*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/
42*b*e*x^5*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

3.88.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.71

$$\int x^4(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{48ac^7x^5(7d+5ex^2)-bcx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(75e+2c^2(63d+25ex^2)+c^4(84dx^2+40ex^4)\right)+48bc^7x^5(7d+}{1680c^7}$$

input
```
Integrate[x^4*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

output
```
(48*a*c^7*x^5*(7*d + 5*e*x^2) - b*c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*
(75*e + 2*c^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^
5*(7*d + 5*e*x^2)*ArcSech[c*x] + (3*I)*b*(42*c^2*d + 25*e)*Log[(-2*I)*c*x
+ 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/(1680*c^7)
```

3.88.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6855, 27, 363, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{ 6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x^4(5ex^2+7d)}{35\sqrt{1-c^2x^2}}dx+\frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{35}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x^4(5ex^2+7d)}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 363}$$

$$\frac{1}{35}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\int\frac{x^4}{\sqrt{1-c^2x^2}}dx-\frac{5ex^5\sqrt{1-c^2x^2}}{6c^2}\right)+\frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \; 262$$

$$\frac{1}{35}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\left(\frac{3\int\frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)-\frac{5ex^5\sqrt{1-c^2x^2}}{6c^2}\right)+\frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \; 262$$

$$\frac{1}{35}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)-\frac{5ex^5\sqrt{1-c^2x^2}}{6c^2}\right)+\frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \; 223$$

$$\frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{35}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{6}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\left(\frac{25e}{c^2}+42d\right)-\frac{5ex^5\sqrt{1-c^2x^2}}{6c^2}\right)$$

input
```
Int[x^4*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

output
```
(d*x^5*(a + b*ArcSech[c*x]))/5 + (e*x^7*(a + b*ArcSech[c*x]))/7 + (b*Sqrt[
(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-5*e*x^5*Sqrt[1 - c^2*x^2])/(6*c^2) + ((42
*d + (25*e)/c^2)*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 -
c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/6))/35
```

3.88. $\int x^4(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.88. $\int x^4(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.88.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.93

method	result
parts	$a\left(\frac{1}{7}e\,x^{7}+\frac{1}{5}d\,x^{5}\right)+\frac{b\left(\frac{c^{5}\operatorname{arcsech}(cx)e\,x^{7}}{7}+\frac{\operatorname{arcsech}(cx)x^{5}c^{5}d}{5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,\left(-40e\sqrt{-c^{2}x^{2}+1}\,c^{5}x^{5}-84c^{5}d\sqrt{-c^{2}x\cdots}\right.}{\cdots}\right)}{\cdots}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{5}d\,c^{7}x^{5}+\frac{1}{7}e\,c^{7}x^{7}\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arcsech}(cx)d\,c^{7}x^{5}}{5}+\frac{\operatorname{arcsech}(cx)e\,c^{7}x^{7}}{7}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(84c^{5}d\sqrt{-c^{2}x^{2}+1}\,x^{3}+40e\sqrt{-c^{2}x^{2}+1}\,c^{5}x\cdots\right.}{\cdots}\right)}{c^{2}}}{c^{5}}$
default	$\frac{\frac{a\left(\frac{1}{5}d\,c^{7}x^{5}+\frac{1}{7}e\,c^{7}x^{7}\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arcsech}(cx)d\,c^{7}x^{5}}{5}+\frac{\operatorname{arcsech}(cx)e\,c^{7}x^{7}}{7}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(84c^{5}d\sqrt{-c^{2}x^{2}+1}\,x^{3}+40e\sqrt{-c^{2}x^{2}+1}\,c^{5}x\cdots\right.}{\cdots}\right)}{c^{2}}}{c^{5}}$

input
```
int(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e*x^7+1/5*d*x^5)+b/c^5*(1/7*c^5*arcsech(c*x)*e*x^7+1/5*arcsech(c*x)
*x^5*c^5*d+1/1680/c*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(-40*e*(-c^
2*x^2+1)^(1/2)*c^5*x^5-84*c^5*d*(-c^2*x^2+1)^(1/2)*x^3-50*e*c^3*x^3*(-c^2*
x^2+1)^(1/2)-126*d*c^3*x*(-c^2*x^2+1)^(1/2)+126*d*c^2*arcsin(c*x)-75*e*c*x
*(-c^2*x^2+1)^(1/2)+75*e*arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.13

$$\int x^4\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{240\,ac^7ex^7+336\,ac^7dx^5-6\,(42\,bc^2d+25\,be)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-48\,(7\,bc^7d+5\,bc^7e)\log\left(\frac{cx\sqrt{-\frac{c^2x^2\cdots}{c^2x\cdots}}}{x}\cdots\right.}{\cdots}$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/1680*(240*a*c^7*e*x^7 + 336*a*c^7*d*x^5 - 6*(42*b*c^2*d + 25*b*e)*arctan
((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - 48*(7*b*c^7*d + 5*b*c^7
*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 48*(5*b*c^7*e*x^7 +
7*b*c^7*d*x^5 - 7*b*c^7*d - 5*b*c^7*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2)) + 1)/(c*x)) - (40*b*c^6*e*x^6 + 2*(42*b*c^6*d + 25*b*c^4*e)*x^4 + 3*(
42*b*c^4*d + 25*b*c^2*e)*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7
```

3.88.6 Sympy [F]

$$\int x^4(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^4(a+b\,\text{asech}\,(cx))\left(d+ex^2\right)\,dx$$

input
```
integrate(x**4*(e*x**2+d)*(a+b*asech(c*x)),x)
```

output
```
Integral(x**4*(a + b*asech(c*x))*(d + e*x**2), x)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.07

$$\int x^4(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\frac{1}{7}\,aex^7+\frac{1}{5}\,adx^5+\frac{1}{40}\left(8\,x^5\,\text{arsech}\,(cx)-\frac{\frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+5\sqrt{\frac{1}{c^2x^2}-1}}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}+\frac{3\,\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4}}{c}\right)bd+\frac{1}{336}\left(48\,x^7\,\text{arsech}\,(cx)-\frac{\frac{15\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+40\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^2x^2}-1}}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\,\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^6}}{c}\right)be$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/40*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) -
1)^(3/2) + 5*sqrt(1/(c^2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c
^2*x^2) - 1) + c^4) + 3*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b*d + 1/336*
(48*x^7*arcsech(c*x) - ((15*(1/(c^2*x^2) - 1)^(5/2) + 40*(1/(c^2*x^2) - 1)
^(3/2) + 33*sqrt(1/(c^2*x^2) - 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^
2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*arctan(sqrt(1/(c^2*x^2
) - 1))/c^6)/c)*b*e
```

3.88.8 Giac [F]

$$\int x^4 (d+ex^2) \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (ex^2+d)(b \,\text{arsech}\,(cx)+a)x^4 \, dx$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^4, x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int x^4 (d+ex^2) \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int x^4 \left(e\,x^2+d\right) \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int(x^4*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^4*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

3.89 $\int x^2(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.89.1 Optimal result

Integrand size = 19, antiderivative size = 174

$$\int x^2(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=-\frac{b(20c^2d+9e)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{120c^4}-\frac{bex^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{20c^2}+\frac{1}{3}dx^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{b(20c^2d+9e)\,\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{120c^5}$$

output
```
1/3*d*x^3*(a+b*arcsech(c*x))+1/5*e*x^5*(a+b*arcsech(c*x))+1/120*b*(20*c^2*
d+9*e)*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^5-1/120*b*(20*c^2*d+9
*e)*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/20*b*e*x^3*
(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

3.89.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.83

$$\int x^2(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{8ac^5x^3(5d+3ex^2)-bcx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(9e+c^2(20d+6ex^2)\right)+8bc^5x^3(5d+3ex^2)\operatorname{sech}^{-1}(cx)+ib(20c^2c}{120c^5}$$

input
```
Integrate[x^2*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

output
```
(8*a*c^5*x^3*(5*d + 3*e*x^2) - b*c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(
9*e + c^2*(20*d + 6*e*x^2)) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcSech[c*x] + I
*b*(20*c^2*d + 9*e)*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)
])/(120*c^5)
```

3.89.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6855, 27, 363, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{x^2(3ex^2+5d)}{15\sqrt{1-c^2x^2}}dx+\frac{1}{3}dx^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\operatorname{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{x^2(3ex^2+5d)}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}dx^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\operatorname{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{363}$$

$$\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{4}\left(\frac{9e}{c^2}+20d\right)\int\frac{x^2}{\sqrt{1-c^2x^2}}dx-\frac{3ex^3\sqrt{1-c^2x^2}}{4c^2}\right)+$$
$$\frac{1}{3}dx^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{4}\left(\frac{9e}{c^2}+20d\right)\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)-\frac{3ex^3\sqrt{1-c^2x^2}}{4c^2}\right)+$$
$$\frac{1}{3}dx^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{3}dx^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{sech}^{-1}(cx)\right)+$$
$$\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{4}\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\left(\frac{9e}{c^2}+20d\right)-\frac{3ex^3\sqrt{1-c^2x^2}}{4c^2}\right)$$

input
```
Int[x^2*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

output
```
(d*x^3*(a + b*ArcSech[c*x]))/3 + (e*x^5*(a + b*ArcSech[c*x]))/5 + (b*Sqrt[
(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-3*e*x^3*Sqrt[1 - c^2*x^2])/(4*c^2) + ((20
*d + (9*e)/c^2)*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/4)
)/15
```

3.89.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.89.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98

method	result
parts	$a\left(\frac{1}{5}e\,x^5+\frac{1}{3}d\,x^3\right)+\frac{b\left(\frac{c^3\operatorname{arcsech}(cx)e\,x^5}{5}+\frac{\operatorname{arcsech}(cx)x^3c^3d}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,(-6e\,c^3x^3\sqrt{-c^2x^2+1}-20d\,c^3x\sqrt{-c^2\ldots}}{120c\sqrt{-\ldots}}\right.}{c^3}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\operatorname{arcsech}(cx)d\,c^5x^3}{3}+\frac{\operatorname{arcsech}(cx)e\,c^5x^5}{5}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(20d\,c^3x\sqrt{-c^2x^2+1}+6e\,c^3x^3\sqrt{-c^2x^2+1}-\ldots}{120\sqrt{-c^2x^2+\ldots}}\right.}{c^2}}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\operatorname{arcsech}(cx)d\,c^5x^3}{3}+\frac{\operatorname{arcsech}(cx)e\,c^5x^5}{5}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(20d\,c^3x\sqrt{-c^2x^2+1}+6e\,c^3x^3\sqrt{-c^2x^2+1}-\ldots}{120\sqrt{-c^2x^2+\ldots}}\right.}{c^2}}{c^3}$

input
```
int(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e*x^5+1/3*d*x^3)+b/c^3*(1/5*c^3*arcsech(c*x)*e*x^5+1/3*arcsech(c*x)
*x^3*c^3*d+1/120/c*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(-6*e*c^3*x^
3*(-c^2*x^2+1)^(1/2)-20*d*c^3*x*(-c^2*x^2+1)^(1/2)+20*d*c^2*arcsin(c*x)-9*
e*c*x*(-c^2*x^2+1)^(1/2)+9*e*arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(100) = 200$.

Time = 0.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.37

$$\int x^2(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$= \frac{24\,ac^5ex^5+40\,ac^5dx^3-2\,(20\,bc^2d+9\,be)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-8\,(5\,bc^5d+3\,bc^5e)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)}{}$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/120*(24*a*c^5*e*x^5 + 40*a*c^5*d*x^3 - 2*(20*b*c^2*d + 9*b*e)*arctan((c*
x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - 8*(5*b*c^5*d + 3*b*c^5*e)*l
og((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 8*(3*b*c^5*e*x^5 + 5*b*c^
5*d*x^3 - 5*b*c^5*d - 3*b*c^5*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) +
 1)/(c*x)) - (6*b*c^4*e*x^4 + (20*b*c^4*d + 9*b*c^2*e)*x^2)*sqrt(-(c^2*x^2
 - 1)/(c^2*x^2)))/c^5
```

3.89.6 Sympy [F]

$$\int x^2(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^2(a+b\,\text{asech}\,(cx))\left(d+ex^2\right)\,dx$$

input
```
integrate(x**2*(e*x**2+d)*(a+b*asech(c*x)),x)
```

output
```
Integral(x**2*(a + b*asech(c*x))*(d + e*x**2), x)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05

$$\int x^2(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{5}\,aex^5+\frac{1}{3}\,adx^3+\frac{1}{6}\left(2\,x^3\operatorname{arsech}\left(cx\right)-\frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)bd$$
$$+\frac{1}{40}\left(8\,x^5\operatorname{arsech}\left(cx\right)-\frac{\frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+5\sqrt{\frac{1}{c^2x^2}-1}}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}+\frac{3\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4}}{c}\right)be$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/6*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) -
1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b
*d + 1/40*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1)^(3/2) + 5*sqrt(1/(c^
2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) + 3
*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b*e
```

3.89.8 Giac [F]

$$\int x^2(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int (ex^2+d)(b\operatorname{arsech}\left(cx\right)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^2, x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^2\left(e\,x^2+d\right)\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

3.90 $\int (d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.90.1 Optimal result

Integrand size = 16, antiderivative size = 112

$$\int (d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx = -\frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + dx\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{sech}^{-1}(cx)\right) + \frac{b(6c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{6c^3}$$

output
```
d*x*(a+b*arcsech(c*x))+1/3*e*x^3*(a+b*arcsech(c*x))+1/6*b*(6*c^2*d+e)*arcs
in(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^3-1/6*b*e*x*(1/(c*x+1))^(1/2)*(c
*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

3.90.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.69

$$\int (d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = adx+\frac{1}{3}aex^3+be\sqrt{\frac{1-cx}{1+cx}}\left(-\frac{x}{6c^2}-\frac{x^2}{6c}\right) + bdx\operatorname{sech}^{-1}(cx)+\frac{1}{3}bex^3\operatorname{sech}^{-1}(cx) + \frac{2bd\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\arctan\left(\frac{\sqrt{1-c^2x^2}}{1-cx}\right)}{c-c^2x} + \frac{ibe\log\left(-2icx+2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)}{6c^3}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

output
```
a*d*x + (a*e*x^3)/3 + b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(-1/6*x/c^2 - x^2/(6*c
)) + b*d*x*ArcSech[c*x] + (b*e*x^3*ArcSech[c*x])/3 + (2*b*d*Sqrt[(1 - c*x)
/(1 + c*x)]*Sqrt[1 - c^2*x^2]*ArcTan[Sqrt[1 - c^2*x^2]/(1 - c*x)])/(c - c^
2*x) + ((I/6)*b*e*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])
/c^3
```

3.90.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6845, 27, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6845}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{ex^2+3d}{3\sqrt{1-c^2x^2}}dx+dx\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\operatorname{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{ex^2+3d}{\sqrt{1-c^2x^2}}dx+dx\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{299}$$

$$\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{(6c^2d+e)\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{ex\sqrt{1-c^2x^2}}{2c^2}\right)+dx\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{223}$$

$$dx\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\arcsin(cx)\,(6c^2d+e)}{2c^3}-\frac{ex\sqrt{1-c^2x^2}}{2c^2}\right)$$

input
```
Int[(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

output
```
d*x*(a + b*ArcSech[c*x]) + (e*x^3*(a + b*ArcSech[c*x]))/3 + (b*Sqrt[(1 + c
*x)^(-1)]*Sqrt[1 + c*x]*(-1/2*(e*x*Sqrt[1 - c^2*x^2])/c^2 + ((6*c^2*d + e)
*ArcSin[c*x])/(2*c^3)))/3
```

3.90.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 6845
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSech[c*x])   u
, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]   Int[SimplifyIntegrand[u/(x*
Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (
IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

3.90.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

method	result
parts	$a\left(\frac{1}{3}e\,x^3+dx\right)+\frac{b\left(\frac{c\,\operatorname{arcsech}(cx)e\,x^3}{3}+\operatorname{arcsech}(cx)dxc+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(6d\,c^2\arcsin(cx)-ecx\sqrt{-c^2x^2+1}+e\arcsin(c\right.}{6c\sqrt{-c^2x^2+1}}\right.}{c}$
derivativedivides	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\operatorname{arcsech}(cx)d\,c^3x+\frac{\operatorname{arcsech}(cx)e\,c^3x^3}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(6d\,c^2\arcsin(cx)-ecx\sqrt{-c^2x^2+1}+e\arcsin(cx)\right)}{6\sqrt{-c^2x^2+1}}\right.}{c^2}}{c}$
default	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\operatorname{arcsech}(cx)d\,c^3x+\frac{\operatorname{arcsech}(cx)e\,c^3x^3}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(6d\,c^2\arcsin(cx)-ecx\sqrt{-c^2x^2+1}+e\arcsin(cx)\right)}{6\sqrt{-c^2x^2+1}}\right.}{c^2}}{c}$

input
```
int((e*x^2+d)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e*x^3+d*x)+b/c*(1/3*c*arcsech(c*x)*e*x^3+arcsech(c*x)*d*x*c+1/6/c*(
-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(6*d*c^2*arcsin(c*x)-e*c*x*(-c^2
*x^2+1)^(1/2)+e*arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(64) = 128$.

Time = 0.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.87

$$\int (d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{2\,ac^3ex^3-bc^2ex^2\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+6\,ac^3dx-2\left(6\,bc^2d+be\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-2\left(3\,bc^3d+bc^3e\right)\log\left(\frac{c}{-}\right.}{6\,c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e*x^3 - b*c^2*e*x^2*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 6*a*c^3*
d*x - 2*(6*b*c^2*d + b*e)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/
(c*x)) - 2*(3*b*c^3*d + b*c^3*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) -
 1)/x) + 2*(b*c^3*e*x^3 + 3*b*c^3*d*x - 3*b*c^3*d - b*c^3*e)*log((c*x*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c^3
```

3.90.6 Sympy [F]

$$\int (d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \int (a+b\,\text{asech}\,(cx))\,(d+ex^2)\,dx$$

input
```
integrate((e*x**2+d)*(a+b*asech(c*x)),x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2), x)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int (d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \frac{1}{3}\,aex^3 + \frac{1}{6}\left(2\,x^3\,\text{arsech}\,(cx) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)be + adx + \frac{\left(cx\,\text{arsech}\,(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)bd}{c}$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e*x^3 + 1/6*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^
2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b*e + a*d*x + (
c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d/c
```

3.90.8 Giac [F]

$$\int (d+ex^2) \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (ex^2+d)(b\,\text{arsech}\,(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsech(c*x) + a), x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^2) \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (e\,x^2+d) \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int((d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int((d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

3.91 $\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx$

3.91.1 Optimal result

Integrand size = 19, antiderivative size = 96

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx = \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x} - \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{x} + ex\left(a+b\text{sech}^{-1}(cx)\right) + \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c}$$

output
```
-d*(a+b*arcsech(c*x))/x+e*x*(a+b*arcsech(c*x))+b*e*arcsin(c*x)*(1/(c*x+1))
^(1/2)*(c*x+1)^(1/2)/c+b*d*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1
/2)/x
```

3.91.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx = -\frac{ad}{x} + aex + bd\left(c+\frac{1}{x}\right)\sqrt{\frac{1-cx}{1+cx}} - \frac{bd\text{sech}^{-1}(cx)}{x} + bex\text{sech}^{-1}(cx) + \frac{2be\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\arctan\left(\frac{\sqrt{1-c^2x^2}}{1-cx}\right)}{c-c^2x}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^2,x]
```

output
```
-((a*d)/x) + a*e*x + b*d*(c + x^(-1))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*d*Arc
Sech[c*x])/x + b*e*x*ArcSech[c*x] + (2*b*e*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[
1 - c^2*x^2]*ArcTan[Sqrt[1 - c^2*x^2]/(1 - c*x)])/(c - c^2*x)
```

3.91.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6855, 25, 358, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}dx$$

$$\Big\downarrow \text{6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{d-ex^2}{x^2\sqrt{1-c^2x^2}}dx-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{x}+ex\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{25}$$

$$-b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{d-ex^2}{x^2\sqrt{1-c^2x^2}}dx-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{x}+ex\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{358}$$

$$-b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-e\int \frac{1}{\sqrt{1-c^2x^2}}dx-\frac{d\sqrt{1-c^2x^2}}{x}\right)-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{x}+ ex\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{223}$$

$$-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{x}+ex\left(a+b\text{sech}^{-1}(cx)\right)-b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{e\arcsin(cx)}{c}-\frac{d\sqrt{1-c^2x^2}}{x}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcSech[c*x]))/x) + e*x*(a + b*ArcSech[c*x]) - b*Sqrt[(1 + c*x
)^(-1)]*Sqrt[1 + c*x]*(-((d*Sqrt[1 - c^2*x^2])/x) - (e*ArcSin[c*x])/c)
```

3.91.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.91.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

method	result	si
parts	$a\left(ex-\frac{d}{x}\right)+bc\left(\frac{\operatorname{arcsech}(cx)ex}{c}-\frac{\operatorname{arcsech}(cx)d}{xc}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(\sqrt{-c^2x^2+1}\,c^2d+\arcsin(cx)ecx\right)}{c^2\sqrt{-c^2x^2+1}}\right)$	1
derivativedivides	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\operatorname{arcsech}(cx)ex-\frac{\operatorname{arcsech}(cx)dc}{x}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(\sqrt{-c^2x^2+1}\,c^2d+\arcsin(cx)ecx\right)}{\sqrt{-c^2x^2+1}}\right)}{c^2}\right)$	1
default	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\operatorname{arcsech}(cx)ex-\frac{\operatorname{arcsech}(cx)dc}{x}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(\sqrt{-c^2x^2+1}\,c^2d+\arcsin(cx)ecx\right)}{\sqrt{-c^2x^2+1}}\right)}{c^2}\right)$	1

input
```
int((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(e*x-d/x)+b*c*(1/c*arcsech(c*x)*e*x-arcsech(c*x)*d/x/c+1/c^2*(-(c*x-1)/c
/x)^(1/2)*((c*x+1)/c/x)^(1/2)*((-c^2*x^2+1)^(1/2)*c^2*d+arcsin(c*x)*e*c*x)
/(-c^2*x^2+1)^(1/2))
```

3.91.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(54) = 108$.

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.90

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^2}\,dx = \frac{bc^2dx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + acex^2 - 2\,bex\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) - acd + (bcd-bce)x\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) + (bce}{cx}$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x, algorithm="fricas")
```

output
```
(b*c^2*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + a*c*e*x^2 - 2*b*e*x*arctan((c*
x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - a*c*d + (b*c*d - b*c*e)*x*l
og((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c*e*x^2 - b*c*d + (b*c
*d - b*c*e)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/(c*x)
```

3.91.6 Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(a+b\operatorname{asech}(cx))\,(d+ex^2)}{x^2}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*asech(c*x))/x**2,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)/x**2, x)
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^2}\,dx = \left(c\sqrt{\frac{1}{c^2x^2}-1}-\frac{\operatorname{arsech}(cx)}{x}\right)bd+aex + \frac{\left(cx\operatorname{arsech}(cx)-\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)be}{c}-\frac{ad}{x}$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x, algorithm="maxima")
```

output
```
(c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*d + a*e*x + (c*x*arcsech(c*x)
 - arctan(sqrt(1/(c^2*x^2) - 1)))*b*e/c - a*d/x
```

3.91.8 Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(ex^2+d)(b\operatorname{arsech}(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^2, x)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^2}\,dx = a\,e\,x-\frac{a\,d}{x}+b\,c\,d\left(\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}-\frac{\operatorname{acosh}\left(\frac{1}{c\,x}\right)}{c\,x}\right) + \frac{b\,e\,\operatorname{atan}\left(\frac{1}{\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}\right)}{c}+b\,e\,x\,\operatorname{acosh}\left(\frac{1}{c\,x}\right)$$

input
```
int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^2,x)
```

output
```
a*e*x - (a*d)/x + b*c*d*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) - acosh(1
/(c*x))/(c*x)) + (b*e*atan(1/((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2))))/c
 + b*e*x*acosh(1/(c*x))
```

3.91. $\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^2}\,dx$

3.92 $\int \frac{(d+ex^2)\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^4}\,dx$

3.92.1 Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{(d+ex^2)\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x^3} + \frac{b(2c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x} - \frac{d\left(a+b\,\text{sech}^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}$$

output
```
-1/3*d*(a+b*arcsech(c*x))/x^3-e*(a+b*arcsech(c*x))/x+1/9*b*d*(1/(c*x+1))^(
1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+1/9*b*(2*c^2*d+9*e)*(1/(c*x+1))^
(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

3.92.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex^2)\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \frac{-3a(d+3ex^2)+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(d+2c^2dx^2+9ex^2\right)-3b(d+3ex^2)\,\text{sech}^{-1}(cx)}{9x^3}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^4,x]
```

output
```
(-3*a*(d + 3*e*x^2) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + 2*c^2*d*x
^2 + 9*e*x^2) - 3*b*(d + 3*e*x^2)*ArcSech[c*x])/(9*x^3)
```

3.92.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6855, 27, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}dx$$

$$\Big\downarrow \text{6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{3ex^2+d}{3x^4\sqrt{1-c^2x^2}}dx-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{3ex^2+d}{x^4\sqrt{1-c^2x^2}}dx-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \text{359}$$

$$-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}(2c^2d+9e)\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx-\frac{d\sqrt{1-c^2x^2}}{3x^3}\right)-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \text{242}$$

$$-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{x}-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\sqrt{1-c^2x^2}(2c^2d+9e)}{3x}-\frac{d\sqrt{1-c^2x^2}}{3x^3}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^4,x]
```

output
```
-1/3*(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-1/3*(d*Sqrt[1 - c^2*x^2])/x^3
 - ((2*c^2*d + 9*e)*Sqrt[1 - c^2*x^2])/(3*x))) - (d*(a + b*ArcSech[c*x]))/
(3*x^3) - (e*(a + b*ArcSech[c*x]))/x
```

3.92.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.92.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

method	result	size
parts	$a\left(-\frac{e}{x}-\frac{d}{3x^3}\right)+b\,c^3\left(-\frac{\operatorname{arcsech}(cx)e}{c^3x}-\frac{\operatorname{arcsech}(cx)d}{3x^3c^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(2c^4d\,x^2+9e\,c^2x^2+c^2d\right)}{9c^4x^2}\right)$	110
derivativedivides	$c^3\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)e}{cx}-\frac{\operatorname{arcsech}(cx)d}{3c\,x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(2c^4d\,x^2+9e\,c^2x^2+c^2d\right)}{9c^2x^2}\right)}{c^2}\right)$	123
default	$c^3\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)e}{cx}-\frac{\operatorname{arcsech}(cx)d}{3c\,x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(2c^4d\,x^2+9e\,c^2x^2+c^2d\right)}{9c^2x^2}\right)}{c^2}\right)$	123

input
```
int((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(-e/x-1/3*d/x^3)+b*c^3*(-1/c^3*arcsech(c*x)*e/x-1/3*arcsech(c*x)*d/x^3/c
^3+1/9/c^4*(-(c*x-1)/c/x)^(1/2)/x^2*((c*x+1)/c/x)^(1/2)*(2*c^4*d*x^2+9*c^2
*e*x^2+c^2*d))
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^4}\,dx = -\frac{9\,aex^2+3\,ad+3\,(3\,bex^2+bd)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(bcdx+(2\,bc^3d+9\,bce)x^3)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{9\,x^3}$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*log((c*x*sqrt(-(c^2*x^2 - 1)
/(c^2*x^2)) + 1)/(c*x)) - (b*c*d*x + (2*b*c^3*d + 9*b*c*e)*x^3)*sqrt(-(c^2
*x^2 - 1)/(c^2*x^2)))/x^3
```

3.92.6 Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)}{x^4}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*asech(c*x))/x**4,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)/x**4, x)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \left(c\sqrt{\frac{1}{c^2x^2}-1}-\frac{\text{arsech}\,(cx)}{x}\right)be + \frac{1}{9}\,bd\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+3\,c^4\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{3\,\text{arsech}\,(cx)}{x^3}\right)-\frac{ae}{x}-\frac{ad}{3\,x^3}$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x, algorithm="maxima")
```

output
```
(c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*e + 1/9*b*d*((c^4*(1/(c^2*x^2
) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(c*x)/x^3) - a*e/
x - 1/3*a*d/x^3
```

3.92.8 Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{(ex^2+d)(b\,\text{arsech}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^4, x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx=\int \frac{\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^4, x)
```

3.93 $\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx$

3.93.1 Optimal result

Integrand size = 19, antiderivative size = 183

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5} + \frac{b(12c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x^3} + \frac{2bc^2(12c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x} - \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}$$

output
```
-1/5*d*(a+b*arcsech(c*x))/x^5-1/3*e*(a+b*arcsech(c*x))/x^3+1/25*b*d*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^5+1/225*b*(12*c^2*d+25*e)*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+2/225*b*c^2*(12*c^2*
d+25*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

3.93.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex^2)(a+b\operatorname{sech}^{-1}(cx))}{x^6}\,dx$$
$$=\frac{-15a(3d+5ex^2)+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(25ex^2(1+2c^2x^2)+3d(3+4c^2x^2+8c^4x^4)\right)-15b(3d+5ex^2)\operatorname{sech}^{-1}}{225x^5}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^6,x]
```

output
```
(-15*a*(3*d + 5*e*x^2) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(25*e*x^2*(
1 + 2*c^2*x^2) + 3*d*(3 + 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*(3*d + 5*e*x^2)*A
rcSech[c*x])/(225*x^5)
```

3.93.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6855, 27, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\operatorname{sech}^{-1}(cx))}{x^6}\,dx$$
$$\Big\downarrow \text{6855}$$
$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{5ex^2+3d}{15x^6\sqrt{1-c^2x^2}}dx-\frac{d(a+b\operatorname{sech}^{-1}(cx))}{5x^5}-\frac{e(a+b\operatorname{sech}^{-1}(cx))}{3x^3}$$
$$\Big\downarrow \text{27}$$
$$-\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{5ex^2+3d}{x^6\sqrt{1-c^2x^2}}dx-\frac{d(a+b\operatorname{sech}^{-1}(cx))}{5x^5}-\frac{e(a+b\operatorname{sech}^{-1}(cx))}{3x^3}$$
$$\Big\downarrow \text{359}$$
$$-\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}(12c^2d+25e)\int \frac{1}{x^4\sqrt{1-c^2x^2}}dx-\frac{3d\sqrt{1-c^2x^2}}{5x^5}\right)-\frac{d(a+b\operatorname{sech}^{-1}(cx))}{5x^5}-\frac{e(a+b\operatorname{sech}^{-1}(cx))}{3x^3}$$

$$\Big\downarrow \; 245$$

$$-\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(12c^2d+25e\right)\left(\frac{2}{3}c^2\int\frac{1}{x^2\sqrt{1-c^2x^2}}dx-\frac{\sqrt{1-c^2x^2}}{3x^3}\right)-\frac{3d\sqrt{1-c^2x^2}}{5x^5}\right)-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}$$

$$\Big\downarrow \; 242$$

$$-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(-\frac{2c^2\sqrt{1-c^2x^2}}{3x}-\frac{\sqrt{1-c^2x^2}}{3x^3}\right)\left(12c^2d+25e\right)-\frac{3d\sqrt{1-c^2x^2}}{5x^5}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^6,x]
```

output
```
-1/15*(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-3*d*Sqrt[1 - c^2*x^2])/(5*x
^5) + ((12*c^2*d + 25*e)*(-1/3*Sqrt[1 - c^2*x^2]/x^3 - (2*c^2*Sqrt[1 - c^2
*x^2])/(3*x)))/5)) - (d*(a + b*ArcSech[c*x]))/(5*x^5) - (e*(a + b*ArcSech[
c*x]))/(3*x^3)
```

3.93.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.93.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

method	result
parts	$a\left(-\frac{e}{3x^3}-\frac{d}{5x^5}\right)+b\,c^5\left(-\frac{\operatorname{arcsech}(cx)e}{3c^5x^3}-\frac{\operatorname{arcsech}(cx)d}{5x^5c^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+}{225c^6x^4}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{e}{3c^3x^3}-\frac{d}{5c^3x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)e}{3c^3x^3}-\frac{\operatorname{arcsech}(cx)d}{5c^3x^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25e\,c^2x^2+9\right.}{225c^4x^4}\right.}{c^2}\right.$
default	$c^5\left(\frac{a\left(-\frac{e}{3c^3x^3}-\frac{d}{5c^3x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)e}{3c^3x^3}-\frac{\operatorname{arcsech}(cx)d}{5c^3x^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25e\,c^2x^2+9\right.}{225c^4x^4}\right.}{c^2}\right.$

input
```
int((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3*e/x^3-1/5*d/x^5)+b*c^5*(-1/3/c^5*arcsech(c*x)*e/x^3-1/5*arcsech(c*
x)*d/x^5/c^5+1/225/c^6*(-(c*x-1)/c/x)^(1/2)/x^4*((c*x+1)/c/x)^(1/2)*(24*c^
6*d*x^4+50*c^4*e*x^4+12*c^4*d*x^2+25*c^2*e*x^2+9*c^2*d))
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx = -\frac{75\,aex^2+45\,ad+15\,(5\,bex^2+3\,bd)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(2\,(12\,bc^5d+25\,bc^3e)x^5+9\,bcdx+(12\,bc^3d}{225\,x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/225*(75*a*e*x^2 + 45*a*d + 15*(5*b*e*x^2 + 3*b*d)*log((c*x*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (2*(12*b*c^5*d + 25*b*c^3*e)*x^5 + 9*b*c*
d*x + (12*b*c^3*d + 25*b*c*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^5
```

3.93.6 Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(a+b\operatorname{asech}(cx))\,(d+ex^2)}{x^6}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*asech(c*x))/x**6,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)/x**6, x)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx = \frac{1}{75}\,bd\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+10\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{15\,\operatorname{arsech}(cx)}{x^5}\right) + \frac{1}{9}\,be\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+3\,c^4\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{3\,\operatorname{arsech}(cx)}{x^3}\right)-\frac{ae}{3\,x^3}-\frac{ad}{5\,x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x, algorithm="maxima")
```

output
```
1/75*b*d*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c^2*x^2) - 1)^(3/2)
+ 15*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5) + 1/9*b*e*((c^4*(
1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(c*x)/x
^3) - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

3.93.8 Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(ex^2+d)(b\,\text{arsech}\,(cx)+a)}{x^6}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^6, x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^6, x)
```

3.94 $\int \frac{(d+ex^2)\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^8}\,dx$

3.94.1 Optimal result

Integrand size = 19, antiderivative size = 238

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^8}\,dx &= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7} \\ &+ \frac{b(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1225x^5} \\ &+ \frac{4bc^2(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3675x^3} \\ &+ \frac{8bc^4(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3675x} \\ &- \frac{d\left(a+b\,\text{sech}^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\,\text{sech}^{-1}(cx)\right)}{5x^5}\end{aligned}$$

output
```
-1/7*d*(a+b*arcsech(c*x))/x^7-1/5*e*(a+b*arcsech(c*x))/x^5+1/49*b*d*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^7+1/1225*b*(30*c^2*d+49*e)*
(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^5+4/3675*b*c^2*(30*c^
2*d+49*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+8/3675*b*
c^4*(30*c^2*d+49*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

3.94.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.49

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx$$
$$=\frac{-105a(5d+7ex^2)+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(49ex^2(3+4c^2x^2+8c^4x^4)+15d(5+6c^2x^2+8c^4x^4+16c^6x^6)\right)-1}{3675x^7}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^8,x]
```

output
```
(-105*a*(5*d + 7*e*x^2) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(49*e*x^2*
(3 + 4*c^2*x^2 + 8*c^4*x^4) + 15*d*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6
)) - 105*b*(5*d + 7*e*x^2)*ArcSech[c*x])/(3675*x^7)
```

3.94.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6855, 27, 359, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx$$
$$\Big\downarrow \text{6855}$$
$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{7ex^2+5d}{35x^8\sqrt{1-c^2x^2}}dx-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}$$
$$\Big\downarrow \text{27}$$
$$-\frac{1}{35}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{7ex^2+5d}{x^8\sqrt{1-c^2x^2}}dx-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}$$
$$\Big\downarrow \text{359}$$
$$-\frac{1}{35}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}(30c^2d+49e)\int \frac{1}{x^6\sqrt{1-c^2x^2}}dx-\frac{5d\sqrt{1-c^2x^2}}{7x^7}\right)-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}$$

$$\Big\downarrow \text{245}$$

$$-\frac{1}{35}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(30c^2d+49e\right)\left(\frac{4}{5}c^2\int\frac{1}{x^4\sqrt{1-c^2x^2}}dx-\frac{\sqrt{1-c^2x^2}}{5x^5}\right)-\frac{5d\sqrt{1-c^2x^2}}{7x^7}\right)-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}$$

$$\Big\downarrow \text{245}$$

$$-\frac{1}{35}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(30c^2d+49e\right)\left(\frac{4}{5}c^2\left(\frac{2}{3}c^2\int\frac{1}{x^2\sqrt{1-c^2x^2}}dx-\frac{\sqrt{1-c^2x^2}}{3x^3}\right)-\frac{\sqrt{1-c^2x^2}}{5x^5}\right)-\frac{5d\sqrt{1-}}{7x}\right.$$
$$\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}$$

$$\Big\downarrow \text{242}$$

$$-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{1}{35}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(\frac{4}{5}c^2\left(-\frac{2c^2\sqrt{1-c^2x^2}}{3x}-\frac{\sqrt{1-c^2x^2}}{3x^3}\right)-\frac{\sqrt{1-c^2x^2}}{5x^5}\right)\left(30c^2d+49e\right)-\frac{5d\sqrt{1-c^2x^2}}{7x^7}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^8,x]
```

output
```
-1/35*(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-5*d*Sqrt[1 - c^2*x^2])/(7*x
^7) + ((30*c^2*d + 49*e)*(-1/5*Sqrt[1 - c^2*x^2]/x^5 + (4*c^2*(-1/3*Sqrt[1
 - c^2*x^2]/x^3 - (2*c^2*Sqrt[1 - c^2*x^2])/(3*x)))/5))/7)) - (d*(a + b*Ar
cSech[c*x]))/(7*x^7) - (e*(a + b*ArcSech[c*x]))/(5*x^5)
```

3.94.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.94.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.62

method	result
parts	$a\left(-\frac{e}{5x^5}-\frac{d}{7x^7}\right)+b\,c^7\left(-\frac{\operatorname{arcsech}(cx)e}{5c^7x^5}-\frac{\operatorname{arcsech}(cx)d}{7x^7c^7}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x \cdots}{3675c^8x \cdots}\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)d}{7c^5x^7}-\frac{\operatorname{arcsech}(cx)e}{5c^5x^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x \cdots\right.}{3675c^6x^6}\right.}{c^2}\right.$
default	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)d}{7c^5x^7}-\frac{\operatorname{arcsech}(cx)e}{5c^5x^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x \cdots\right.}{3675c^6x^6}\right.}{c^2}\right.$

input
```
int((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/5*e/x^5-1/7*d/x^7)+b*c^7*(-1/5/c^7*arcsech(c*x)*e/x^5-1/7*arcsech(c*
x)*d/x^7/c^7+1/3675/c^8*(-(c*x-1)/c/x)^(1/2)/x^6*((c*x+1)/c/x)^(1/2)*(240*
c^8*d*x^6+392*c^6*e*x^6+120*c^6*d*x^4+196*c^4*e*x^4+90*c^4*d*x^2+147*c^2*e
*x^2+75*c^2*d))
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx =$$
$$-\frac{735\,aex^2+525\,ad+105\,(7\,bex^2+5\,bd)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(8\,(30\,bc^7d+49\,bc^5e)x^7+4\,(30\,bc^5d+4}{3675\,x^7}$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/3675*(735*a*e*x^2 + 525*a*d + 105*(7*b*e*x^2 + 5*b*d)*log((c*x*sqrt(-(c
^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (8*(30*b*c^7*d + 49*b*c^5*e)*x^7 + 4*
(30*b*c^5*d + 49*b*c^3*e)*x^5 + 75*b*c*d*x + 3*(30*b*c^3*d + 49*b*c*e)*x^3
)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^7
```

3.94.6 Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)}{x^8}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*asech(c*x))/x**8,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)/x**8, x)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx$$
$$= \frac{1}{245}bd\left(\frac{5\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{7}{2}}+21\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+35\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+35\,c^8\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{35\,\mathrm{arsech}\,(cx)}{x^7}\right)$$
$$+\frac{1}{75}be\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+10\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{15\,\mathrm{arsech}\,(cx)}{x^5}\right)$$
$$-\frac{ae}{5\,x^5}-\frac{ad}{7\,x^7}$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x, algorithm="maxima")
```

output
```
1/245*b*d*((5*c^8*(1/(c^2*x^2) - 1)^(7/2) + 21*c^8*(1/(c^2*x^2) - 1)^(5/2)
 + 35*c^8*(1/(c^2*x^2) - 1)^(3/2) + 35*c^8*sqrt(1/(c^2*x^2) - 1))/c - 35*a
rcsech(c*x)/x^7) + 1/75*b*e*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c
^2*x^2) - 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5
) - 1/5*a*e/x^5 - 1/7*a*d/x^7
```

3.94.8 Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arsech}\,(cx)+a)}{x^8}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^8, x)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^8, x)
```

3.95 $\int x^5(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.95.1 Optimal result

Integrand size = 19, antiderivative size = 232

$$\int x^5(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx = -\frac{b(4c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{24c^8} + \frac{b(8c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{72c^8} - \frac{b(4c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{120c^8} + \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{7/2}}{56c^8} + \frac{1}{6}dx^6\left(a+b\text{sech}^{-1}(cx)\right) + \frac{1}{8}ex^8\left(a+b\text{sech}^{-1}(cx)\right)$$

output
```
1/6*d*x^6*(a+b*arcsech(c*x))+1/8*e*x^8*(a+b*arcsech(c*x))+1/72*b*(8*c^2*d+
9*e)*(-c^2*x^2+1)^(3/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8-1/120*b*(4*c^2
*d+9*e)*(-c^2*x^2+1)^(5/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8+1/56*b*e*(-
c^2*x^2+1)^(7/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8-1/24*b*(4*c^2*d+3*e)*
(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^8
```

3.95.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.54

$$\int x^5\left(d+ex^2\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{24}ax^6\left(4d+3ex^2\right)-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(144e+8c^2(28d+9ex^2)+2c^4(56dx^2+27ex^4)+c^6(84dx^4+45ex^6)\right)}{2520c^8}+\frac{1}{24}bx^6\left(4d+3ex^2\right)\operatorname{sech}^{-1}(cx)$$

input
```
Integrate[x^5*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

output
```
(a*x^6*(4*d + 3*e*x^2))/24 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(144*e
 + 8*c^2*(28*d + 9*e*x^2) + 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 +
45*e*x^6)))/(2520*c^8) + (b*x^6*(4*d + 3*e*x^2)*ArcSech[c*x])/24
```

3.95.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6855, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(d+ex^2\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{x^5\left(3ex^2+4d\right)}{24\sqrt{1-c^2x^2}}dx+\frac{1}{6}dx^6\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\operatorname{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{x^5\left(3ex^2+4d\right)}{\sqrt{1-c^2x^2}}dx+\frac{1}{6}dx^6\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\operatorname{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{48}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{x^4(3ex^2+4d)}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{6}dx^6\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\operatorname{sech}^{-1}(cx)\right)$$

$\big\downarrow$ 86

$$\frac{1}{48}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\left(-\frac{3e(1-c^2x^2)^{5/2}}{c^6}+\frac{(4dc^2+9e)(1-c^2x^2)^{3/2}}{c^6}+\frac{(-8dc^2-9e)\sqrt{1-c^2x^2}}{c^6}+\frac{4dc^2+3}{c^6\sqrt{1-c}}\right.$$
$$\frac{1}{6}dx^6\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\operatorname{sech}^{-1}(cx)\right)$$

$\big\downarrow$ 2009

$$\frac{1}{6}dx^6\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\operatorname{sech}^{-1}(cx)\right)+$$
$$\frac{1}{48}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{2(1-c^2x^2)^{5/2}(4c^2d+9e)}{5c^8}+\frac{2(1-c^2x^2)^{3/2}(8c^2d+9e)}{3c^8}-\frac{2\sqrt{1-c^2x^2}(4c^2d+3e)}{c^8}+\frac{6}{}\right.$$

input
```
Int[x^5*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

output
```
(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-2*(4*c^2*d + 3*e)*Sqrt[1 - c^2*x^
2])/c^8 + (2*(8*c^2*d + 9*e)*(1 - c^2*x^2)^(3/2))/(3*c^8) - (2*(4*c^2*d +
9*e)*(1 - c^2*x^2)^(5/2))/(5*c^8) + (6*e*(1 - c^2*x^2)^(7/2))/(7*c^8)))/48
 + (d*x^6*(a + b*ArcSech[c*x]))/6 + (e*x^8*(a + b*ArcSech[c*x]))/8
```

3.95.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.95.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.60

method	result
parts	$a\left(\frac{1}{8}e\,x^{8}+\frac{1}{6}d\,x^{6}\right)+\frac{b\left(\frac{c^{6}\operatorname{arcsech}(cx)e\,x^{8}}{8}+\frac{\operatorname{arcsech}(cx)d\,x^{6}c^{6}}{6}-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}+54c^{4}e\,x^{4}+112c^{4}\cdots\right.}{2520c}\cdots\right.}{c^{6}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{6}c^{8}d\,x^{6}+\frac{1}{8}e\,c^{8}x^{8}\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arcsech}(cx)d\,c^{8}x^{6}}{6}+\frac{\operatorname{arcsech}(cx)e\,c^{8}x^{8}}{8}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}+54c^{4}e\,x^{4}+112c^{4}d\,x^{2}\cdots\right.}{2520}\cdots\right.}{c^{2}}\cdots}{c^{6}}$
default	$\frac{\frac{a\left(\frac{1}{6}c^{8}d\,x^{6}+\frac{1}{8}e\,c^{8}x^{8}\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arcsech}(cx)d\,c^{8}x^{6}}{6}+\frac{\operatorname{arcsech}(cx)e\,c^{8}x^{8}}{8}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}+54c^{4}e\,x^{4}+112c^{4}d\,x^{2}\cdots\right.}{2520}\cdots\right.}{c^{2}}\cdots}{c^{6}}$

input
```
int(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/8*e*x^8+1/6*d*x^6)+b/c^6*(1/8*c^6*arcsech(c*x)*e*x^8+1/6*arcsech(c*x)
*d*x^6*c^6-1/2520/c*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(45*c^6*e*x
^6+84*c^6*d*x^4+54*c^4*e*x^4+112*c^4*d*x^2+72*c^2*e*x^2+224*c^2*d+144*e))
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.72

$$\int x^5(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{315\,ac^7ex^8+420\,ac^7dx^6+105\,(3\,bc^7ex^8+4\,bc^7dx^6)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(45\,bc^6ex^7+6\,(14\,bc^6d+9\,bc^4}{2520\,c^7}$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/2520*(315*a*c^7*e*x^8 + 420*a*c^7*d*x^6 + 105*(3*b*c^7*e*x^8 + 4*b*c^7*d
*x^6)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (45*b*c^6*e*x^
7 + 6*(14*b*c^6*d + 9*b*c^4*e)*x^5 + 8*(14*b*c^4*d + 9*b*c^2*e)*x^3 + 16*(
14*b*c^2*d + 9*b*e)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7
```

3.95.6 Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.98

$$\int x^5(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\begin{cases}\frac{adx^6}{6}+\frac{aex^8}{8}+\frac{bdx^6\operatorname{asech}(cx)}{6}+\frac{bex^8\operatorname{asech}(cx)}{8}-\frac{bdx^4\sqrt{-c^2x^2+1}}{30c^2}-\frac{bex^6\sqrt{-c^2x^2+1}}{56c^2}-\frac{2bdx^2\sqrt{-c^2x^2+1}}{45c^4}-\frac{3bex^4\sqrt{-c^2x^2+1}}{140c^4}-\\ (a+\infty b)\left(\frac{dx^6}{6}+\frac{ex^8}{8}\right)\end{cases}$$

input
```
integrate(x**5*(e*x**2+d)*(a+b*asech(c*x)),x)
```

output
```
Piecewise((a*d*x**6/6 + a*e*x**8/8 + b*d*x**6*asech(c*x)/6 + b*e*x**8*asec
h(c*x)/8 - b*d*x**4*sqrt(-c**2*x**2 + 1)/(30*c**2) - b*e*x**6*sqrt(-c**2*x
**2 + 1)/(56*c**2) - 2*b*d*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 3*b*e*x**
4*sqrt(-c**2*x**2 + 1)/(140*c**4) - 4*b*d*sqrt(-c**2*x**2 + 1)/(45*c**6) -
 b*e*x**2*sqrt(-c**2*x**2 + 1)/(35*c**6) - 2*b*e*sqrt(-c**2*x**2 + 1)/(35*
c**8), Ne(c, 0)), ((a + oo*b)*(d*x**6/6 + e*x**8/8), True))
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.76

$$\int x^5(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\frac{1}{8}\,aex^8+\frac{1}{6}\,adx^6+\frac{1}{90}\left(15\,x^6\,\text{arsech}\,(cx)-\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^5}\right)bd+\frac{1}{280}\left(35\,x^8\,\text{arsech}\,(cx)+\frac{5\,c^6x^7\left(\frac{1}{c^2x^2}-1\right)^{\frac{7}{2}}-21\,c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+35\,c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-35\,x\sqrt{\frac{1}{c^2x^2}-}}{c^7}\right.$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e*x^8 + 1/6*a*d*x^6 + 1/90*(15*x^6*arcsech(c*x) - (3*c^4*x^5*(1/(c^2
*x^2) - 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) + 15*x*sqrt(1/(c^2*x
^2) - 1))/c^5)*b*d + 1/280*(35*x^8*arcsech(c*x) + (5*c^6*x^7*(1/(c^2*x^2)
- 1)^(7/2) - 21*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2)
- 1)^(3/2) - 35*x*sqrt(1/(c^2*x^2) - 1))/c^7)*b*e
```

3.95.8 Giac [F]

$$\int x^5(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (ex^2+d)(b\,\text{arsech}\,(cx)+a)x^5\,dx$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^5, x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int x^5 \left(d+ex^2\right) \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int x^5 \left(e\,x^2+d\right) \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int(x^5*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^5*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

3.96 $\int x^3(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.96.1 Optimal result

Integrand size = 19, antiderivative size = 180

$$\int x^3(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=-\frac{b(3c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^6}+\frac{b(3c^2d+4e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{36c^6}-\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{30c^6}+\frac{1}{4}dx^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{sech}^{-1}(cx)\right)$$

output
```
1/4*d*x^4*(a+b*arcsech(c*x))+1/6*e*x^6*(a+b*arcsech(c*x))+1/36*b*(3*c^2*d+
4*e)*(-c^2*x^2+1)^(3/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^6-1/30*b*e*(-c^2
*x^2+1)^(5/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^6-1/12*b*(3*c^2*d+2*e)*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6
```

3.96.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.59

$$\int x^3\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{180}\left(15ax^4\left(3d+2ex^2\right)-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(16e+c^2(30d+8ex^2)+3c^4(5dx^2+2ex^4)\right)}{c^6}+15bx^4\left(3d+2ex^2\right)\text{sech}^{-1}(cx)\right)$$

input
```
Integrate[x^3*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

output
```
(15*a*x^4*(3*d + 2*e*x^2) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(16*e +
 c^2*(30*d + 8*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^6 + 15*b*x^4*(3*d +
2*e*x^2)*ArcSech[c*x])/180
```

3.96.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6855, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$\Big\downarrow \text{ 6855}$$
$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x^3\left(2ex^2+3d\right)}{12\sqrt{1-c^2x^2}}dx+\frac{1}{4}dx^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{sech}^{-1}(cx)\right)$$
$$\Big\downarrow \text{ 27}$$
$$\frac{1}{12}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x^3\left(2ex^2+3d\right)}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}dx^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{sech}^{-1}(cx)\right)$$
$$\Big\downarrow \text{ 354}$$

$$\frac{1}{24}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x^2(2ex^2+3d)}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{4}dx^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{24}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \left(\frac{2e(1-c^2x^2)^{3/2}}{c^4}+\frac{(-3dc^2-4e)\sqrt{1-c^2x^2}}{c^4}+\frac{3dc^2+2e}{c^4\sqrt{1-c^2x^2}}\right)dx^2+ \frac{1}{4}dx^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}dx^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{sech}^{-1}(cx)\right)+ \frac{1}{24}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2(1-c^2x^2)^{3/2}(3c^2d+4e)}{3c^6}-\frac{2\sqrt{1-c^2x^2}(3c^2d+2e)}{c^6}-\frac{4e(1-c^2x^2)^{5/2}}{5c^6}\right)$$

input
```
Int[x^3*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

output
```
(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-2*(3*c^2*d + 2*e)*Sqrt[1 - c^2*x^
2])/c^6 + (2*(3*c^2*d + 4*e)*(1 - c^2*x^2)^(3/2))/(3*c^6) - (4*e*(1 - c^2*
x^2)^(5/2))/(5*c^6)))/24 + (d*x^4*(a + b*ArcSech[c*x]))/4 + (e*x^6*(a + b*
ArcSech[c*x]))/6
```

3.96.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.96.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.67

method	result
parts	$a\left(\frac{1}{6}e\,x^6+\frac{1}{4}d\,x^4\right)+\frac{b\left(\frac{c^4\,\mathrm{arcsech}(cx)e\,x^6}{6}+\frac{\mathrm{arcsech}(cx)x^4c^4d}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,\left(6c^4e\,x^4+15c^4d\,x^2+8e\,c^2x^2+30c^2d+\right.}{180c}\right.}{c^4}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^2d\left(e\,c^2x^2+c^2d\right)^2}{2}-\frac{\left(e\,c^2x^2+c^2d\right)^3}{3}\right)}{2c^2e^2}+\frac{b\left(-\frac{\mathrm{arcsech}(cx)c^6d^3}{12e^2}+\frac{\mathrm{arcsech}(cx)c^6d\,x^4}{4}+\frac{e\,\mathrm{arcsech}(cx)c^6x^6}{6}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}}{}\right.}{}}{}$
default	$\frac{-\frac{a\left(\frac{c^2d\left(e\,c^2x^2+c^2d\right)^2}{2}-\frac{\left(e\,c^2x^2+c^2d\right)^3}{3}\right)}{2c^2e^2}+\frac{b\left(-\frac{\mathrm{arcsech}(cx)c^6d^3}{12e^2}+\frac{\mathrm{arcsech}(cx)c^6d\,x^4}{4}+\frac{e\,\mathrm{arcsech}(cx)c^6x^6}{6}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}}{}\right.}{}}{}$

input
```
int(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/6*e*x^6+1/4*d*x^4)+b/c^4*(1/6*c^4*arcsech(c*x)*e*x^6+1/4*arcsech(c*x)
*x^4*c^4*d-1/180/c*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(6*c^4*e*x^4
+15*c^4*d*x^2+8*c^2*e*x^2+30*c^2*d+16*e))
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.82

$$\int x^3(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{30\,ac^5ex^6+45\,ac^5dx^4+15\,(2\,bc^5ex^6+3\,bc^5dx^4)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(6\,bc^4ex^5+(15\,bc^4d+8\,bc^2e)x^3+}{180\,c^5}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/180*(30*a*c^5*e*x^6 + 45*a*c^5*d*x^4 + 15*(2*b*c^5*e*x^6 + 3*b*c^5*d*x^4
)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (6*b*c^4*e*x^5 + (
15*b*c^4*d + 8*b*c^2*e)*x^3 + 2*(15*b*c^2*d + 8*b*e)*x)*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)))/c^5
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98

$$\int x^3(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\begin{cases}\frac{adx^4}{4}+\frac{aex^6}{6}+\frac{bdx^4\operatorname{asech}(cx)}{4}+\frac{bex^6\operatorname{asech}(cx)}{6}-\frac{bdx^2\sqrt{-c^2x^2+1}}{12c^2}-\frac{bex^4\sqrt{-c^2x^2+1}}{30c^2}-\frac{bd\sqrt{-c^2x^2+1}}{6c^4}-\frac{2bex^2\sqrt{-c^2x^2+1}}{45c^4}-4\\(a+\infty b)\left(\frac{dx^4}{4}+\frac{ex^6}{6}\right)\end{cases}$$

input
```
integrate(x**3*(e*x**2+d)*(a+b*asech(c*x)),x)
```

output
```
Piecewise((a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*asech(c*x)/4 + b*e*x**6*asec
h(c*x)/6 - b*d*x**2*sqrt(-c**2*x**2 + 1)/(12*c**2) - b*e*x**4*sqrt(-c**2*x
**2 + 1)/(30*c**2) - b*d*sqrt(-c**2*x**2 + 1)/(6*c**4) - 2*b*e*x**2*sqrt(-
c**2*x**2 + 1)/(45*c**4) - 4*b*e*sqrt(-c**2*x**2 + 1)/(45*c**6), Ne(c, 0))
, ((a + oo*b)*(d*x**4/4 + e*x**6/6), True))
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.77

$$\int x^3(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{6}aex^6+\frac{1}{4}adx^4+\frac{1}{12}\left(3x^4\,\text{arsech}\,(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)bd$$
$$+\frac{1}{90}\left(15x^6\,\text{arsech}\,(cx)-\frac{3c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}-10c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15x\sqrt{\frac{1}{c^2x^2}-1}}{c^5}\right)be$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^
2) - 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*d + 1/90*(15*x^6*arcsech
(c*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) - 1)^
(3/2) + 15*x*sqrt(1/(c^2*x^2) - 1))/c^5)*b*e
```

3.96.8 Giac [F]

$$\int x^3(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int(ex^2+d)(b\,\text{arsech}\,(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^3, x)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^3\,(e\,x^2+d)\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

3.97 $\int x(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.97.1 Optimal result

Integrand size = 17, antiderivative size = 164

$$\int x(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx = -\frac{b(2c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{4c^4}+\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{12c^4}+\frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e}-\frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{4e}$$

output
```
1/4*(e*x^2+d)^2*(a+b*arcsech(c*x))/e+1/12*b*e*(-c^2*x^2+1)^(3/2)*(1/(c*x+1
))^(1/2)*(c*x+1)^(1/2)/c^4-1/4*b*d^2*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*x+1
))^(1/2)*(c*x+1)^(1/2)/e-1/4*b*(2*c^2*d+e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)
*(-c^2*x^2+1)^(1/2)/c^4
```

3.97.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.52

$$\int x\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\frac{1}{12}\left(3ax^2\left(2d+ex^2\right)-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(2e+c^2\left(6d+ex^2\right)\right)}{c^4}+3bx^2\left(2d+ex^2\right)\text{sech}^{-1}(cx)\right)$$

input
```
Integrate[x*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

output
```
(3*a*x^2*(2*d + e*x^2) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(2*e + c^2
*(6*d + e*x^2)))/c^4 + 3*b*x^2*(2*d + e*x^2)*ArcSech[c*x])/12
```

3.97.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6853, 2036, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6853}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{\left(ex^2+d\right)^2}{x\sqrt{1-cx}\sqrt{cx+1}}dx}{4e}+\frac{\left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{2036}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{\left(ex^2+d\right)^2}{x\sqrt{1-c^2x^2}}dx}{4e}+\frac{\left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{354}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(ex^2+d)^2}{x^2\sqrt{1-c^2x^2}}dx^2}{8e}+\frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \left(\frac{d^2}{x^2\sqrt{1-c^2x^2}}-\frac{e^2\sqrt{1-c^2x^2}}{c^2}+\frac{e(2dc^2+e)}{c^2\sqrt{1-c^2x^2}}\right)dx^2}{8e}+\frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-2d^2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{2e\sqrt{1-c^2x^2}(2c^2d+e)}{c^4}+\frac{2e^2(1-c^2x^2)^{3/2}}{3c^4}\right)}{8e}$$

input
```
Int[x*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

output
```
((d + e*x^2)^2*(a + b*ArcSech[c*x]))/(4*e) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[
1 + c*x]*((-2*e*(2*c^2*d + e)*Sqrt[1 - c^2*x^2])/c^4 + (2*e^2*(1 - c^2*x^2
)^(3/2))/(3*c^4) - 2*d^2*ArcTanh[Sqrt[1 - c^2*x^2]]))/(8*e)
```

3.97.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2036
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

rule 6853
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Simp[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x
^2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e
, p}, x] && NeQ[p, -1]
```

3.97.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.18

method	result
parts	$\frac{a(e\,x^2+d)^2}{4e}+\frac{b\left(\frac{c^2 e\,\operatorname{arcsech}(cx)x^4}{4}+\frac{\operatorname{arcsech}(cx)x^2c^2d}{2}+\frac{c^2\operatorname{arcsech}(cx)d^2}{4e}-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(3c^4d^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)\ldots\right.}{12c\ldots}\ldots\right)}{c^2}$
derivativedivides	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^2}{4c^2e}+\frac{b\left(\frac{\operatorname{arcsech}(cx)c^4d^2}{4e}+\frac{\operatorname{arcsech}(cx)c^4d\,x^2}{2}+\frac{e\operatorname{arcsech}(cx)c^4x^4}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(3c^4d^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\ldots\right.\right.}{12\ldots}\ldots\right)}{c^2}}{c^2}$
default	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^2}{4c^2e}+\frac{b\left(\frac{\operatorname{arcsech}(cx)c^4d^2}{4e}+\frac{\operatorname{arcsech}(cx)c^4d\,x^2}{2}+\frac{e\operatorname{arcsech}(cx)c^4x^4}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(3c^4d^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\ldots\right.\right.}{12\ldots}\ldots\right)}{c^2}}{c^2}$

input
```
int(x*(e*x^2+d)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x^2+d)^2/e+b/c^2*(1/4*c^2*e*arcsech(c*x)*x^4+1/2*arcsech(c*x)*x^2
*c^2*d+1/4*c^2/e*arcsech(c*x)*d^2-1/12/c/e*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)
/c/x)^(1/2)*(3*c^4*d^2*arctanh(1/(-c^2*x^2+1)^(1/2))+6*c^2*d*e*(-c^2*x^2+1
)^(1/2)+e^2*(-c^2*x^2+1)^(1/2)*c^2*x^2+2*e^2*(-c^2*x^2+1)^(1/2))/(-c^2*x^2
+1)^(1/2))
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.76

$$\int x\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{3\,ac^3ex^4+6\,ac^3dx^2+3\,(bc^3ex^4+2\,bc^3dx^2)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(bc^2ex^3+2\,(3\,bc^2d+be)x)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{12\,c^3}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*a*c^3*e*x^4 + 6*a*c^3*d*x^2 + 3*(b*c^3*e*x^4 + 2*b*c^3*d*x^2)*log(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*e*x^3 + 2*(3*b*c^
2*d + b*e)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^3
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.77

$$\int x\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\begin{cases}\frac{adx^2}{2}+\frac{aex^4}{4}+\frac{bdx^2\operatorname{asech}(cx)}{2}+\frac{bex^4\operatorname{asech}(cx)}{4}-\frac{bd\sqrt{-c^2x^2+1}}{2c^2}-\frac{bex^2\sqrt{-c^2x^2+1}}{12c^2}-\frac{be\sqrt{-c^2x^2+1}}{6c^4} & \text{for } c\neq 0\\ (a+\infty b)\left(\frac{dx^2}{2}+\frac{ex^4}{4}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x*(e*x**2+d)*(a+b*asech(c*x)),x)
```

output
```
Piecewise((a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*asech(c*x)/2 + b*e*x**4*asec
h(c*x)/4 - b*d*sqrt(-c**2*x**2 + 1)/(2*c**2) - b*e*x**2*sqrt(-c**2*x**2 +
1)/(12*c**2) - b*e*sqrt(-c**2*x**2 + 1)/(6*c**4), Ne(c, 0)), ((a + oo*b)*(
d*x**2/2 + e*x**4/4), True))
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.59

$$\int x(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{4}aex^4+\frac{1}{2}adx^2+\frac{1}{2}\left(x^2\operatorname{arsech}(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)bd$$
$$+\frac{1}{12}\left(3x^4\operatorname{arsech}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)be$$

input
```
integrate(x*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1
)/c)*b*d + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 3
*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*e
```

3.97.8 Giac [F]

$$\int x(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int(ex^2+d)(b\operatorname{arsech}(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x, x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int x\left(e\,x^2+d\right)\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

3.98 $\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx$

3.98.1 Optimal result

Integrand size = 19, antiderivative size = 296

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx = -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c}+\frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}+\frac{1}{2}ex^2\left(a+b\text{sech}^{-1}(cx)\right)-\frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}+\frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}-d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

output
```
1/2*e*x^2*(a+b*arcsech(c*x))-d*(a+b*arcsech(c*x))*ln(1/x)+1/2*I*b*d*arccsc
(c*x)^2*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-b*d*arccsc(c*
x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2
)/(1+1/c/x)^(1/2)+b*d*arccsc(c*x)*ln(1/x)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(
1/2)/(1+1/c/x)^(1/2)+1/2*I*b*d*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1
-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-1/2*b*e*x*(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2)/c
```

3.98.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.36

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx = \frac{1}{2}aex^2+be\left(-\frac{1}{2c^2}-\frac{x}{2c}\right)\sqrt{\frac{1-cx}{1+cx}}+\frac{1}{2}bex^2\text{sech}^{-1}(cx) + ad\log(x)+\frac{1}{2}bd\left(-\text{sech}^{-1}(cx)\left(\text{sech}^{-1}(cx) + 2\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)\right) + \text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x,x]
```

output
```
(a*e*x^2)/2 + b*e*(-1/2*1/c^2 - x/(2*c))*Sqrt[(1 - c*x)/(1 + c*x)] + (b*e*
x^2*ArcSech[c*x])/2 + a*d*Log[x] + (b*d*(-(ArcSech[c*x]*(ArcSech[c*x] + 2*
Log[1 + E^(-2*ArcSech[c*x])])) + PolyLog[2, -E^(-2*ArcSech[c*x])]))/2
```

3.98.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6857, 6373, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx$$

$$\Big\downarrow \text{6857}$$

$$-\int \left(\frac{d}{x^2}+e\right) x^3 \left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right) d\frac{1}{x}$$

$\Big\downarrow$ 6373

$$\frac{b\int -\frac{ex^2-2d\log\left(\frac{1}{x}\right)}{2\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}d\frac{1}{x}}{c}-d\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)$$

$\Big\downarrow$ 27

$$-\frac{b\int \frac{ex^2-2d\log\left(\frac{1}{x}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}d\frac{1}{x}}{2c}-d\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)$$

$\Big\downarrow$ 7293

$$-\frac{b\int \left(\frac{ex^2}{\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}-\frac{2d\log\left(\frac{1}{x}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}\right)d\frac{1}{x}}{2c}-d\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)$$

$\Big\downarrow$ 2009

$$-d\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)-\frac{b\left(-\frac{icd\sqrt{1-\frac{1}{c^2x^2}}\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{icd\sqrt{1-\frac{1}{c^2x^2}}\arcsin\left(\frac{1}{cx}\right)^2}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{2cd\sqrt{1-\frac{1}{c^2x^2}}\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{2cd\sqrt{1-\frac{}{}}}{\sqrt{}}\right.}{2c}$$

input
```
Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x,x]
```

output
```
(e*x^2*(a + b*ArcCosh[1/(c*x)]))/2 - d*(a + b*ArcCosh[1/(c*x)])*Log[x^(-1)
] - (b*(e*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x - (I*c*d*Sqrt[1 - 1/(c^2*
x^2)]*ArcSin[1/(c*x)]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (2*c*d*S
qrt[1 - 1/(c^2*x^2)]*ArcSin[1/(c*x)]*Log[1 - E^((2*I)*ArcSin[1/(c*x)])])/(
Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - (2*c*d*Sqrt[1 - 1/(c^2*x^2)]*ArcSi
n[1/(c*x)]*Log[x^(-1)])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - (I*c*d*Sq
rt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*I)*ArcSin[1/(c*x)])])/(Sqrt[-1 + 1/(c
*x)]*Sqrt[1 + 1/(c*x)])))/(2*c)
```

3.98. $\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx$

3.98.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.98.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.54

method	result
parts	$\frac{ae\,x^2}{2}+ad\ln(x)+b\left(\frac{\operatorname{arcsech}(cx)^2 d}{2}+\frac{e\left(-\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+c^2x^2\operatorname{arcsech}(cx)+1\right)}{2c^2}-d\operatorname{arcsech}(cx\right.$
derivativedivides	$\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{b\left(\frac{c^2d\operatorname{arcsech}(cx)^2}{2}+\frac{e\left(-\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+c^2x^2\operatorname{arcsech}(cx)+1\right)}{2}-\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1\right.\right.\right.}{c^2}$
default	$\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{b\left(\frac{c^2d\operatorname{arcsech}(cx)^2}{2}+\frac{e\left(-\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+c^2x^2\operatorname{arcsech}(cx)+1\right)}{2}-\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1\right.\right.\right.}{c^2}$

3.98. $\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx$

input
```
int((e*x^2+d)*(a+b*arcsech(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*e*x^2+a*d*ln(x)+b*(1/2*arcsech(c*x)^2*d+1/2*e*(-(-(c*x-1)/c/x)^(1/2)
*c*x*((c*x+1)/c/x)^(1/2)+c^2*x^2*arcsech(c*x)+1)/c^2-d*arcsech(c*x)*ln(1+(
1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-1/2*d*polylog(2,-(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))^2))
```

3.98.5 Fricas [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{(ex^2+d)(b\,\text{arsech}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))/x, x)
```

3.98.6 Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)}{x}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*asech(c*x))/x,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)/x, x)
```

3.98.7 Maxima [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{(ex^2+d)(b\,\text{arsech}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x,x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + a*d*log(x) + integrate(b*e*x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c
*x) - 1) + 1/(c*x)) + b*d*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x
))/x, x)
```

3.98.8 Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)(b\operatorname{arsech}(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x, x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x, x)
```

3.99 $\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx$

3.99.1 Optimal result

Integrand size = 19, antiderivative size = 309

$$\begin{aligned}
\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx &= \frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} + \frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\
&\quad + \frac{1}{4}bc^2d\operatorname{sech}^{-1}(cx) - \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2} \\
&\quad - \frac{be\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\
&\quad + \frac{be\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\
&\quad - e\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\
&\quad + \frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}
\end{aligned}$$

output
```
1/4*b*c^2*d*arcsech(c*x)-1/2*d*(a+b*arcsech(c*x))/x^2-e*(a+b*arcsech(c*x))
*ln(1/x)+1/2*I*b*e*arccsc(c*x)^2*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1
/c/x)^(1/2)-b*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x
^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+b*e*arccsc(c*x)*ln(1/x)*(1-1/c^
2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/2*I*b*e*polylog(2,(I/c/x+(
1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2
)+1/4*b*c*d*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/x
```

3.99.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.55

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx &= -\frac{ad}{2x^2}+bd\left(\frac{1}{4x^2}+\frac{c}{4x}\right)\sqrt{\frac{1-cx}{1+cx}} \\ &\quad -\frac{bd\text{sech}^{-1}(cx)}{2x^2}-\frac{1}{4}bc^2d\log(x)+ae\log(x) \\ &\quad +\frac{1}{4}bc^2d\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right) \\ &\quad +\frac{1}{2}be\Big(-\text{sech}^{-1}(cx)\left(\text{sech}^{-1}(cx)\right. \\ &\quad \left.+2\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)\right) \\ &\quad +\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)\Big)\end{aligned}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^3,x]
```

output
```
-1/2*(a*d)/x^2 + b*d*(1/(4*x^2) + c/(4*x))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*
d*ArcSech[c*x])/(2*x^2) - (b*c^2*d*Log[x])/4 + a*e*Log[x] + (b*c^2*d*Log[1
 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/4 + (b*e*(-
(ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])])) + PolyLog[2
, -E^(-2*ArcSech[c*x])]))/2
```

3.99.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6857, 6373, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}dx$$

$$\Big\downarrow \text{ 6857}$$

$$-\int \left(\frac{d}{x^2}+e\right)x\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 6373}$$

$$\frac{b\int \frac{\frac{d}{x^2}+2e\log\left(\frac{1}{x}\right)}{2\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}d\frac{1}{x}}{c}-\frac{d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\int \frac{\frac{d}{x^2}+2e\log\left(\frac{1}{x}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}d\frac{1}{x}}{2c}-\frac{d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{b\int \left(\frac{d}{\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}x^2}+\frac{2e\log\left(\frac{1}{x}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}\right)d\frac{1}{x}}{2c}-\frac{d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+\frac{b\left(\frac{1}{2}c^3d\text{arccosh}\left(\frac{1}{cx}\right)+\frac{ice\sqrt{1-\frac{1}{c^2x^2}}\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{ice\sqrt{1-\frac{1}{c^2x^2}}\arcsin\left(\frac{1}{cx}\right)^2}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{2ce\sqrt{1-\frac{1}{c^2x^2}}\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin\dots}\right.}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}\dots}{2c}$$

input
```
Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^3,x]
```

output
```
-1/2*(d*(a + b*ArcCosh[1/(c*x)]))/x^2 - e*(a + b*ArcCosh[1/(c*x)])*Log[x^(
-1)] + (b*((c^2*d*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(2*x) + (c^3*d*Arc
Cosh[1/(c*x)])/2 + (I*c*e*Sqrt[1 - 1/(c^2*x^2)]*ArcSin[1/(c*x)]^2)/(Sqrt[-
1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - (2*c*e*Sqrt[1 - 1/(c^2*x^2)]*ArcSin[1/(c
*x)]*Log[1 - E^((2*I)*ArcSin[1/(c*x)])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c
*x)]) + (2*c*e*Sqrt[1 - 1/(c^2*x^2)]*ArcSin[1/(c*x)]*Log[x^(-1)])/(Sqrt[-1
 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (I*c*e*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E
^((2*I)*ArcSin[1/(c*x)])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])))/(2*c)
```

3.99.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.99. $\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx$

3.99.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.54

method	result
parts	$-\frac{ad}{2x^2}+ae\ln\left(x\right)+\frac{be\,\mathrm{arcsech}(cx)^2}{2}+\frac{b\,c^2d\,\mathrm{arcsech}(cx)}{4}+\frac{bcd\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}}{4x}-\frac{bd\,\mathrm{arcsech}(cx)}{2x^2}-be\,\mathrm{ar}$
derivativedivides	$c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}+\frac{be\,\mathrm{arcsech}(cx)^2}{2c^2}+\frac{bd\,\mathrm{arcsech}(cx)}{4}+\frac{bd\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}}{4cx}-\frac{b\,\mathrm{arcsech}(cx)d}{2c^2x^2}-\frac{be\,\mathrm{a}}{}\right.$
default	$c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}+\frac{be\,\mathrm{arcsech}(cx)^2}{2c^2}+\frac{bd\,\mathrm{arcsech}(cx)}{4}+\frac{bd\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}}{4cx}-\frac{b\,\mathrm{arcsech}(cx)d}{2c^2x^2}-\frac{be\,\mathrm{a}}{}\right.$

input
```
int((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*d/x^2+a*e*ln(x)+1/2*b*e*arcsech(c*x)^2+1/4*b*c^2*d*arcsech(c*x)+1/4
*b*c*d/x*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)-1/2*b*d/x^2*arcsech(c*x)
-b*e*arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-1/2*b*e
*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)
```

3.99.5 Fricas [F]

$$\int \frac{(d+ex^2)\left(a+b\mathrm{sech}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{(ex^2+d)(b\,\mathrm{arsech}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))/x^3, x)
```

3.99.6 Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(a+b\operatorname{asech}(cx))\,(d+ex^2)}{x^3}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*asech(c*x))/x**3,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)/x**3, x)
```

3.99.7 Maxima [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)(b\operatorname{arsech}(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x, algorithm="maxima")
```

output
```
-1/8*b*d*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) -
 c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) - 1
) - 1))/c + 4*arcsech(c*x)/x^2) + b*e*integrate(log(sqrt(1/(c*x) + 1)*sqrt
(1/(c*x) - 1) + 1/(c*x))/x, x) + a*e*log(x) - 1/2*a*d/x^2
```

3.99.8 Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)(b\operatorname{arsech}(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^3, x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^3, x)
```

3.100 $\int x^2(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right) dx$

3.100.1 Optimal result

Integrand size = 21, antiderivative size = 275

$$\begin{aligned}
&\int x^2(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right) dx \\
&= -\frac{b(280c^4d^2+252c^2de+75e^2)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1680c^6} \\
&\quad -\frac{be(84c^2d+25e)\,x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{840c^4}-\frac{be^2x^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2} \\
&\quad +\frac{1}{3}d^2x^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{sech}^{-1}(cx)\right) \\
&\quad +\frac{b(280c^4d^2+252c^2de+75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{1680c^7}
\end{aligned}$$

output
```
1/3*d^2*x^3*(a+b*arcsech(c*x))+2/5*d*e*x^5*(a+b*arcsech(c*x))+1/7*e^2*x^7*
(a+b*arcsech(c*x))+1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*arcsin(c*x)*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^7-1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^
2)*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6-1/840*b*e*(84*
c^2*d+25*e)*x^3*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/4
2*b*e^2*x^5*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

3.100.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.75

$$\int x^2(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{16ac^7x^3(35d^2+42dex^2+15e^2x^4)-bcx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(75e^2+2c^2e(126d+25ex^2)+8c^4(35d^2+21dex^2\right.}{}$$

input
```
Integrate[x^2*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

output
```
(16*a*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) - b*c*x*Sqrt[(1 - c*x)/(1
 + c*x)]*(1 + c*x)*(75*e^2 + 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 +
21*d*e*x^2 + 5*e^2*x^4)) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)
*ArcSech[c*x] + I*b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*Log[(-2*I)*c*x +
2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/(1680*c^7)
```

3.100.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6855, 27, 1590, 27, 363, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$\Big\downarrow \text{6855}$$
$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{x^2(15e^2x^4+42dex^2+35d^2)}{105\sqrt{1-c^2x^2}}dx+\frac{1}{3}d^2x^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{sech}^{-1}(cx)\right)$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{105}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{x^2(15e^2x^4+42dex^2+35d^2)}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}d^2x^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\downarrow \text{1590}$$

$$\frac{1}{105}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{3x^2(70c^2d^2+e(84dc^2+25e)x^2)}{\sqrt{1-c^2x^2}}dx}{6c^2}-\frac{5e^2x^5\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{3}d^2x^3(a+b\text{sech}^{-1}(cx))+\frac{2}{5}dex^5(a+b\text{sech}^{-1}(cx))+\frac{1}{7}e^2x^7(a+b\text{sech}^{-1}(cx))$$

$$\downarrow \text{27}$$

$$\frac{1}{105}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{x^2(70c^2d^2+e(84dc^2+25e)x^2)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{5e^2x^5\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{3}d^2x^3(a+b\text{sech}^{-1}(cx))+\frac{2}{5}dex^5(a+b\text{sech}^{-1}(cx))+\frac{1}{7}e^2x^7(a+b\text{sech}^{-1}(cx))$$

$$\downarrow \text{363}$$

$$\frac{1}{105}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{(280c^4d^2+252c^2de+75e^2)\int \frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{ex^3\sqrt{1-c^2x^2}(84c^2d+25e)}{4c^2}}{2c^2}-\frac{5e^2x^5\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{3}d^2x^3(a+b\text{sech}^{-1}(cx))+\frac{2}{5}dex^5(a+b\text{sech}^{-1}(cx))+\frac{1}{7}e^2x^7(a+b\text{sech}^{-1}(cx))$$

$$\downarrow \text{262}$$

$$\frac{1}{105}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{(280c^4d^2+252c^2de+75e^2)\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{ex^3\sqrt{1-c^2x^2}(84c^2d+25e)}{4c^2}}{2c^2}-\frac{5e^2x^5\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{3}d^2x^3(a+b\text{sech}^{-1}(cx))+\frac{2}{5}dex^5(a+b\text{sech}^{-1}(cx))+\frac{1}{7}e^2x^7(a+b\text{sech}^{-1}(cx))$$

$$\downarrow \text{223}$$

$$\frac{1}{3}d^2x^3(a+b\text{sech}^{-1}(cx))+\frac{2}{5}dex^5(a+b\text{sech}^{-1}(cx))+\frac{1}{7}e^2x^7(a+b\text{sech}^{-1}(cx))+\frac{1}{105}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)(280c^4d^2+252c^2de+75e^2)}{4c^2}-\frac{ex^3\sqrt{1-c^2x^2}(84c^2d+25e)}{4c^2}}{2c^2}-\frac{5e^2x^5\sqrt{1-c^2x^2}}{2c^2}\right)$$

input
```
Int[x^2*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

3.100. $\int x^2(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

output
```
(d^2*x^3*(a + b*ArcSech[c*x]))/3 + (2*d*e*x^5*(a + b*ArcSech[c*x]))/5 + (e
^2*x^7*(a + b*ArcSech[c*x]))/7 + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-
5*e^2*x^5*Sqrt[1 - c^2*x^2])/(2*c^2) + (-1/4*(e*(84*c^2*d + 25*e)*x^3*Sqrt
[1 - c^2*x^2])/c^2 + ((280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*(-1/2*(x*Sqrt[1
 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2))/(2*c^2)))/105
```

3.100.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.100.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.04

method	result
parts	$a\left(\frac{1}{7}e^2x^7+\frac{2}{5}de\,x^5+\frac{1}{3}x^3d^2\right)+\frac{b\left(\frac{c^3\,\mathrm{arcsech}(cx)e^2x^7}{7}+\frac{2c^3\,\mathrm{arcsech}(cx)de\,x^5}{5}+\frac{\mathrm{arcsech}(cx)d^2x^3c^3}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx}{c}}}{}\right.}{}$
derivativedivides	$\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\mathrm{arcsech}(cx)d^2c^7x^3}{3}+\frac{2\,\mathrm{arcsech}(cx)d\,c^7e\,x^5}{5}+\frac{\mathrm{arcsech}(cx)e^2c^7x^7}{7}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(28}{}\right.}{}$
default	$\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\mathrm{arcsech}(cx)d^2c^7x^3}{3}+\frac{2\,\mathrm{arcsech}(cx)d\,c^7e\,x^5}{5}+\frac{\mathrm{arcsech}(cx)e^2c^7x^7}{7}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(28}{}\right.}{}$

input
```
int(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^2*x^7+2/5*d*e*x^5+1/3*x^3*d^2)+b/c^3*(1/7*c^3*arcsech(c*x)*e^2*x^
7+2/5*c^3*arcsech(c*x)*d*e*x^5+1/3*arcsech(c*x)*d^2*x^3*c^3+1/1680/c^3*(-(
c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(-40*e^2*(-c^2*x^2+1)^(1/2)*c^5*x^
5-168*c^5*d*e*(-c^2*x^2+1)^(1/2)*x^3-280*d^2*c^5*x*(-c^2*x^2+1)^(1/2)+280*
d^2*c^4*arcsin(c*x)-50*e^2*c^3*x^3*(-c^2*x^2+1)^(1/2)-252*d*c^3*e*x*(-c^2*
x^2+1)^(1/2)+252*d*c^2*e*arcsin(c*x)-75*e^2*c*x*(-c^2*x^2+1)^(1/2)+75*e^2*
arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.24

$$\int x^2(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{240\,ac^7e^2x^7+672\,ac^7dex^5+560\,ac^7d^2x^3-2\left(280\,bc^4d^2+252\,bc^2de+75\,be^2\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-1}{}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/1680*(240*a*c^7*e^2*x^7 + 672*a*c^7*d*e*x^5 + 560*a*c^7*d^2*x^3 - 2*(280
*b*c^4*d^2 + 252*b*c^2*d*e + 75*b*e^2)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)) - 1)/(c*x)) - 16*(35*b*c^7*d^2 + 42*b*c^7*d*e + 15*b*c^7*e^2)*log(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 16*(15*b*c^7*e^2*x^7 + 42*b*
c^7*d*e*x^5 + 35*b*c^7*d^2*x^3 - 35*b*c^7*d^2 - 42*b*c^7*d*e - 15*b*c^7*e^
2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (40*b*c^6*e^2*x^6
 + 2*(84*b*c^6*d*e + 25*b*c^4*e^2)*x^4 + (280*b*c^6*d^2 + 252*b*c^4*d*e +
75*b*c^2*e^2)*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7
```

3.100.6 Sympy [F]

$$\int x^2(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^2(a+b\,\text{asech}\,(cx))\left(d+ex^2\right)^2\,dx$$

input
```
integrate(x**2*(e*x**2+d)**2*(a+b*asech(c*x)),x)
```

output
```
Integral(x**2*(a + b*asech(c*x))*(d + e*x**2)**2, x)
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.19

$$\int x^2(d+ex^2)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{7}ae^2x^7+\frac{2}{5}adex^5+\frac{1}{3}ad^2x^3+\frac{1}{6}\left(2\,x^3\operatorname{arsech}(cx)-\frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)bd^2$$
$$+\frac{1}{20}\left(8\,x^5\operatorname{arsech}(cx)-\frac{\frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+5\sqrt{\frac{1}{c^2x^2}-1}}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}+\frac{3\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4}}{c}\right)bde$$
$$+\frac{1}{336}\left(48\,x^7\operatorname{arsech}(cx)-\frac{\frac{15\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+40\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^2x^2}-1}}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^6}}{c}\right)be^2$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/6*(2*x^3*arcsech(c*x) -
(sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*
x^2) - 1))/c^2)/c)*b*d^2 + 1/20*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1
)^(3/2) + 5*sqrt(1/(c^2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^
2*x^2) - 1) + c^4) + 3*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b*d*e + 1/336
*(48*x^7*arcsech(c*x) - ((15*(1/(c^2*x^2) - 1)^(5/2) + 40*(1/(c^2*x^2) - 1
)^(3/2) + 33*sqrt(1/(c^2*x^2) - 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c
^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*arctan(sqrt(1/(c^2*x^
2) - 1))/c^6)/c)*b*e^2
```

3.100.8 Giac [F]

$$\int x^2 \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^2 (b\,\text{arsech}\,(cx)+a)x^2 \, dx$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*x^2, x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int x^2 \left(e\,x^2+d\right)^2 \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int(x^2*(d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)
```

3.101 $\int (d+ex^2)^2 (a+b\text{sech}^{-1}(cx)) \, dx$

3.101.1 Optimal result

Integrand size = 18, antiderivative size = 204

$$\int (d+ex^2)^2 (a+b\text{sech}^{-1}(cx)) \, dx = -\frac{be(40c^2d+9e)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{120c^4} - \frac{be^2x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{20c^2} + d^2x(a+b\text{sech}^{-1}(cx)) + \frac{2}{3}dex^3(a+b\text{sech}^{-1}(cx)) + \frac{1}{5}e^2x^5(a+b\text{sech}^{-1}(cx)) + \frac{b(120c^4d^2+40c^2de+9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{120c^5}$$

output
```
d^2*x*(a+b*arcsech(c*x))+2/3*d*e*x^3*(a+b*arcsech(c*x))+1/5*e^2*x^5*(a+b*a
rcsech(c*x))+1/120*b*(120*c^4*d^2+40*c^2*d*e+9*e^2)*arcsin(c*x)*(1/(c*x+1)
)^(1/2)*(c*x+1)^(1/2)/c^5-1/120*b*e*(40*c^2*d+9*e)*x*(1/(c*x+1))^(1/2)*(c*
x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/20*b*e^2*x^3*(1/(c*x+1))^(1/2)*(c*x+1)
^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

3.101.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.85

$$\int (d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right) dx$$
$$= \frac{8ac^5x(15d^2+10dex^2+3e^2x^4)-bcex\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(9e+c^2(40d+6ex^2)\right)+8bc^5x(15d^2+10dex^2+3e^2}{120c^5}$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

output
```
(8*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - b*c*e*x*Sqrt[(1 - c*x)/(1 +
 c*x)]*(1 + c*x)*(9*e + c^2*(40*d + 6*e*x^2)) + 8*b*c^5*x*(15*d^2 + 10*d*e
*x^2 + 3*e^2*x^4)*ArcSech[c*x] + I*b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*Lo
g[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/(120*c^5)
```

3.101.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6845, 27, 1473, 25, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right) dx$$
$$\Big\downarrow \; 6845$$
$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{3e^2x^4+10dex^2+15d^2}{15\sqrt{1-c^2x^2}}dx+d^2x\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\text{sech}^{-1}(cx)\right)$$
$$\Big\downarrow \; 27$$
$$\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{3e^2x^4+10dex^2+15d^2}{\sqrt{1-c^2x^2}}dx+d^2x\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\text{sech}^{-1}(cx)\right)$$
$$\Big\downarrow \; 1473$$

$$\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{60c^2d^2+e(40dc^2+9e)x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{3e^2x^3\sqrt{1-c^2x^2}}{4c^2}\right)+d^2x(a+b\text{sech}^{-1}(cx))+\frac{2}{3}dex^3(a+b\text{sech}^{-1}(cx))+\frac{1}{5}e^2x^5(a+b\text{sech}^{-1}(cx))$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{60c^2d^2+e(40dc^2+9e)x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{3e^2x^3\sqrt{1-c^2x^2}}{4c^2}\right)+d^2x(a+b\text{sech}^{-1}(cx))+\frac{2}{3}dex^3(a+b\text{sech}^{-1}(cx))+\frac{1}{5}e^2x^5(a+b\text{sech}^{-1}(cx))$$

$$\Big\downarrow \text{299}$$

$$\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{(120c^4d^2+40c^2de+9e^2)\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{ex\sqrt{1-c^2x^2}(40c^2d+9e)}{2c^2}}{4c^2}-\frac{3e^2x^3\sqrt{1-c^2x^2}}{4c^2}\right)+d^2x(a+b\text{sech}^{-1}(cx))+\frac{2}{3}dex^3(a+b\text{sech}^{-1}(cx))+\frac{1}{5}e^2x^5(a+b\text{sech}^{-1}(cx))$$

$$\Big\downarrow \text{223}$$

$$d^2x(a+b\text{sech}^{-1}(cx))+\frac{2}{3}dex^3(a+b\text{sech}^{-1}(cx))+\frac{1}{5}e^2x^5(a+b\text{sech}^{-1}(cx))+\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\arcsin(cx)(120c^4d^2+40c^2de+9e^2)}{2c^3}-\frac{ex\sqrt{1-c^2x^2}(40c^2d+9e)}{2c^2}}{4c^2}-\frac{3e^2x^3\sqrt{1-c^2x^2}}{4c^2}\right)$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

output
```
d^2*x*(a + b*ArcSech[c*x]) + (2*d*e*x^3*(a + b*ArcSech[c*x]))/3 + (e^2*x^5
*(a + b*ArcSech[c*x]))/5 + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-3*e^2*
x^3*Sqrt[1 - c^2*x^2])/(4*c^2) + (-1/2*(e*(40*c^2*d + 9*e)*x*Sqrt[1 - c^2*
x^2])/c^2 + ((120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*ArcSin[c*x])/(2*c^3))/(4*c
^2)))/15
```

3.101. $\int (d+ex^2)^2 (a+b\text{sech}^{-1}(cx))\, dx$

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 6845
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSech[c*x])   u
, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]   Int[SimplifyIntegrand[u/(x*
Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (
IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

3.101. $\int (d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right) dx$

3.101.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02

method	result
parts	$a\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+x\,d^2\right)+\frac{b\left(\frac{c\,\text{arcsech}(cx)e^2x^5}{5}+\frac{2c\,\text{arcsech}(cx)de\,x^3}{3}+\text{arcsech}(cx)xc\,d^2+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,(1}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\text{arcsech}(cx)d^2c^5x+\frac{2\,\text{arcsech}(cx)d\,c^5e\,x^3}{3}+\frac{\text{arcsech}(cx)e^2c^5x^5}{5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(120d^2c^4}{}\right.}{}}{c}$
default	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\text{arcsech}(cx)d^2c^5x+\frac{2\,\text{arcsech}(cx)d\,c^5e\,x^3}{3}+\frac{\text{arcsech}(cx)e^2c^5x^5}{5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(120d^2c^4}{}\right.}{}}{c}$

input
```
int((e*x^2+d)^2*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^2*x^5+2/3*d*e*x^3+x*d^2)+b/c*(1/5*c*arcsech(c*x)*e^2*x^5+2/3*c*ar
csech(c*x)*d*e*x^3+arcsech(c*x)*x*c*d^2+1/120/c^3*(-(c*x-1)/c/x)^(1/2)*x*(
(c*x+1)/c/x)^(1/2)*(120*d^2*c^4*arcsin(c*x)-40*d*c^3*e*x*(-c^2*x^2+1)^(1/2
)-6*e^2*c^3*x^3*(-c^2*x^2+1)^(1/2)+40*d*c^2*e*arcsin(c*x)-9*e^2*c*x*(-c^2*
x^2+1)^(1/2)+9*e^2*arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(130) = 260.

Time = 0.37 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.50

$$\int \left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

$$=\frac{24\,ac^5e^2x^5+80\,ac^5dex^3+120\,ac^5d^2x-2\left(120\,bc^4d^2+40\,bc^2de+9\,be^2\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-8\,(15\,bc}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/120*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x - 2*(120*b*c^
4*d^2 + 40*b*c^2*d*e + 9*b*e^2)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
 - 1)/(c*x)) - 8*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*log((c*x*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 8*(3*b*c^5*e^2*x^5 + 10*b*c^5*d*e*x^3
 + 15*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e - 3*b*c^5*e^2)*log((c*x*sq
rt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (6*b*c^4*e^2*x^4 + (40*b*c^4*d*
e + 9*b*c^2*e^2)*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^5
```

3.101.6 Sympy [F]

$$\int \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)\, dx = \int \left(a+b\,\text{asech}\,(cx)\right)\left(d+ex^2\right)^2\, dx$$

input
```
integrate((e*x**2+d)**2*(a+b*asech(c*x)),x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)**2, x)
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.10

$$\int \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)\, dx$$
$$= \frac{1}{5}\,ae^2x^5 + \frac{2}{3}\,adex^3 + \frac{1}{3}\left(2\,x^3\,\text{arsech}\,(cx) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)bde$$
$$+ \frac{1}{40}\left(8\,x^5\,\text{arsech}\,(cx) - \frac{\frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+5\sqrt{\frac{1}{c^2x^2}-1}}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4} + \frac{3\,\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4}}{c}\right)be^2$$
$$+ ad^2x + \frac{\left(cx\,\text{arsech}\,(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)bd^2}{c}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/3*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2
) - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/
c)*b*d*e + 1/40*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1)^(3/2) + 5*sqrt
(1/(c^2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^
4) + 3*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b*e^2 + a*d^2*x + (c*x*arcsec
h(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d^2/c
```

3.101.8 Giac [F]

$$\int \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (ex^2+d)^2(b\,\text{arsech}\,(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a), x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (e\,x^2+d)^2\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
```

output
```
int((d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)
```

3.102 $\int \frac{(d+ex^2)^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^2}\,dx$

3.102.1 Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \frac{(d+ex^2)^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^2}\,dx = \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} - \frac{d^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{x} + 2dex\left(a+b\,\text{sech}^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\,\text{sech}^{-1}(cx)\right) + \frac{be(12c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{6c^3}$$

output
```
-d^2*(a+b*arcsech(c*x))/x+2*d*e*x*(a+b*arcsech(c*x))+1/3*e^2*x^3*(a+b*arcs
ech(c*x))+1/6*b*e*(12*c^2*d+e)*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)
/c^3+b*d^2*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x-1/6*b*e^2*
x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

3.102.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^2}\,dx$$
$$= \frac{-bc\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(-6c^2d^2+e^2x^2\right)+2ac^3(-3d^2+6dex^2+e^2x^4)+2bc^3(-3d^2+6dex^2+e^2x^4)\,\text{sech}^{-1}(c}{6c^3x}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^2,x]
```

output
```
(-(b*c*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(-6*c^2*d^2 + e^2*x^2)) + 2*a*c
^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)
*ArcSech[c*x] + I*b*e*(12*c^2*d + e)*x*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(
1 + c*x)]*(1 + c*x)])/(6*c^3*x)
```

3.102.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6855, 27, 1588, 27, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^2}\,dx$$
$$\Big\downarrow \text{ 6855}$$
$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{-e^2x^4-6dex^2+3d^2}{3x^2\sqrt{1-c^2x^2}}dx-\frac{d^2(a+b\text{sech}^{-1}(cx))}{x}+2dex(a+b\text{sech}^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\text{sech}^{-1}(cx))$$
$$\Big\downarrow \text{ 27}$$
$$-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{-e^2x^4-6dex^2+3d^2}{x^2\sqrt{1-c^2x^2}}dx-\frac{d^2(a+b\text{sech}^{-1}(cx))}{x}+2dex(a+b\text{sech}^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\text{sech}^{-1}(cx))$$

$$\Big\downarrow \text{1588}$$

$$-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\int\frac{e(ex^2+6d)}{\sqrt{1-c^2x^2}}dx-\frac{3d^2\sqrt{1-c^2x^2}}{x}\right)-\frac{d^2(a+b\text{sech}^{-1}(cx))}{x}+2dex(a+b\text{sech}^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\text{sech}^{-1}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-e\int\frac{ex^2+6d}{\sqrt{1-c^2x^2}}dx-\frac{3d^2\sqrt{1-c^2x^2}}{x}\right)-\frac{d^2(a+b\text{sech}^{-1}(cx))}{x}+2dex(a+b\text{sech}^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\text{sech}^{-1}(cx))$$

$$\Big\downarrow \text{299}$$

$$-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-e\left(\frac{(12c^2d+e)\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{ex\sqrt{1-c^2x^2}}{2c^2}\right)-\frac{3d^2\sqrt{1-c^2x^2}}{x}\right)-\frac{d^2(a+b\text{sech}^{-1}(cx))}{x}+2dex(a+b\text{sech}^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\text{sech}^{-1}(cx))$$

$$\Big\downarrow \text{223}$$

$$-\frac{d^2(a+b\text{sech}^{-1}(cx))}{x}+2dex(a+b\text{sech}^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\text{sech}^{-1}(cx))-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-e\left(\frac{\arcsin(cx)(12c^2d+e)}{2c^3}-\frac{ex\sqrt{1-c^2x^2}}{2c^2}\right)-\frac{3d^2\sqrt{1-c^2x^2}}{x}\right)$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^2,x]
```

output
```
-((d^2*(a + b*ArcSech[c*x]))/x) + 2*d*e*x*(a + b*ArcSech[c*x]) + (e^2*x^3*
(a + b*ArcSech[c*x]))/3 - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-3*d^2*S
qrt[1 - c^2*x^2])/x - e*(-1/2*(e*x*Sqrt[1 - c^2*x^2])/c^2 + ((12*c^2*d + e
)*ArcSin[c*x])/(2*c^3))))/3
```

3.102.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.102.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.04

method	result
parts	$a\left(\frac{e^2x^3}{3}+2dex-\frac{d^2}{x}\right)+bc\left(\frac{\operatorname{arcsech}(cx)e^2x^3}{3c}+\frac{2\operatorname{arcsech}(cx)dex}{c}-\frac{\operatorname{arcsech}(cx)d^2}{xc}-\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(\cdots\right.}{\cdots}\right.$
derivativedivides	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b\left(2\operatorname{arcsech}(cx)c^3dex+\frac{\operatorname{arcsech}(cx)e^2c^3x^3}{3}-\frac{\operatorname{arcsech}(cx)c^3d^2}{x}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(6\sqrt{-\cdots}\right.}{\cdots}\right.}{c^4}\right.$
default	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b\left(2\operatorname{arcsech}(cx)c^3dex+\frac{\operatorname{arcsech}(cx)e^2c^3x^3}{3}-\frac{\operatorname{arcsech}(cx)c^3d^2}{x}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(6\sqrt{-\cdots}\right.}{\cdots}\right.}{c^4}\right.$

input
```
int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e^2*x^3+2*d*e*x-d^2/x)+b*c*(1/3/c*arcsech(c*x)*e^2*x^3+2/c*arcsech(
c*x)*d*e*x-arcsech(c*x)*d^2/x/c-1/6/c^4*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)
^(1/2)*(-6*(-c^2*x^2+1)^(1/2)*c^4*d^2-12*arcsin(c*x)*c^3*d*e*x+e^2*(-c^2*x
^2+1)^(1/2)*c^2*x^2-arcsin(c*x)*e^2*c*x)/(-c^2*x^2+1)^(1/2))
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(107) = 214$.

Time = 0.32 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^2}\,dx$$

$$= \frac{2\,ac^3e^2x^4 + 12\,ac^3dex^2 - 6\,ac^3d^2 - 2\,(12\,bc^2de + be^2)x\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + 2\,(3\,bc^3d^2 - 6\,bc^3de - bc}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e^2*x^4 + 12*a*c^3*d*e*x^2 - 6*a*c^3*d^2 - 2*(12*b*c^2*d*e +
b*e^2)*x*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + 2*(3*b*c
^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
) - 1)/x) + 2*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2 + (3*b*c^3*d^
2 - 6*b*c^3*d*e - b*c^3*e^2)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) +
1)/(c*x)) + (6*b*c^4*d^2*x - b*c^2*e^2*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
)/(c^3*x)
```

3.102.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)^2}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**2,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**2, x)
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx$$
$$= \frac{1}{3}ae^2x^3 + \left(c\sqrt{\frac{1}{c^2x^2}-1} - \frac{\operatorname{arsech}(cx)}{x}\right)bd^2$$
$$+ \frac{1}{6}\left(2\,x^3\operatorname{arsech}(cx) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)be^2$$
$$+ 2\,adex + \frac{2\left(cx\operatorname{arsech}(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)bde}{c} - \frac{ad^2}{x}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 + (c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*d^2 + 1/6*(2*
x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) +
arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b*e^2 + 2*a*d*e*x + 2*(c*x*arcsech(c
*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d*e/c - a*d^2/x
```

3.102.8 Giac [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(ex^2+d)^2(b\operatorname{arsech}(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^2, x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^2}\,dx = \int \frac{(e\,x^2+d)^2 \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^2, x)
```

3.103 $\int \frac{(d+ex^2)^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^4}\,dx$

3.103.1 Optimal result

Integrand size = 21, antiderivative size = 176

$$\int \frac{(d+ex^2)^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x^3} + \frac{2bd(c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x} - \frac{d^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\,\text{sech}^{-1}(cx)\right)}{x} + e^2x\left(a+b\,\text{sech}^{-1}(cx)\right) + \frac{be^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\arcsin(cx)}{c}$$

output
```
-1/3*d^2*(a+b*arcsech(c*x))/x^3-2*d*e*(a+b*arcsech(c*x))/x+e^2*x*(a+b*arcs
ech(c*x))+b*e^2*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c+1/9*b*d^2*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+2/9*b*d*(c^2*d+9*e)*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

3.103.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^4} dx$$
$$= \frac{bcd\sqrt{\frac{1-cx}{1+cx}}(1+cx)(d+2c^2dx^2+18ex^2)-3ac(d^2+6dex^2-3e^2x^4)-3bc(d^2+6dex^2-3e^2x^4)\operatorname{sech}^{-1}(cx}{9cx^3}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^4,x]
```

output
```
(b*c*d*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + 2*c^2*d*x^2 + 18*e*x^2) -
3*a*c*(d^2 + 6*d*e*x^2 - 3*e^2*x^4) - 3*b*c*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*
ArcSech[c*x] + (9*I)*b*e^2*x^3*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)
]*(1 + c*x)])/(9*c*x^3)
```

3.103.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6855, 27, 1588, 25, 358, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^4} dx$$
$$\Big\downarrow \text{ 6855}$$
$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{-3e^2x^4+6dex^2+d^2}{3x^4\sqrt{1-c^2x^2}}dx - \frac{d^2(a+b\text{sech}^{-1}(cx))}{3x^3} - \frac{2de(a+b\text{sech}^{-1}(cx))}{x} + e^2x(a+b\text{sech}^{-1}(cx))$$
$$\Big\downarrow \text{ 27}$$
$$-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{-3e^2x^4+6dex^2+d^2}{x^4\sqrt{1-c^2x^2}}dx - \frac{d^2(a+b\text{sech}^{-1}(cx))}{3x^3} - \frac{2de(a+b\text{sech}^{-1}(cx))}{x} + e^2x(a+b\text{sech}^{-1}(cx))$$

$$\Big\downarrow \text{1588}$$

$$-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{1}{3}\int -\frac{2d(dc^2+9e)-9e^2x^2}{x^2\sqrt{1-c^2x^2}}dx-\frac{d^2\sqrt{1-c^2x^2}}{3x^3}\right)-\frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{3x^3}-\frac{2de(a+b\operatorname{sech}^{-1}(cx))}{x}+e^2x(a+b\operatorname{sech}^{-1}(cx))$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\int \frac{2d(dc^2+9e)-9e^2x^2}{x^2\sqrt{1-c^2x^2}}dx-\frac{d^2\sqrt{1-c^2x^2}}{3x^3}\right)-\frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{3x^3}-\frac{2de(a+b\operatorname{sech}^{-1}(cx))}{x}+e^2x(a+b\operatorname{sech}^{-1}(cx))$$

$$\Big\downarrow \text{358}$$

$$-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(-9e^2\int \frac{1}{\sqrt{1-c^2x^2}}dx-\frac{2d\sqrt{1-c^2x^2}(c^2d+9e)}{x}\right)-\frac{d^2\sqrt{1-c^2x^2}}{3x^3}\right)-\frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{3x^3}-\frac{2de(a+b\operatorname{sech}^{-1}(cx))}{x}+e^2x(a+b\operatorname{sech}^{-1}(cx))$$

$$\Big\downarrow \text{223}$$

$$-\frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{3x^3}-\frac{2de(a+b\operatorname{sech}^{-1}(cx))}{x}+e^2x(a+b\operatorname{sech}^{-1}(cx))-\frac{1}{3}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(-\frac{9e^2\arcsin(cx)}{c}-\frac{2d\sqrt{1-c^2x^2}(c^2d+9e)}{x}\right)-\frac{d^2\sqrt{1-c^2x^2}}{3x^3}\right)$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(a + b*ArcSech[c*x]))/x^3 - (2*d*e*(a + b*ArcSech[c*x]))/x + e^2
*x*(a + b*ArcSech[c*x]) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-1/3*(d^2
*Sqrt[1 - c^2*x^2])/x^3 + ((-2*d*(c^2*d + 9*e)*Sqrt[1 - c^2*x^2])/x - (9*e
^2*ArcSin[c*x])/c)/3))/3
```

3.103. $\int \frac{(d+ex^2)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^4}\,dx$

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.103. $\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx$

3.103.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.14

method	result
parts	$a\left(e^2x-\frac{2de}{x}-\frac{d^2}{3x^3}\right)+b\,c^3\left(\frac{\operatorname{arcsech}(cx)e^2x}{c^3}-\frac{2\operatorname{arcsech}(cx)de}{c^3x}-\frac{\operatorname{arcsech}(cx)d^2}{3x^3c^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(2\sqrt{\ }\right.}{}\right.$
derivativedivides	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\left(\operatorname{arcsech}(cx)e^2cx-\frac{\operatorname{arcsech}(cx)c\,d^2}{3x^3}-\frac{2\operatorname{arcsech}(cx)cde}{x}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(2\sqrt{-c^2x^2+1}\,c^6d\right.}{}\right.}{c^4}\right.$
default	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\left(\operatorname{arcsech}(cx)e^2cx-\frac{\operatorname{arcsech}(cx)c\,d^2}{3x^3}-\frac{2\operatorname{arcsech}(cx)cde}{x}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(2\sqrt{-c^2x^2+1}\,c^6d\right.}{}\right.}{c^4}\right.$

input
```
int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(e^2*x-2*d*e/x-1/3*d^2/x^3)+b*c^3*(1/c^3*arcsech(c*x)*e^2*x-2/c^3*arcsec
h(c*x)*d*e/x-1/3*arcsech(c*x)*d^2/x^3/c^3+1/9/c^6*(-(c*x-1)/c/x)^(1/2)/x^2
*((c*x+1)/c/x)^(1/2)*(2*(-c^2*x^2+1)^(1/2)*c^6*d^2*x^2+(-c^2*x^2+1)^(1/2)*
c^4*d^2+18*(-c^2*x^2+1)^(1/2)*c^4*d*e*x^2+9*arcsin(c*x)*e^2*c^3*x^3)/(-c^2
*x^2+1)^(1/2))
```

3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(106) = 212$.

Time = 0.31 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^4}\,dx$$
$$= \frac{9\,ace^2x^4-18\,be^2x^3\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-18\,acdex^2+3\,(bcd^2+6\,bcde-3\,bce^2)x^3\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right.}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/9*(9*a*c*e^2*x^4 - 18*b*e^2*x^3*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2
)) - 1)/(c*x)) - 18*a*c*d*e*x^2 + 3*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*
log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 3*a*c*d^2 + 3*(3*b*c*e^2
*x^4 - 6*b*c*d*e*x^2 - b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*lo
g((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + (b*c^2*d^2*x + 2*(b*c^
4*d^2 + 9*b*c^2*d*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c*x^3)
```

3.103.6 Sympy [F]

$$\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx=\int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)^2}{x^4}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**4,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**4, x)
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.76

$$\begin{aligned}&\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx\\&=2\left(c\sqrt{\frac{1}{c^2x^2}-1}-\frac{\text{arsech}\,(cx)}{x}\right)bde+ae^2x\\&\quad+\frac{1}{9}\,bd^2\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+3\,c^4\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{3\,\text{arsech}\,(cx)}{x^3}\right)\\&\quad+\frac{\left(cx\,\text{arsech}\,(cx)-\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)be^2}{c}-\frac{2\,ade}{x}-\frac{ad^2}{3\,x^3}\end{aligned}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x, algorithm="maxima")
```

output
```
2*(c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*d*e + a*e^2*x + 1/9*b*d^2*(
(c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(
c*x)/x^3) + (c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*e^2/c - 2
*a*d*e/x - 1/3*a*d^2/x^3
```

3.103.8 Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^4} dx = \int \frac{(ex^2+d)^2 (b\,\text{arsech}\,(cx)+a)}{x^4} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^4, x)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^4} dx = \int \frac{(e\,x^2+d)^2 \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^4} dx$$

input
```
int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^4, x)
```

3.104 $\int \frac{(d+ex^2)^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^6}\,dx$

3.104.1 Optimal result

Integrand size = 21, antiderivative size = 213

$$\begin{aligned}\int \frac{(d+ex^2)^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^6}\,dx &= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5}+\frac{2bd(6c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x^3}\\ &\quad+\frac{b(24c^4d^2+100c^2de+225e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x}\\ &\quad-\frac{d^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\,\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}\end{aligned}$$

output
```
-1/5*d^2*(a+b*arcsech(c*x))/x^5-2/3*d*e*(a+b*arcsech(c*x))/x^3-e^2*(a+b*ar
csech(c*x))/x+1/25*b*d^2*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2
)/x^5+2/225*b*d*(6*c^2*d+25*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1
)^(1/2)/x^3+1/225*b*(24*c^4*d^2+100*c^2*d*e+225*e^2)*(1/(c*x+1))^(1/2)*(c*
x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

3.104.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx$$
$$=\frac{-15a(3d^2+10dex^2+15e^2x^4)+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(225e^2x^4+50dex^2(1+2c^2x^2)+3d^2(3+4c^2x^2+8c^4x^4\right)}{225x^5}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^6,x]
```

output
```
(-15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1
+ c*x)*(225*e^2*x^4 + 50*d*e*x^2*(1 + 2*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 +
8*c^4*x^4)) - 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcSech[c*x])/(225*x^
5)
```

3.104.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6855, 27, 1588, 25, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx$$

$$\Big\downarrow \text{ 6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{15e^2x^4+10dex^2+3d^2}{15x^6\sqrt{1-c^2x^2}}dx-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{15e^2x^4+10dex^2+3d^2}{x^6\sqrt{1-c^2x^2}}dx-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \text{ 1588}$$

$$-\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{1}{5}\int -\frac{75e^2x^2+2d(6dc^2+25e)}{x^4\sqrt{1-c^2x^2}}dx-\frac{3d^2\sqrt{1-c^2x^2}}{5x^5}\right)-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\int \frac{75e^2x^2+2d(6dc^2+25e)}{x^4\sqrt{1-c^2x^2}}dx-\frac{3d^2\sqrt{1-c^2x^2}}{5x^5}\right)-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}(24c^4d^2+100c^2de+225e^2)\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx-\frac{2d\sqrt{1-c^2x^2}(6c^2d+25e)}{3x^3}\right)-\frac{3d^2\sqrt{1-c^2x^2}}{5x^5}\right)-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{x}-\frac{1}{15}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(-\frac{2d\sqrt{1-c^2x^2}(6c^2d+25e)}{3x^3}-\frac{\sqrt{1-c^2x^2}(24c^4d^2+100c^2de+225e^2)}{3x}\right)-\frac{3d^2\sqrt{1-c^2x^2}}{5x^5}\right)$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^6,x]
```

output
```
-1/15*(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-3*d^2*Sqrt[1 - c^2*x^2])/(5
*x^5) + ((-2*d*(6*c^2*d + 25*e)*Sqrt[1 - c^2*x^2])/(3*x^3) - ((24*c^4*d^2
+ 100*c^2*d*e + 225*e^2)*Sqrt[1 - c^2*x^2])/(3*x))/5)) - (d^2*(a + b*ArcSe
ch[c*x]))/(5*x^5) - (2*d*e*(a + b*ArcSech[c*x]))/(3*x^3) - (e^2*(a + b*Arc
Sech[c*x]))/x
```

3.104. $\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx$

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.104. $\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx$

3.104.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.83

method	result
parts	$a\left(-\frac{e^2}{x}-\frac{2de}{3x^3}-\frac{d^2}{5x^5}\right)+b\,c^5\left(-\frac{\operatorname{arcsech}(cx)e^2}{c^5x}-\frac{2\operatorname{arcsech}(cx)de}{3c^5x^3}-\frac{\operatorname{arcsech}(cx)d^2}{5x^5c^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,(2}{}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)e^2}{cx}-\frac{\operatorname{arcsech}(cx)d^2}{5c\,x^5}-\frac{2\operatorname{arcsech}(cx)de}{3c\,x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(24c^8d^2x^4+100c^6de\right.}{}\right.}{c^4}\right.$
default	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)e^2}{cx}-\frac{\operatorname{arcsech}(cx)d^2}{5c\,x^5}-\frac{2\operatorname{arcsech}(cx)de}{3c\,x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(24c^8d^2x^4+100c^6de\right.}{}\right.}{c^4}\right.$

input
```
int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*(-e^2/x-2/3*d*e/x^3-1/5*d^2/x^5)+b*c^5*(-1/c^5*arcsech(c*x)*e^2/x-2/3/c^
5*arcsech(c*x)*d*e/x^3-1/5*arcsech(c*x)*d^2/x^5/c^5+1/225/c^8*(-(c*x-1)/c/
x)^(1/2)/x^4*((c*x+1)/c/x)^(1/2)*(24*c^8*d^2*x^4+100*c^6*d*e*x^4+12*c^6*d^
2*x^2+225*c^4*e^2*x^4+50*c^4*d*e*x^2+9*c^4*d^2))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx =$$

$$-\frac{225\,ae^2x^4+150\,adex^2+45\,ad^2+15\left(15\,be^2x^4+10\,bdex^2+3\,bd^2\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(\left(24\,bc^5d^2+1\right.\right.}{225\,x^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/225*(225*a*e^2*x^4 + 150*a*d*e*x^2 + 45*a*d^2 + 15*(15*b*e^2*x^4 + 10*b
*d*e*x^2 + 3*b*d^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) -
((24*b*c^5*d^2 + 100*b*c^3*d*e + 225*b*c*e^2)*x^5 + 9*b*c*d^2*x + 2*(6*b*c
^3*d^2 + 25*b*c*d*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^5
```

3.104.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx=\int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)^2}{x^6}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**6,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**6, x)
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx$$
$$=\left(c\sqrt{\frac{1}{c^2x^2}-1}-\frac{\text{arsech}\,(cx)}{x}\right)be^2+\frac{1}{75}\,bd^2\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+10\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{15\ \text{arsech}\,(cx)}{x^5}\right)+\frac{2}{9}\,bde\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+3\,c^4\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{3\ \text{arsech}\,(cx)}{x^3}\right)-\frac{ae^2}{x}-\frac{2\,ade}{3\,x^3}-\frac{ad^2}{5\,x^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x, algorithm="maxima")
```

output
```
(c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*e^2 + 1/75*b*d^2*((3*c^6*(1/(
c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c^2*x^2) - 1)^(3/2) + 15*c^6*sqrt(1/(c^2*
x^2) - 1))/c - 15*arcsech(c*x)/x^5) + 2/9*b*d*e*((c^4*(1/(c^2*x^2) - 1)^(3
/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(c*x)/x^3) - a*e^2/x - 2/3
*a*d*e/x^3 - 1/5*a*d^2/x^5
```

3.104.8 Giac [F]

$$\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arsech}\,(cx)+a)}{x^6}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^6, x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(e\,x^2+d)^2\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^6, x)
```

3.105 $\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx$

3.105.1 Optimal result

Integrand size = 21, antiderivative size = 281

$$\begin{aligned}\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx &= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7}+\frac{2bd(15c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1225x^5}\\&+\frac{b(360c^4d^2+1176c^2de+1225e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{11025x^3}\\&+\frac{2bc^2(360c^4d^2+1176c^2de+1225e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{11025x}\\&-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}\end{aligned}$$

output
```
-1/7*d^2*(a+b*arcsech(c*x))/x^7-2/5*d*e*(a+b*arcsech(c*x))/x^5-1/3*e^2*(a+
b*arcsech(c*x))/x^3+1/49*b*d^2*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1
)^(1/2)/x^7+2/1225*b*d*(15*c^2*d+49*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c
^2*x^2+1)^(1/2)/x^5+1/11025*b*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(1/(c*x+
1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+2/11025*b*c^2*(360*c^4*d^2+
1176*c^2*d*e+1225*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/
x
```

3.105.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.57

$$\int \frac{(d+ex^2)^2\,(a+b\operatorname{sech}^{-1}(cx))}{x^8}\,dx$$
$$= \frac{-105a(15d^2+42dex^2+35e^2x^4)+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\,(1225e^2x^4(1+2c^2x^2)+294dex^2(3+4c^2x^2+8c^4x^4)\,\cdots}{11025x^7}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^8,x]
```

output
```
(-105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(
1 + c*x)*(1225*e^2*x^4*(1 + 2*c^2*x^2) + 294*d*e*x^2*(3 + 4*c^2*x^2 + 8*c^
4*x^4) + 45*d^2*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*(15*d^2
+ 42*d*e*x^2 + 35*e^2*x^4)*ArcSech[c*x])/(11025*x^7)
```

3.105.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.75, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6855, 27, 1588, 25, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2\,(a+b\operatorname{sech}^{-1}(cx))}{x^8}\,dx$$
$$\Big\downarrow \; 6855$$
$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{35e^2x^4+42dex^2+15d^2}{105x^8\sqrt{1-c^2x^2}}dx-\frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{7x^7}-\frac{2de(a+b\operatorname{sech}^{-1}(cx))}{5x^5}-\frac{e^2(a+b\operatorname{sech}^{-1}(cx))}{3x^3}$$
$$\Big\downarrow \; 27$$
$$-\frac{1}{105}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{35e^2x^4+42dex^2+15d^2}{x^8\sqrt{1-c^2x^2}}dx-\frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{7x^7}-\frac{2de(a+b\operatorname{sech}^{-1}(cx))}{5x^5}-\frac{e^2(a+b\operatorname{sech}^{-1}(cx))}{3x^3}$$
$$\Big\downarrow \; 1588$$

$$-\frac{1}{105}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{1}{7}\int -\frac{245e^2x^2+6d(15dc^2+49e)}{x^6\sqrt{1-c^2x^2}}dx-\frac{15d^2\sqrt{1-c^2x^2}}{7x^7}\right)-\frac{d^2(a+b\text{sech}^{-1}(cx))}{7x^7}-\frac{2de(a+b\text{sech}^{-1}(cx))}{5x^5}-\frac{e^2(a+b\text{sech}^{-1}(cx))}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{105}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\int \frac{245e^2x^2+6d(15dc^2+49e)}{x^6\sqrt{1-c^2x^2}}dx-\frac{15d^2\sqrt{1-c^2x^2}}{7x^7}\right)-\frac{d^2(a+b\text{sech}^{-1}(cx))}{7x^7}-\frac{2de(a+b\text{sech}^{-1}(cx))}{5x^5}-\frac{e^2(a+b\text{sech}^{-1}(cx))}{3x^3}$$

$$\Big\downarrow \text{359}$$

$$-\frac{1}{105}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(\frac{1}{5}(360c^4d^2+1176c^2de+1225e^2)\int \frac{1}{x^4\sqrt{1-c^2x^2}}dx-\frac{6d\sqrt{1-c^2x^2}(15c^2d+49e)}{5x^5}\right.\right.$$
$$\frac{d^2(a+b\text{sech}^{-1}(cx))}{7x^7}-\frac{2de(a+b\text{sech}^{-1}(cx))}{5x^5}-\frac{e^2(a+b\text{sech}^{-1}(cx))}{3x^3}$$

$$\Big\downarrow \text{245}$$

$$-\frac{1}{105}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(\frac{1}{5}(360c^4d^2+1176c^2de+1225e^2)\left(\frac{2}{3}c^2\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx-\frac{\sqrt{1-c^2x^2}}{3x^3}\right)-\frac{6d\sqrt{1-}}{}\right.\right.$$
$$\frac{d^2(a+b\text{sech}^{-1}(cx))}{7x^7}-\frac{2de(a+b\text{sech}^{-1}(cx))}{5x^5}-\frac{e^2(a+b\text{sech}^{-1}(cx))}{3x^3}$$

$$\Big\downarrow \text{242}$$

$$-\frac{d^2(a+b\text{sech}^{-1}(cx))}{7x^7}-\frac{2de(a+b\text{sech}^{-1}(cx))}{5x^5}-\frac{e^2(a+b\text{sech}^{-1}(cx))}{3x^3}-$$
$$\frac{1}{105}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(\frac{1}{5}\left(-\frac{2c^2\sqrt{1-c^2x^2}}{3x}-\frac{\sqrt{1-c^2x^2}}{3x^3}\right)(360c^4d^2+1176c^2de+1225e^2)-\frac{6d\sqrt{1-c^2x^2}(1}{5x^5}\right.\right.$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^8,x]
```

output
```
-1/105*(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-15*d^2*Sqrt[1 - c^2*x^2])/
(7*x^7) + ((-6*d*(15*c^2*d + 49*e)*Sqrt[1 - c^2*x^2])/(5*x^5) + ((360*c^4*
d^2 + 1176*c^2*d*e + 1225*e^2)*(-1/3*Sqrt[1 - c^2*x^2]/x^3 - (2*c^2*Sqrt[1
 - c^2*x^2])/(3*x)))/5)/7)) - (d^2*(a + b*ArcSech[c*x]))/(7*x^7) - (2*d*e*
(a + b*ArcSech[c*x]))/(5*x^5) - (e^2*(a + b*ArcSech[c*x]))/(3*x^3)
```

3.105. $\int \frac{(d+ex^2)^2(a+b\text{sech}^{-1}(cx))}{x^8}\,dx$

3.105.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.105. $\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx$

3.105.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.74

method	result
parts	$a\left(-\frac{e^2}{3x^3}-\frac{2de}{5x^5}-\frac{d^2}{7x^7}\right)+b\,c^7\left(-\frac{\operatorname{arcsech}(cx)e^2}{3c^7x^3}-\frac{2\operatorname{arcsech}(cx)de}{5c^7x^5}-\frac{\operatorname{arcsech}(cx)d^2}{7x^7c^7}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{}\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{2de}{5c^3x^5}-\frac{d^2}{7c^3x^7}-\frac{e^2}{3c^3x^3}\right)}{c^4}+\frac{b\left(-\frac{2\operatorname{arcsech}(cx)de}{5c^3x^5}-\frac{\operatorname{arcsech}(cx)d^2}{7c^3x^7}-\frac{\operatorname{arcsech}(cx)e^2}{3c^3x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(720c^{10}d^2x^6+\right.}{}\right.}{}\right.$
default	$c^7\left(\frac{a\left(-\frac{2de}{5c^3x^5}-\frac{d^2}{7c^3x^7}-\frac{e^2}{3c^3x^3}\right)}{c^4}+\frac{b\left(-\frac{2\operatorname{arcsech}(cx)de}{5c^3x^5}-\frac{\operatorname{arcsech}(cx)d^2}{7c^3x^7}-\frac{\operatorname{arcsech}(cx)e^2}{3c^3x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(720c^{10}d^2x^6+\right.}{}\right.}{}\right.$

input
```
int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3*e^2/x^3-2/5*d*e/x^5-1/7*d^2/x^7)+b*c^7*(-1/3/c^7*arcsech(c*x)*e^2/
x^3-2/5/c^7*arcsech(c*x)*d*e/x^5-1/7*arcsech(c*x)*d^2/x^7/c^7+1/11025/c^10
*(-(c*x-1)/c/x)^(1/2)/x^6*((c*x+1)/c/x)^(1/2)*(720*c^10*d^2*x^6+2352*c^8*d
*e*x^6+360*c^8*d^2*x^4+2450*c^6*e^2*x^6+1176*c^6*d*e*x^4+270*c^6*d^2*x^2+1
225*c^4*e^2*x^4+882*c^4*d*e*x^2+225*c^4*d^2))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex^2)^2\,(a+b\operatorname{sech}^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{3675\,ae^2x^4+4410\,adex^2+1575\,ad^2+105\,(35\,be^2x^4+42\,bdex^2+15\,bd^2)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(2\,(360}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4
 + 42*b*d*e*x^2 + 15*b*d^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(
c*x)) - (2*(360*b*c^7*d^2 + 1176*b*c^5*d*e + 1225*b*c^3*e^2)*x^7 + (360*b*
c^5*d^2 + 1176*b*c^3*d*e + 1225*b*c*e^2)*x^5 + 225*b*c*d^2*x + 18*(15*b*c^
3*d^2 + 49*b*c*d*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^7
```

3.105.6 Sympy [F]

$$\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx=\int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)^2}{x^8}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**8,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**8, x)
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx$$
$$=\frac{1}{245}bd^2\left(\frac{5\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{7}{2}}+21\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+35\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+35\,c^8\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{35\,\text{arsech}\,(cx)}{x^7}\right)$$
$$+\frac{2}{75}bde\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+10\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{15\,\text{arsech}\,(cx)}{x^5}\right)$$
$$+\frac{1}{9}be^2\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+3\,c^4\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{3\,\text{arsech}\,(cx)}{x^3}\right)-\frac{ae^2}{3\,x^3}-\frac{2\,ade}{5\,x^5}-\frac{ad^2}{7\,x^7}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x, algorithm="maxima")
```

output
```
1/245*b*d^2*((5*c^8*(1/(c^2*x^2) - 1)^(7/2) + 21*c^8*(1/(c^2*x^2) - 1)^(5/
2) + 35*c^8*(1/(c^2*x^2) - 1)^(3/2) + 35*c^8*sqrt(1/(c^2*x^2) - 1))/c - 35
*arcsech(c*x)/x^7) + 2/75*b*d*e*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(
1/(c^2*x^2) - 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)
/x^5) + 1/9*b*e^2*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) -
 1))/c - 3*arcsech(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x
^7
```

3.105.8 Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^8}\,dx=\int \frac{(ex^2+d)^2(b\,\text{arsech}\,(cx)+a)}{x^8}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^8, x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^8}\,dx=\int \frac{(e\,x^2+d)^2\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^8, x)
```

3.106 $\int x^3(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.106.1 Optimal result

Integrand size = 21, antiderivative size = 278

$$\begin{aligned}\int x^3(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx &= -\frac{b(6c^4d^2+8c^2de+3e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{24c^8}\\&\quad+\frac{b(6c^4d^2+16c^2de+9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{72c^8}\\&\quad-\frac{be(8c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{120c^8}+\frac{be^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{7/2}}{56c^8}\\&\quad+\frac{1}{4}d^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{sech}^{-1}(cx)\right)\end{aligned}$$

output
```
1/4*d^2*x^4*(a+b*arcsech(c*x))+1/3*d*e*x^6*(a+b*arcsech(c*x))+1/8*e^2*x^8*
(a+b*arcsech(c*x))+1/72*b*(6*c^4*d^2+16*c^2*d*e+9*e^2)*(-c^2*x^2+1)^(3/2)*
(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8-1/120*b*e*(8*c^2*d+9*e)*(-c^2*x^2+1)^(
5/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8+1/56*b*e^2*(-c^2*x^2+1)^(7/2)*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8-1/24*b*(6*c^4*d^2+8*c^2*d*e+3*e^2)*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^8
```

3.106.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.60

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\frac{1}{24}\left(6ad^2x^4+8adex^6+3ae^2x^8\right.$$
$$-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(144e^2+8c^2e(56d+9ex^2)+c^4(420d^2+224dex^2+54e^2x^4)+3c^6(70d^2x^2+56dex^4+\right.}{105c^8}$$
$$\left.+bx^4\left(6d^2+8dex^2+3e^2x^4\right)\operatorname{sech}^{-1}(cx)\right)$$

input
```
Integrate[x^3*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

output
```
(6*a*d^2*x^4 + 8*a*d*e*x^6 + 3*a*e^2*x^8 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1
 + c*x)*(144*e^2 + 8*c^2*e*(56*d + 9*e*x^2) + c^4*(420*d^2 + 224*d*e*x^2 +
 54*e^2*x^4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/(105*c^8) +
b*x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcSech[c*x])/24
```

3.106.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6855, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \ 6855$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{x^3\left(3e^2x^4+8dex^2+6d^2\right)}{24\sqrt{1-c^2x^2}}dx+\frac{1}{4}d^2x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\operatorname{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{24}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{x^3(3e^2x^4+8dex^2+6d^2)}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}d^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{48}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{x^2(3e^2x^4+8dex^2+6d^2)}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{4}d^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{1195}$$

$$\frac{1}{48}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\left(-\frac{3e^2(1-c^2x^2)^{5/2}}{c^6}+\frac{e(8dc^2+9e)\,(1-c^2x^2)^{3/2}}{c^6}+\frac{(-6d^2c^4-16dec^2-9e^2)\,\sqrt{1-c^2x}}{c^6}\right.$$
$$\frac{1}{4}d^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{sech}^{-1}(cx)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}d^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{sech}^{-1}(cx)\right)+$$
$$\frac{1}{48}b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{2e(1-c^2x^2)^{5/2}\,(8c^2d+9e)}{5c^8}+\frac{6e^2(1-c^2x^2)^{7/2}}{7c^8}+\frac{2(1-c^2x^2)^{3/2}\,(6c^4d^2+16c^2de+9e^2}{3c^8}\right.$$

input
```
Int[x^3*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

output
```
(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-2*(6*c^4*d^2 + 8*c^2*d*e + 3*e^2)
*Sqrt[1 - c^2*x^2])/c^8 + (2*(6*c^4*d^2 + 16*c^2*d*e + 9*e^2)*(1 - c^2*x^2
)^(3/2))/(3*c^8) - (2*e*(8*c^2*d + 9*e)*(1 - c^2*x^2)^(5/2))/(5*c^8) + (6*
e^2*(1 - c^2*x^2)^(7/2))/(7*c^8)))/48 + (d^2*x^4*(a + b*ArcSech[c*x]))/4 +
 (d*e*x^6*(a + b*ArcSech[c*x]))/3 + (e^2*x^8*(a + b*ArcSech[c*x]))/8
```

3.106. $\int x^3(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.106.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.106.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.71

method	result
parts	$a\left(\frac{1}{8}e^2x^8+\frac{1}{3}de\,x^6+\frac{1}{4}x^4d^2\right)+\frac{b\left(\frac{c^4\operatorname{arcsech}(cx)e^2x^8}{8}+\frac{c^4\operatorname{arcsech}(cx)de\,x^6}{3}+\frac{\operatorname{arcsech}(cx)d^2x^4c^4}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+}{cx}}}{}\right.}{}$
derivativedivides	$-\frac{a\left(\frac{c^2d\left(e\,c^2x^2+c^2d\right)^3}{3}-\frac{\left(e\,c^2x^2+c^2d\right)^4}{4}\right)}{2c^4e^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)c^8d^4}{24e^2}+\frac{\operatorname{arcsech}(cx)c^8d^2x^4}{4}+\frac{e\operatorname{arcsech}(cx)c^8d\,x^6}{3}+\frac{e^2\operatorname{arcsech}(cx)c^8x}{8}\right.}{}$
default	$-\frac{a\left(\frac{c^2d\left(e\,c^2x^2+c^2d\right)^3}{3}-\frac{\left(e\,c^2x^2+c^2d\right)^4}{4}\right)}{2c^4e^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)c^8d^4}{24e^2}+\frac{\operatorname{arcsech}(cx)c^8d^2x^4}{4}+\frac{e\operatorname{arcsech}(cx)c^8d\,x^6}{3}+\frac{e^2\operatorname{arcsech}(cx)c^8x}{8}\right.}{}$

input
```
int(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/8*e^2*x^8+1/3*d*e*x^6+1/4*x^4*d^2)+b/c^4*(1/8*c^4*arcsech(c*x)*e^2*x^
8+1/3*c^4*arcsech(c*x)*d*e*x^6+1/4*arcsech(c*x)*d^2*x^4*c^4-1/2520/c^3*(-(
c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(45*c^6*e^2*x^6+168*c^6*d*e*x^4+21
0*c^6*d^2*x^2+54*c^4*e^2*x^4+224*c^4*d*e*x^2+420*c^4*d^2+72*c^2*e^2*x^2+44
8*c^2*d*e+144*e^2))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.82

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$

$$=\frac{315\,ac^7e^2x^8+840\,ac^7dex^6+630\,ac^7d^2x^4+105\left(3\,bc^7e^2x^8+8\,bc^7dex^6+6\,bc^7d^2x^4\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-}{}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/2520*(315*a*c^7*e^2*x^8 + 840*a*c^7*d*e*x^6 + 630*a*c^7*d^2*x^4 + 105*(3
*b*c^7*e^2*x^8 + 8*b*c^7*d*e*x^6 + 6*b*c^7*d^2*x^4)*log((c*x*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (45*b*c^6*e^2*x^7 + 6*(28*b*c^6*d*e + 9*b*
c^4*e^2)*x^5 + 2*(105*b*c^6*d^2 + 112*b*c^4*d*e + 36*b*c^2*e^2)*x^3 + 4*(1
05*b*c^4*d^2 + 112*b*c^2*d*e + 36*b*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
)/c^7
```

3.106.6 Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.19

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\begin{cases}\frac{ad^2x^4}{4}+\frac{adex^6}{3}+\frac{ae^2x^8}{8}+\frac{bd^2x^4\operatorname{asech}(cx)}{4}+\frac{bdex^6\operatorname{asech}(cx)}{3}+\frac{be^2x^8\operatorname{asech}(cx)}{8}-\frac{bd^2x^2\sqrt{-c^2x^2+1}}{12c^2}-\frac{bdex^4\sqrt{-c^2x^2+1}}{15c^2}-\frac{be^2}{} \\ (a+\infty b)\left(\frac{d^2x^4}{4}+\frac{dex^6}{3}+\frac{e^2x^8}{8}\right)\end{cases}$$

input
```
integrate(x**3*(e*x**2+d)**2*(a+b*asech(c*x)),x)
```

output
```
Piecewise((a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*asec
h(c*x)/4 + b*d*e*x**6*asech(c*x)/3 + b*e**2*x**8*asech(c*x)/8 - b*d**2*x**
2*sqrt(-c**2*x**2 + 1)/(12*c**2) - b*d*e*x**4*sqrt(-c**2*x**2 + 1)/(15*c**
2) - b*e**2*x**6*sqrt(-c**2*x**2 + 1)/(56*c**2) - b*d**2*sqrt(-c**2*x**2 +
 1)/(6*c**4) - 4*b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 3*b*e**2*x**4
*sqrt(-c**2*x**2 + 1)/(140*c**4) - 8*b*d*e*sqrt(-c**2*x**2 + 1)/(45*c**6)
- b*e**2*x**2*sqrt(-c**2*x**2 + 1)/(35*c**6) - 2*b*e**2*sqrt(-c**2*x**2 +
1)/(35*c**8), Ne(c, 0)), ((a + oo*b)*(d**2*x**4/4 + d*e*x**6/3 + e**2*x**8
/8), True))
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.88

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\frac{1}{8}ae^2x^8+\frac{1}{3}adex^6+\frac{1}{4}ad^2x^4$$
$$+\frac{1}{12}\left(3x^4\operatorname{arsech}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)bd^2$$
$$+\frac{1}{45}\left(15x^6\operatorname{arsech}(cx)-\frac{3c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}-10c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15x\sqrt{\frac{1}{c^2x^2}-1}}{c^5}\right)bde$$
$$+\frac{1}{280}\left(35x^8\operatorname{arsech}(cx)+\frac{5c^6x^7\left(\frac{1}{c^2x^2}-1\right)^{\frac{7}{2}}-21c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+35c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-35x\sqrt{\frac{1}{c^2x^2}-}}{c^7}\right.$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arcsech(c*x) +
 (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*d^2
+ 1/45*(15*x^6*arcsech(c*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) - 10*c^2*
x^3*(1/(c^2*x^2) - 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) - 1))/c^5)*b*d*e + 1/2
80*(35*x^8*arcsech(c*x) + (5*c^6*x^7*(1/(c^2*x^2) - 1)^(7/2) - 21*c^4*x^5*
(1/(c^2*x^2) - 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 35*x*sqrt(1
/(c^2*x^2) - 1))/c^7)*b*e^2
```

3.106.8 Giac [F]

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^2 (b \operatorname{arsech}(cx)+a)x^3 \, dx$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*x^3, x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int x^3 \left(e\,x^2+d\right)^2 \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int(x^3*(d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)
```

3.107 $\int x(d+ex^2)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$

3.107.1 Optimal result

Integrand size = 19, antiderivative size = 230

$$\begin{aligned}\int x(d+ex^2)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = &-\frac{b(3c^4d^2+3c^2de+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^6}\\&+\frac{be(3c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{18c^6}\\&-\frac{be^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{30c^6}\\&+\frac{(d+ex^2)^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{6e}\\&-\frac{bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{6e}\end{aligned}$$

output
```
1/6*(e*x^2+d)^3*(a+b*arcsech(c*x))/e+1/18*b*e*(3*c^2*d+2*e)*(-c^2*x^2+1)^(
3/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^6-1/30*b*e^2*(-c^2*x^2+1)^(5/2)*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^6-1/6*b*d^3*arctanh((-c^2*x^2+1)^(1/2))*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)/e-1/6*b*(3*c^4*d^2+3*c^2*d*e+e^2)*(1/(c*x+1))
^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6
```

3.107.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.60

$$\int x\left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{6}ax^2\left(3d^2+3dex^2+e^2x^4\right)$$
$$-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(8e^2+2c^2e(15d+2ex^2)+3c^4(15d^2+5dex^2+e^2x^4)\right)}{90c^6}$$
$$+\frac{1}{6}bx^2\left(3d^2+3dex^2+e^2x^4\right)\text{sech}^{-1}(cx)$$

input
```
Integrate[x*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

output
```
(a*x^2*(3*d^2 + 3*d*e*x^2 + e^2*x^4))/6 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1
+ c*x)*(8*e^2 + 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2
*x^4)))/(90*c^6) + (b*x^2*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcSech[c*x])/6
```

3.107.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6853, 2036, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$\Big\downarrow \text{6853}$$
$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{\left(ex^2+d\right)^3}{x\sqrt{1-cx}\sqrt{cx+1}}dx}{6e}+\frac{\left(d+ex^2\right)^3\left(a+b\text{sech}^{-1}(cx)\right)}{6e}$$
$$\Big\downarrow \text{2036}$$
$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{\left(ex^2+d\right)^3}{x\sqrt{1-c^2x^2}}dx}{6e}+\frac{\left(d+ex^2\right)^3\left(a+b\text{sech}^{-1}(cx)\right)}{6e}$$
$$\Big\downarrow \text{354}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(ex^2+d)^3}{x^2\sqrt{1-c^2x^2}}dx^2}{12e}+\frac{(d+ex^2)^3\,(a+b\text{sech}^{-1}(cx))}{6e}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \left(\frac{d^3}{x^2\sqrt{1-c^2x^2}}+\frac{e^3(1-c^2x^2)^{3/2}}{c^4}-\frac{e^2(3dc^2+2e)\sqrt{1-c^2x^2}}{c^4}+\frac{e(3d^2c^4+3dec^2+e^2)}{c^4\sqrt{1-c^2x^2}}\right)dx^2}{12e}+\frac{(d+ex^2)^3\,(a+b\text{sech}^{-1}(cx))}{6e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(d+ex^2)^3\,(a+b\text{sech}^{-1}(cx))}{6e}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-2d^3\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2e^2(1-c^2x^2)^{3/2}(3c^2d+2e)}{3c^6}-\frac{2e^3(1-c^2x^2)^{5/2}}{5c^6}-\frac{2e\sqrt{1-c^2x^2}(3c^4d^2+3c^2de+e^2)}{c^6}\right)}{12e}$$

input
```
Int[x*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

output
```
((d + e*x^2)^3*(a + b*ArcSech[c*x]))/(6*e) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[
1 + c*x]*((-2*e*(3*c^4*d^2 + 3*c^2*d*e + e^2)*Sqrt[1 - c^2*x^2])/c^6 + (2*
e^2*(3*c^2*d + 2*e)*(1 - c^2*x^2)^(3/2))/(3*c^6) - (2*e^3*(1 - c^2*x^2)^(5
/2))/(5*c^6) - 2*d^3*ArcTanh[Sqrt[1 - c^2*x^2]]))/(12*e)
```

3.107.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2036
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

rule 6853
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Simp[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x
^2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e
, p}, x] && NeQ[p, -1]
```

3.107.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.23

method	result
parts	$\frac{a(e\,x^2+d)^3}{6e}+\frac{b\left(\frac{c^2e^2\operatorname{arcsech}(cx)x^6}{6}+\frac{c^2e\operatorname{arcsech}(cx)x^4d}{2}+\frac{\operatorname{arcsech}(cx)x^2c^2d^2}{2}+\frac{c^2\operatorname{arcsech}(cx)d^3}{6e}-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(15c^6\cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^3}{6c^4e}+\frac{b\left(\frac{\operatorname{arcsech}(cx)c^6d^3}{6e}+\frac{\operatorname{arcsech}(cx)c^6d^2x^2}{2}+\frac{e\operatorname{arcsech}(cx)c^6d\,x^4}{2}+\frac{e^2\operatorname{arcsech}(cx)c^6x^6}{6}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(15c^6\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^3}{6c^4e}+\frac{b\left(\frac{\operatorname{arcsech}(cx)c^6d^3}{6e}+\frac{\operatorname{arcsech}(cx)c^6d^2x^2}{2}+\frac{e\operatorname{arcsech}(cx)c^6d\,x^4}{2}+\frac{e^2\operatorname{arcsech}(cx)c^6x^6}{6}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(15c^6\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$

input
```
int(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*(e*x^2+d)^3/e+b/c^2*(1/6*c^2*e^2*arcsech(c*x)*x^6+1/2*c^2*e*arcsech(
c*x)*x^4*d+1/2*arcsech(c*x)*x^2*c^2*d^2+1/6*c^2/e*arcsech(c*x)*d^3-1/90/c^
3/e*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(15*c^6*d^3*arctanh(1/(-c^2
*x^2+1)^(1/2))+45*c^4*d^2*e*(-c^2*x^2+1)^(1/2)+15*c^4*d*e^2*(-c^2*x^2+1)^(
1/2)*x^2+3*e^3*(-c^2*x^2+1)^(1/2)*c^4*x^4+30*c^2*d*e^2*(-c^2*x^2+1)^(1/2)+
4*e^3*c^2*x^2*(-c^2*x^2+1)^(1/2)+8*e^3*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1
/2))
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.83

$$\int x(d+ex^2)^2(a+b\operatorname{sech}^{-1}(cx))\,dx$$
$$=\frac{15\,ac^5e^2x^6+45\,ac^5dex^4+45\,ac^5d^2x^2+15\,(bc^5e^2x^6+3\,bc^5dex^4+3\,bc^5d^2x^2)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(3\,bc^4}{90\,c^5}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
1/90*(15*a*c^5*e^2*x^6 + 45*a*c^5*d*e*x^4 + 45*a*c^5*d^2*x^2 + 15*(b*c^5*e
^2*x^6 + 3*b*c^5*d*e*x^4 + 3*b*c^5*d^2*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)) + 1)/(c*x)) - (3*b*c^4*e^2*x^5 + (15*b*c^4*d*e + 4*b*c^2*e^2)*x^
3 + (45*b*c^4*d^2 + 30*b*c^2*d*e + 8*b*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^
2)))/c^5
```

3.107.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.10

$$\int x(d+ex^2)^2(a+b\operatorname{sech}^{-1}(cx))\,dx$$
$$=\begin{cases}\frac{ad^2x^2}{2}+\frac{adex^4}{2}+\frac{ae^2x^6}{6}+\frac{bd^2x^2\operatorname{asech}(cx)}{2}+\frac{bdex^4\operatorname{asech}(cx)}{2}+\frac{be^2x^6\operatorname{asech}(cx)}{6}-\frac{bd^2\sqrt{-c^2x^2+1}}{2c^2}-\frac{bdex^2\sqrt{-c^2x^2+1}}{6c^2}-\frac{be^2x^4}{}\\(a+\infty b)\left(\frac{d^2x^2}{2}+\frac{dex^4}{2}+\frac{e^2x^6}{6}\right)\end{cases}$$

input
```
integrate(x*(e*x**2+d)**2*(a+b*asech(c*x)),x)
```

output
```
Piecewise((a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*asec
h(c*x)/2 + b*d*e*x**4*asech(c*x)/2 + b*e**2*x**6*asech(c*x)/6 - b*d**2*sqr
t(-c**2*x**2 + 1)/(2*c**2) - b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(6*c**2) - b*
e**2*x**4*sqrt(-c**2*x**2 + 1)/(30*c**2) - b*d*e*sqrt(-c**2*x**2 + 1)/(3*c
**4) - 2*b*e**2*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 4*b*e**2*sqrt(-c**2*
x**2 + 1)/(45*c**6), Ne(c, 0)), ((a + oo*b)*(d**2*x**2/2 + d*e*x**4/2 + e*
*2*x**6/6), True))
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.80

$$\int x(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{6}ae^2x^6+\frac{1}{2}adex^4+\frac{1}{2}ad^2x^2+\frac{1}{2}\left(x^2\,\text{arsech}\,(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)bd^2$$
$$+\frac{1}{6}\left(3\,x^4\,\text{arsech}\,(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)bde$$
$$+\frac{1}{90}\left(15\,x^6\,\text{arsech}\,(cx)-\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^5}\right)be^2$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arcsech(c*x) - x*
sqrt(1/(c^2*x^2) - 1)/c)*b*d^2 + 1/6*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^
2*x^2) - 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*d*e + 1/90*(15*x^6*a
rcsech(c*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2)
 - 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) - 1))/c^5)*b*e^2
```

3.107.8 Giac [F]

$$\int x(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int\left(ex^2+d\right)^2(b\,\text{arsech}\,(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*x, x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x\left(e\,x^2+d\right)^2\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)
```

3.108 $\int \frac{(d+ex^2)^2 \left(a+b\,\text{sech}^{-1}(cx)\right)}{x}\,dx$

3.108.1 Optimal result

Integrand size = 21, antiderivative size = 370

$$
\begin{aligned}
\int \frac{(d+ex^2)^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}\,dx =& -\frac{be(6c^2d+e)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3} \\
&-\frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c} \\
&+\frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\
&+dex^2\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\,\text{sech}^{-1}(cx)\right) \\
&-\frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\
&+\frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\
&-d^2\left(a+b\,\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\
&+\frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}
\end{aligned}
$$

output
```
d*e*x^2*(a+b*arcsech(c*x))+1/4*e^2*x^4*(a+b*arcsech(c*x))-d^2*(a+b*arcsech
(c*x))*ln(1/x)+1/2*I*b*d^2*arccsc(c*x)^2*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1
/2)/(1+1/c/x)^(1/2)-b*d^2*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*
(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+b*d^2*arccsc(c*x)*ln(
1/x)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/2*I*b*d^2*poly
log(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/
(1+1/c/x)^(1/2)-1/6*b*e*(6*c^2*d+e)*x*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/c^3
-1/12*b*e^2*x^3*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/c
```

3.108.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.48

$$\begin{aligned}\int \frac{(d+ex^2)^2\,(a+b\operatorname{sech}^{-1}(cx))}{x}\,dx &= adex^2+\frac{1}{4}ae^2x^4-\frac{bde\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{c^2}\\ &\quad-\frac{be^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\,(2+c^2x^2)}{12c^4}+bdex^2\operatorname{sech}^{-1}(cx)\\ &\quad+\frac{1}{4}be^2x^4\operatorname{sech}^{-1}(cx)-\frac{1}{2}bd^2\operatorname{sech}^{-1}(cx)\left(\operatorname{sech}^{-1}(cx)\right.\\ &\quad\left.+2\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)\right)\\ &\quad+ad^2\log(x)+\frac{1}{2}bd^2\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(cx)}\right)\end{aligned}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x,x]
```

output
```
a*d*e*x^2 + (a*e^2*x^4)/4 - (b*d*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/c^
2 - (b*e^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(2 + c^2*x^2))/(12*c^4) + b
*d*e*x^2*ArcSech[c*x] + (b*e^2*x^4*ArcSech[c*x])/4 - (b*d^2*ArcSech[c*x]*(
ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])]))/2 + a*d^2*Log[x] + (b*d^2*
PolyLog[2, -E^(-2*ArcSech[c*x])])/2
```

3.108.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6857, 6373, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x} dx$$

$$\Big\downarrow \text{6857}$$

$$-\int \left(\frac{d}{x^2}+e\right)^2 x^5 \left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right) d\frac{1}{x}$$

$$\Big\downarrow \text{6373}$$

$$\frac{b\int -\frac{e\left(\frac{4d}{x^2}+e\right)x^4-4d^2\log\left(\frac{1}{x}\right)}{4\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}d\frac{1}{x}}{c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{e\left(\frac{4d}{x^2}+e\right)x^4-4d^2\log\left(\frac{1}{x}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}d\frac{1}{x}}{4c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int \left(\frac{e\left(\frac{4d}{x^2}+e\right)x^4}{\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}-\frac{4d^2\log\left(\frac{1}{x}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}\right)d\frac{1}{x}}{4c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$-d^2\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)-\frac{b\left(-\frac{2icd^2\sqrt{1-\frac{1}{c^2x^2}}\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{2icd^2\sqrt{1-\frac{1}{c^2x^2}}\arcsin\left(\frac{1}{cx}\right)^2}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{4cd^2\sqrt{1-\frac{1}{c^2x^2}}\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-4cd^2}{4c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x,x]
```

output
```
d*e*x^2*(a + b*ArcCosh[1/(c*x)]) + (e^2*x^4*(a + b*ArcCosh[1/(c*x)]))/4 -
d^2*(a + b*ArcCosh[1/(c*x)])*Log[x^(-1)] - (b*((2*e*(6*d + e/c^2)*Sqrt[-1
+ 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)/3 + (e^2*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*
x)]*x^3)/3 - ((2*I)*c*d^2*Sqrt[1 - 1/(c^2*x^2)]*ArcSin[1/(c*x)]^2)/(Sqrt[-
1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (4*c*d^2*Sqrt[1 - 1/(c^2*x^2)]*ArcSin[1/
(c*x)]*Log[1 - E^((2*I)*ArcSin[1/(c*x)])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/
(c*x)]) - (4*c*d^2*Sqrt[1 - 1/(c^2*x^2)]*ArcSin[1/(c*x)]*Log[x^(-1)])/(Sqr
t[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - ((2*I)*c*d^2*Sqrt[1 - 1/(c^2*x^2)]*Po
lyLog[2, E^((2*I)*ArcSin[1/(c*x)])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]
)))/(4*c)
```

3.108.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.108.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.75

method	result
parts	$a\left(\frac{e^{2}x^{4}}{4}+de\,x^{2}+d^{2}\ln\left(x\right)\right)+b\left(\frac{\operatorname{arcsech}\left(cx\right)^{2}d^{2}}{2}+\frac{e\left(12\ \operatorname{arcsech}\left(cx\right)c^{4}d\,x^{2}+3e\ \operatorname{arcsech}\left(cx\right)c^{4}x^{4}-12\sqrt{-\right.}$
derivativedivides	$ade\,x^{2}+\frac{a\,e^{2}x^{4}}{4}+a\,d^{2}\ln\left(cx\right)+\frac{b\left(\frac{c^{4}d^{2}\operatorname{arcsech}\left(cx\right)^{2}}{2}+\frac{e\left(12\ \operatorname{arcsech}\left(cx\right)c^{4}d\,x^{2}+3e\ \operatorname{arcsech}\left(cx\right)c^{4}x^{4}-12\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\right.}}{}$
default	$ade\,x^{2}+\frac{a\,e^{2}x^{4}}{4}+a\,d^{2}\ln\left(cx\right)+\frac{b\left(\frac{c^{4}d^{2}\operatorname{arcsech}\left(cx\right)^{2}}{2}+\frac{e\left(12\ \operatorname{arcsech}\left(cx\right)c^{4}d\,x^{2}+3e\ \operatorname{arcsech}\left(cx\right)c^{4}x^{4}-12\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\right.}}{}$

input
```
int((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*e^2*x^4+d*e*x^2+d^2*ln(x))+b*(1/2*arcsech(c*x)^2*d^2+1/12/c^4*e*(12
*arcsech(c*x)*c^4*d*x^2+3*e*arcsech(c*x)*c^4*x^4-12*(-(c*x-1)/c/x)^(1/2)*(
(c*x+1)/c/x)^(1/2)*c^3*d*x-(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*e*c^3*
x^3-2*((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*e*c*x+12*c^2*d+2*e)-d^2*arc
sech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-1/2*d^2*polylog
(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))
```

3.108.5 Fricas [F]

$$\int \frac{(d+ex^2)^2\left(a+b\mathrm{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{(ex^2+d)^2(b\,\mathrm{arsech}\left(cx\right)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arcsech(c*x))/x, x)
```

3.108.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x} \, dx = \int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)^2}{x} \, dx$$

input
```
integrate((e*x**2+d)**2*(a+b*asech(c*x))/x,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)**2/x, x)
```

3.108.7 Maxima [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x} \, dx = \int \frac{(ex^2+d)^2 (b\,\text{arsech}\,(cx)+a)}{x} \, dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) + integrate(b*e^2*x^3*log(sqrt(1/
(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x)) + 2*b*d*e*x*log(sqrt(1/(c*x) + 1)*
sqrt(1/(c*x) - 1) + 1/(c*x)) + b*d^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) -
1) + 1/(c*x))/x, x)
```

3.108.8 Giac [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x} \, dx = \int \frac{(ex^2+d)^2 (b\,\text{arsech}\,(cx)+a)}{x} \, dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x, x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x}\,dx = \int \frac{(e\,x^2+d)^2 \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x, x)
```

3.109 $\int \frac{(d+ex^2)^2 \left(a+b\,\text{sech}^{-1}(cx)\right)}{x^3}\,dx$

3.109.1 Optimal result

Integrand size = 21, antiderivative size = 373

$$\begin{aligned}\int \frac{(d+ex^2)^2 \left(a+b\,\text{sech}^{-1}(cx)\right)}{x^3}\,dx &= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} \\ &\quad + \frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{4}bc^2d^2\text{sech}^{-1}(cx) \\ &\quad - \frac{d^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\,\text{sech}^{-1}(cx)\right) \\ &\quad - \frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\ &\quad + \frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\ &\quad - 2de\left(a+b\,\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\ &\quad + \frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}\end{aligned}$$

output
```
1/4*b*c^2*d^2*arcsech(c*x)-1/2*d^2*(a+b*arcsech(c*x))/x^2+1/2*e^2*x^2*(a+b
*arcsech(c*x))-2*d*e*(a+b*arcsech(c*x))*ln(1/x)+I*b*d*e*arccsc(c*x)^2*(1-1
/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-2*b*d*e*arccsc(c*x)*ln(1-
(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c
/x)^(1/2)+2*b*d*e*arccsc(c*x)*ln(1/x)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)
/(1+1/c/x)^(1/2)+I*b*d*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2
/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/4*b*c*d^2*(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2)/x-1/2*b*e^2*x*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/c
```

3.109.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx$$
$$=\frac{1}{4}\left(-\frac{2ad^2}{x^2}+2ae^2x^2-\frac{2be^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{c^2}-\frac{2bd^2\operatorname{sech}^{-1}(cx)}{x^2}+2be^2x^2\operatorname{sech}^{-1}(cx)\right.$$
$$+\frac{bd^2\sqrt{\frac{1-cx}{1+cx}}\left(\sqrt{1-cx}(1+cx)+2c^2x^2\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{x^2\sqrt{1-cx}}$$
$$-4bde\operatorname{sech}^{-1}(cx)\left(\operatorname{sech}^{-1}(cx)+2\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)\right)+8ade\log(x)$$
$$\left.+4bde\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^3,x]
```

output
```
((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*e^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*
x))/c^2 - (2*b*d^2*ArcSech[c*x])/x^2 + 2*b*e^2*x^2*ArcSech[c*x] + (b*d^2*S
qrt[(1 - c*x)/(1 + c*x)]*(Sqrt[1 - c*x]*(1 + c*x) + 2*c^2*x^2*Sqrt[1 + c*x
]*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))/(x^2*Sqrt[1 - c*x]) - 4*b*d*e*Arc
Sech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])]) + 8*a*d*e*Log[x]
 + 4*b*d*e*PolyLog[2, -E^(-2*ArcSech[c*x])])/4
```

3.109.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6857, 6373, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^3} \, dx$$

$$\Big\downarrow \text{6857}$$

$$-\int \left(\frac{d}{x^2}+e\right)^2 x^3 \left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right) d\frac{1}{x}$$

$$\Big\downarrow \text{6373}$$

$$\frac{b\int -\frac{-\frac{d^2}{x^2}-4e\log\left(\frac{1}{x}\right)d+e^2x^2}{2\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}d\frac{1}{x}}{c}-\frac{d^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{-\frac{d^2}{x^2}-4e\log\left(\frac{1}{x}\right)d+e^2x^2}{\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}d\frac{1}{x}}{2c}-\frac{d^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int \left(-\frac{d^2}{\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}x^2}-\frac{4e\log\left(\frac{1}{x}\right)d}{\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}+\frac{e^2x^2}{\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}\right)d\frac{1}{x}}{2c}-\frac{d^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)-\frac{b\left(-\frac{1}{2}c^3d^2\text{arccosh}\left(\frac{1}{cx}\right)-\frac{2icde\sqrt{1-\frac{1}{c^2x^2}}\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{2icde\sqrt{1-\frac{1}{c^2x^2}}\arcsin\left(\frac{1}{cx}\right)^2}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{4cde\sqrt{1-\frac{1}{c^2x^2}}\arcsin\left(\frac{1}{cx}\right)\log\left(1\right.}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}\right.}{2c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^3,x]
```

output
```
-1/2*(d^2*(a + b*ArcCosh[1/(c*x)]))/x^2 + (e^2*x^2*(a + b*ArcCosh[1/(c*x)]
))/2 - 2*d*e*(a + b*ArcCosh[1/(c*x)])*Log[x^(-1)] - (b*(-1/2*(c^2*d^2*Sqrt
[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/x + e^2*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c
*x)]*x - (c^3*d^2*ArcCosh[1/(c*x)])/2 - ((2*I)*c*d*e*Sqrt[1 - 1/(c^2*x^2)]
*ArcSin[1/(c*x)]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (4*c*d*e*Sqrt
[1 - 1/(c^2*x^2)]*ArcSin[1/(c*x)]*Log[1 - E^((2*I)*ArcSin[1/(c*x)])])/(Sqr
t[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - (4*c*d*e*Sqrt[1 - 1/(c^2*x^2)]*ArcSin
[1/(c*x)]*Log[x^(-1)])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - ((2*I)*c*d
*e*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*I)*ArcSin[1/(c*x)])])/(Sqrt[-1 +
 1/(c*x)]*Sqrt[1 + 1/(c*x)])))/(2*c)
```

3.109.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6373
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCosh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(Sqrt[1
 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && Le
Q[m + p, 0]))
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.109.4 Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.67

method	result
parts	$a\left(\frac{e^2x^2}{2}-\frac{d^2}{2x^2}+2de\ln\left(x\right)\right)+bde\,\mathrm{arcsech}\left(cx\right)^2+\frac{b\,c^2d^2\,\mathrm{arcsech}(cx)}{4}+\frac{bc\,d^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{4x}-\frac{b}{}$
derivativedivides	$c^2\left(\frac{a\,x^2e^2}{2c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{2ade\ln(cx)}{c^2}+\frac{bde\,\mathrm{arcsech}(cx)^2}{c^2}+\frac{b\,d^2\,\mathrm{arcsech}(cx)}{4}+\frac{b\,d^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{4cx}-\frac{b\,\mathrm{arcsech}}{2c^2}\right.$
default	$c^2\left(\frac{a\,x^2e^2}{2c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{2ade\ln(cx)}{c^2}+\frac{bde\,\mathrm{arcsech}(cx)^2}{c^2}+\frac{b\,d^2\,\mathrm{arcsech}(cx)}{4}+\frac{b\,d^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{4cx}-\frac{b\,\mathrm{arcsech}}{2c^2}\right.$

input
```
int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e^2*x^2-1/2*d^2/x^2+2*d*e*ln(x))+b*d*e*arcsech(c*x)^2+1/4*b*c^2*d^2
*arcsech(c*x)+1/4*b*c*d^2/x*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)-1/2*b
*d^2/x^2*arcsech(c*x)+1/2*b*e^2*x^2*arcsech(c*x)-1/2*b/c*e^2*(-(c*x-1)/c/x
)^(1/2)*x*((c*x+1)/c/x)^(1/2)+1/2*b/c^2*e^2-2*b*e*d*arcsech(c*x)*ln(1+(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-b*e*d*polylog(2,-(1/c/x+(-1+1/c/x)
^(1/2)*(1+1/c/x)^(1/2))^2)
```

3.109.5 Fricas [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\mathrm{sech}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{\left(ex^2+d\right)^2\left(b\,\mathrm{arsech}\left(cx\right)+a\right)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arcsech(c*x))/x^3, x)
```

3.109.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)^2}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**3,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**3, x)
```

3.109.7 Maxima [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arsech}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*a*e^2*x^2 - 1/8*b*d^2*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2
*x^2) - 1) - 1) - c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqr
t(1/(c^2*x^2) - 1) - 1))/c + 4*arcsech(c*x)/x^2) + 2*a*d*e*log(x) - 1/2*a*
d^2/x^2 + integrate(b*e^2*x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c
*x)) + 2*b*d*e*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x)
```

3.109.8 Giac [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arsech}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^3, x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 \left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(e\,x^2+d\right)^2 \left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^3, x)
```

3.110 $\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$

3.110.1 Optimal result

Integrand size = 21, antiderivative size = 519

$$
\begin{aligned}
\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d+ex^2}\,dx &= \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e} - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce} \\
&+ \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&- \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&+ \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&- \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&- \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&+ \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&- \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&+ \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}
\end{aligned}
$$

output
```
x*(a+b*arcsech(c*x))/e-b*arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/c/e+1/2*
(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1
/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arcsech(c*x))*l
n(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+
e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2
)/e^(3/2)-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*b*poly
log(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)
-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*b*polylog(2,c*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1
/2)/e^(3/2)
```

3.110.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 921, normalized size of antiderivative = 1.77

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$$

$$= \frac{4ac\sqrt{e}x-4ac\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+b\left(4\sqrt{e}\left(cx\operatorname{sech}^{-1}(cx)-2\arctan\left(\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(cx)\right)\right)\right)-2ic\sqrt{d}\left(-4i\right.\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
```

output
```
(4*a*c*Sqrt[e]*x - 4*a*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + b*(4*Sqrt[e
]*(c*x*ArcSech[c*x] - 2*ArcTan[Tanh[ArcSech[c*x]/2]]) - (2*I)*c*Sqrt[d]*((
-4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt
[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1
 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d +
e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcS
ech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[
d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[
2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] +
PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] +
PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])
 + (2*I)*c*Sqrt[d]*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2
]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d +
e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*
(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[S
qrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*
d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt[e] +
Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 - (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(...
```

3.110.3 Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6857, 6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{sech}^{-1}(cx))}{d+ex^2}\,dx$$

$$\Big\downarrow \text{6857}$$

$$-\int \frac{x^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{\frac{d}{x^2}+e}d\frac{1}{x}$$

$$\Big\downarrow \text{6374}$$

$$-\int \left(\frac{x^2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e}-\frac{d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{-d}\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2e^{3/2}}+\frac{\sqrt{-d}\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2e^{3/2}}+\frac{x\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{e}-\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}}-\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}}-\frac{b\arctan\left(\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}\right)}{ce}$$

input
```
Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
```

output
```
(x*(a + b*ArcCosh[1/(c*x)]))/e - (b*ArcTan[Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(
c*x)]])/(c*e) + (Sqrt[-d]*(a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^A
rcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a
+ b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - S
qrt[c^2*d + e])])/(2*e^(3/2)) + (Sqrt[-d]*(a + b*ArcCosh[1/(c*x)])*Log[1 -
 (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2))
 - (Sqrt[-d]*(a + b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x
)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2, -((
c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*e^(3/2))
+ (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c
^2*d + e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCosh[
1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog
[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/
2))
```

3.110. $\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$

3.110.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.110.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 54.98 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.79

method	result
parts	$\frac{ax}{e}-\frac{ad\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}+\frac{b\,\mathrm{arcsech}(cx)x}{e}-\frac{2b\arctan\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce}+\frac{bcd\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+(2c\cdots\right.}\cdots\right.}{\cdots}$
derivativedivides	$\frac{\frac{ac^3x}{e}-\frac{ac^3d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}+bc^2\left(\frac{\mathrm{arcsech}(cx)cx}{e}+\frac{dc^2\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+(2c^2d+4e)\_Z^2+c^2d\right)}\frac{(\_R1^2c^2d+4\_R1\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{ac^3x}{e}-\frac{ac^3d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}+bc^2\left(\frac{\mathrm{arcsech}(cx)cx}{e}+\frac{dc^2\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+(2c^2d+4e)\_Z^2+c^2d\right)}\frac{(\_R1^2c^2d+4\_R1\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int(x^2*(a+b*arcsech(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/e*x-a*d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*arcsech(c*x)/e*x-2*b/c/e
*arctan(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))+1/8*b*c/e^2*d*sum((_R1^2*c
^2*d+4*_R1^2*e+c^2*d)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/
c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
-1/8*b*c/e^2*d*sum((_R1^2*c^2*d+c^2*d+4*e)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(ar
csech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1
-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^
2*d+4*e)*_Z^2+c^2*d))
```

3.110.5 Fricas [F]

$$\int \frac{x^2(a+b\text{sech}^{-1}(cx))}{d+ex^2}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsech(c*x) + a*x^2)/(e*x^2 + d), x)
```

3.110.6 Sympy [F]

$$\int \frac{x^2(a+b\text{sech}^{-1}(cx))}{d+ex^2}\,dx=\int \frac{x^2(a+b\,\text{asech}\,(cx))}{d+ex^2}\,dx$$

input
```
integrate(x**2*(a+b*asech(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**2*(a + b*asech(c*x))/(d + e*x**2), x)
```

3.110.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\operatorname{sech}^{-1}(cx))}{d + ex^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.110.8 Giac [F]

$$\int \frac{x^2(a + b\operatorname{sech}^{-1}(cx))}{d + ex^2} dx = \int \frac{(b \operatorname{arsech}(cx) + a)x^2}{ex^2 + d} dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d), x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\operatorname{sech}^{-1}(cx))}{d + ex^2} dx = \int \frac{x^2 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)}{e x^2 + d} dx$$

input
```
int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2),x)
```

output
```
int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2), x)
```

3.111 $\int \frac{x\left(a+b\,\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$

3.111.1 Optimal result

Integrand size = 19, antiderivative size = 459

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e}$$
$$+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}$$
$$+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}$$
$$+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e}$$
$$+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e}$$
$$+\frac{b\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(cx)}\right)}{2e}$$
$$+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}$$
$$+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}$$
$$+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e}$$
$$+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e}$$

output
```
-(a+b*arcsech(c*x))^2/b/e-(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))^2)/e+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcs
ech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1
/2)-(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcs
ech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1
/2)+(c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,-1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))^2)/e+1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*b*po
lylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c
^2*d+e)^(1/2)))/e+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2
))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e
```

3.111.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 860, normalized size of antiderivative = 1.87

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$$

$$= \frac{4ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\operatorname{arctanh}\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tanh\left(\frac{1}{2}\text{sech}^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)+4ib\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\operatorname{arctanh}\left(\frac{\left(ic\sqrt{d}+\sqrt{e}\right)\cdots}{\cdots}\right)\cdots}{\cdots}$$

input
```
Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
```

output
```
((4*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*
c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + (4*I)*b*ArcS
in[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt
[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] - 2*b*ArcSech[c*x]*Log[1 + E^(
-2*ArcSech[c*x])] + b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))
/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSec
h[c*x])] + b*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt
[d]*E^ArcSech[c*x])] - (2*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])
] + b*ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^Ar
cSech[c*x])] + (2*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*L
og[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + b*Arc
Sech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x
])] + (2*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I
*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + a*Log[d + e*x^
2] + b*PolyLog[2, -E^(-2*ArcSech[c*x])] - b*PolyLog[2, ((-I)*(-Sqrt[e] + S
qrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - b*PolyLog[2, (I*(-Sqrt[e] +
 Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - b*PolyLog[2, ((-I)*(Sqrt[
e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - b*PolyLog[2, (I*(S...
```

3.111.3 Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6857, 6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{d+ex^2}\,dx$$

$$\Big\downarrow \text{6857}$$

$$-\int \frac{x\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{\frac{d}{x^2}+e}d\frac{1}{x}$$

$$\Big\downarrow \text{6374}$$

$$-\int \left(\frac{x\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e}-\frac{d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)x}\right)d\frac{1}{x}$$

$\downarrow$ 2009

$$\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2e}+\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}+\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2e}-\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)^2}{be}-\frac{\log\left(e^{-2\text{arccosh}\left(\frac{1}{cx}\right)}+1\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e}+\frac{b\,\text{PolyLog}\left(2,-e^{-2\text{arccosh}\left(\frac{1}{cx}\right)}\right)}{2e}$$

input
```
Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
```

output
```
-((a + b*ArcCosh[1/(c*x)])^2/(b*e)) - ((a + b*ArcCosh[1/(c*x)])*Log[1 + E^
(-2*ArcCosh[1/(c*x)])])/e + ((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*
E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcCosh[
1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d +
e])])/(2*e) + ((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcCosh[1/(c
*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcCosh[1/(c*x)])*Log[
1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) +
(b*PolyLog[2, -E^(-2*ArcCosh[1/(c*x)])])/(2*e) + (b*PolyLog[2, -((c*Sqrt[-
d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*e) + (b*PolyLog[2
, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + (b
*PolyLog[2, -((c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e]))
])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c
^2*d + e])])/(2*e)
```

3.111. $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$

3.111.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.111.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.45 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.10

method	result
parts	$\frac{a\ln(e\,x^2+d)}{2e}-\frac{b\,\mathrm{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\right)}{e}-\frac{b\,\mathrm{arcsech}(cx)\ln\left(1-i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\right)}{e}$
derivativedivides	$\frac{\frac{a\,c^2\ln\left(e\,c^2x^2+c^2d\right)}{2e}+b\,c^2\left(\frac{c^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\,\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\left(\_R1^2+1\right)\left(\mathrm{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{}}{\_}\right)\right.}{\_}\right.}{4e}\right.}{}$
default	$\frac{\frac{a\,c^2\ln\left(e\,c^2x^2+c^2d\right)}{2e}+b\,c^2\left(\frac{c^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\,\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\left(\_R1^2+1\right)\left(\mathrm{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{}}{\_}\right)\right.}{\_}\right.}{4e}\right.}{}$

input
```
int(x*(a+b*arcsech(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*a/e*ln(e*x^2+d)-b/e*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2)))-b/e*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2
)))-b/e*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e*dilog(1-I*
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+1/4*b*c^2*d/e*sum((_R1^2+1)/(_R1
^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=
RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/4*b/e*sum((_R1^2*c^2*d+c^2*
d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

3.111.5 Fricas [F]

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x*arcsech(c*x) + a*x)/(e*x^2 + d), x)
```

3.111.6 Sympy [F]

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{x(a+b\,\text{asech}\,(cx))}{d+ex^2}\,dx$$

input
```
integrate(x*(a+b*asech(c*x))/(e*x**2+d),x)
```

output
```
Integral(x*(a + b*asech(c*x))/(d + e*x**2), x)
```

3.111.7 Maxima [F]

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
b*integrate(x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 +
d), x) + 1/2*a*log(e*x^2 + d)/e
```

3.111.8 Giac [F]

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d), x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{d+ex^2}\,dx = \int \frac{x\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

input
```
int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2),x)
```

output
```
int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2), x)
```

3.112 $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{d+ex^2}\,dx$

3.112.1 Optimal result

Integrand size = 18, antiderivative size = 469

$$\begin{aligned}\int \frac{a+b\operatorname{sech}^{-1}(cx)}{d+ex^2}\,dx &= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &- \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &- \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

output
```
1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsech(c*x
))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^
2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^
(1/2)/e^(1/2)-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b*
polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(
1/2)-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,c*(1/c/x+(
-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d
)^(1/2)/e^(1/2)
```

3.112.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 849, normalized size of antiderivative = 1.81

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{d+ex^2}\,dx$$

$$= \frac{2a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)-4b\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\operatorname{arctanh}\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)+4b\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\text{ar}}{}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(d + e*x^2),x]
```

output
```
(2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/S
qrt[c^2*d + e]] + 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Ar
cTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] - I*
b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSec
h[c*x])] - 2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (
I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + I*b*ArcSech[c
*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] +
 2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e
] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + I*b*ArcSech[c*x]*Log[1
 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*b*ArcSi
n[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^
2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - I*b*ArcSech[c*x]*Log[1 + (I*(Sqrt
[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSin[Sqrt[1 +
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])] - I*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d
+ e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + I*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^
2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + I*b*PolyLog[2, ((-I)*(Sqrt[e] + S
qrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - I*b*PolyLog[2, (I*(Sqrt[e]
+ Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/(2*Sqrt[d]*Sqrt[e])
```

3.112.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6847, 6324, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex^2}\,dx$$

$$\Big\downarrow \text{ 6847}$$

$$-\int \frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{\frac{d}{x^2}+e}d\frac{1}{x}$$

$$\Big\downarrow \text{ 6324}$$

$$-\int \left(\frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{2\sqrt{e}\left(\sqrt{e}-\frac{\sqrt{-d}}{x}\right)}+\frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{2\sqrt{e}\left(\frac{\sqrt{-d}}{x}+\sqrt{e}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcSech[c*x])/(d + e*x^2),x]
```

output
```
((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e]
 - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[1/(c*x)])*Log
[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt
[-d]*Sqrt[e]) + ((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcCosh[1/
(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCo
sh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d
 + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCosh[1/(c
*x)])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2,
(c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*
Sqrt[e]) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt
[c^2*d + e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh
[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e])
```

3.112.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6324
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

rule 6847
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

3.112.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.62 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.64

method	result
parts	$\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} - \frac{bc\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)} \frac{\_R1\left(\mathrm{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{\_R1}\right. \cdots\right.}{\_R1^2c^2d+c \cdots}\right. \cdots}{2} \cdots$
derivativedivides	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(-\frac{\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)} \frac{\_R1\left(\mathrm{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1- \cdots}}{\_R1}\right. \cdots\right.}{\_R1^2c^2 \cdots}\right. \cdots}{2} \cdots\right.}{\cdots}$
default	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(-\frac{\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)} \frac{\_R1\left(\mathrm{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1- \cdots}}{\_R1}\right. \cdots\right.}{\_R1^2c^2 \cdots}\right. \cdots}{2} \cdots\right.}{\cdots}$

input
```
int((a+b*arcsech(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/2*b*c*sum(_R1/(_R1^2*c^2*d+c^2*d+2
*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dil
og((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^
4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*b*c*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(ar
csech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1
-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^
2*d+4*e)*_Z^2+c^2*d))
```

3.112.5 Fricas [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{d + ex^2} dx = \int \frac{b \operatorname{arsech}(cx) + a}{ex^2 + d} dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)/(e*x^2 + d), x)
```

3.112.6 Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{d + ex^2} dx = \int \frac{a + b \operatorname{asech}(cx)}{d + ex^2} dx$$

input
```
integrate((a+b*asech(c*x))/(e*x**2+d),x)
```

output
```
Integral((a + b*asech(c*x))/(d + e*x**2), x)
```

3.112.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{d + ex^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.112.8 Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{d + ex^2} dx = \int \frac{b\,\text{arsech}\,(cx) + a}{ex^2 + d} dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(e*x^2 + d), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{d + ex^2} dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{e\,x^2 + d} dx$$

input
```
int((a + b*acosh(1/(c*x)))/(d + e*x^2),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(d + e*x^2), x)
```

3.113 $\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x(d+ex^2)}\,dx$

3.113.1 Optimal result

Integrand size = 21, antiderivative size = 417

$$\begin{aligned}\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x(d+ex^2)}\,dx &= \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{2bd} - \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}\\ &- \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}\\ &- \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}\\ &- \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}\\ &- \frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}\\ &- \frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} - \frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}\end{aligned}$$

output
```
1/2*(a+b*arcsech(c*x))^2/b/d-1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b
*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/
(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d-1/2*(a+b
*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/
(e^(1/2)+(c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,c*(
1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2
)))/d-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
```

3.113.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 841, normalized size of antiderivative = 2.02

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)}\,dx = -\frac{b\operatorname{sech}^{-1}(cx)^2+4ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\operatorname{arctanh}\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)+4ib\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\text{a}}{}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)),x]
```

output
```
-1/2*(b*ArcSech[c*x]^2 + (4*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/
Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^
2*d + e]] + (4*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcT
anh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + b*Ar
cSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*
x])] - (2*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (
I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + b*ArcSech[c*x
]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (
2*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt
[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + b*ArcSech[c*x]*Log[1
 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*b*A
rcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + b*ArcSech[c*x]*Log[1 + (I*(Sq
rt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*b*ArcSin[Sqr
t[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d +
 e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*a*Log[x] + a*Log[d + e*x^2] - b*Poly
Log[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - b
*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] -
 b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x]
)] - b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[...
```

3.113.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6857, 6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)}dx$$

$$\Big\downarrow \text{6857}$$

$$-\int \frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)x}d\frac{1}{x}$$

$$\Big\downarrow \text{6374}$$

$$-\int \left(\frac{\sqrt{-d}\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2d\left(\frac{\sqrt{-d}}{x}+\sqrt{e}\right)}-\frac{\sqrt{-d}\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2d\left(\sqrt{e}-\frac{\sqrt{-d}}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2d}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2d}+\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)^2}{2bd}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d}$$

input
```
Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)),x]
```

output
```
(a + b*ArcCosh[1/(c*x)])^2/(2*b*d) - ((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*
Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b
*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt
[c^2*d + e])])/(2*d) - ((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^Arc
Cosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcCosh[1/(c*
x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])
/(2*d) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c
^2*d + e]))])/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[
e] - Sqrt[c^2*d + e])])/(2*d) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCosh[1/(c
*x)])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]*E^A
rcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d)
```

3.113.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

3.113. $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)}\,dx$

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.113.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.06 (sec) , antiderivative size = 2081, normalized size of antiderivative = 4.99

method	result	size
parts	Expression too large to display	2081
derivativedivides	Expression too large to display	2108
default	Expression too large to display	2108

input
```
int((a+b*arcsech(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d*ln(e*x^2+d)+a/d*ln(x)+b*(1/2/d*arcsech(c*x)^2-1/4*(-c^2*d*(e*(c^2
*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/e/(c^2*d+e)/d*arcsec
h(c*x)^2+1/8*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(
1/2)*e)/e/(c^2*d+e)/d*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-1/2*(c^2*d-2*(e*(c^2*d+e))^(1/2
)+2*e)/c^2/d^2*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2
*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)+1/4*(e*(c^2*d+e))^(1/2)/e/(c^2
*d+e)*c^2*arcsech(c*x)*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/c
^4/d^3*e*arcsech(c*x)^2-1/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/c^4/d^3*e*po
lylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2
*d+e))^(1/2)-2*e))-1/2*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/d*arcsech(c*x)^2+1/4*
(e*(c^2*d+e))^(1/2)/(c^2*d+e)/d*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+1/2*(c^2*d-2*(e*(c^2*
d+e))^(1/2)+2*e)/c^2/d^2*arcsech(c*x)^2-1/4*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2
*e)/c^2/d^2*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c
^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-1/4*(e*(c^2*d+e))^(1/2)/e/(c^2*d+e)*c^2*a
rcsech(c*x)^2+1/8*(e*(c^2*d+e))^(1/2)/e/(c^2*d+e)*c^2*polylog(2,d*c^2*(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))
+(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/(...
```

3.113.5 Fricas [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\,(d + ex^2)}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex^2 + d)x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)/(e*x^3 + d*x), x)
```

3.113.6 Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\,(d + ex^2)}\,dx = \int \frac{a + b\,\text{asech}\,(cx)}{x\,(d + ex^2)}\,dx$$

input
```
integrate((a+b*asech(c*x))/x/(e*x**2+d),x)
```

output
```
Integral((a + b*asech(c*x))/(x*(d + e*x**2)), x)
```

3.113.7 Maxima [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\,(d + ex^2)}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex^2 + d)x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(log(sqrt(1/(c*x) + 1)
*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^3 + d*x), x)
```

3.113.8 Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x(d + ex^2)}\, dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex^2 + d)x}\, dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/((e*x^2 + d)*x), x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x(d + ex^2)}\, dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x\,(e\,x^2 + d)}\, dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)), x)
```

3.114 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)}\,dx$

3.114.1 Optimal result

Integrand size = 21, antiderivative size = 523

$$
\begin{aligned}
\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)}\,dx &= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b\text{sech}^{-1}(cx)}{dx} \\
&+ \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&- \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&+ \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&- \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&- \frac{b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&+ \frac{b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&- \frac{b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&+ \frac{b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}
\end{aligned}
$$

output
```
-a/d/x-b*arcsech(c*x)/d/x+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^
(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^
(3/2)-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2
))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*(a+b*arcse
ch(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))
*e^(1/2)/(-d)^(3/2)-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*b*polyl
og(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d
+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+
1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(
1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+b*c*(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2)/d
```

3.114.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.28 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.78

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2(d+ex^2)}\,dx$$

$$= \frac{-4a\sqrt{d}-4a\sqrt{e}x\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+b\left(4\sqrt{d}\sqrt{\frac{1-cx}{1+cx}}(1+cx)-4\sqrt{d}\operatorname{sech}^{-1}(cx)-2i\sqrt{e}x\left(-4i\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{}}{c\sqrt{}}}}{\sqrt{2}}\right.\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
(-4*a*Sqrt[d] - 4*a*Sqrt[e]*x*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + b*(4*Sqrt[d]*S
qrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) - 4*Sqrt[d]*ArcSech[c*x] - (2*I)*Sqrt[e
]*x*((-4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*
c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]
*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^
2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*
E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c
*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]
/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x
])] + PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x
])] + PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c
*x])]) + (2*I)*Sqrt[e]*x*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/
Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^
2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1
 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*Ar
cSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt
[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1
- (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + ...
```

3.114.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6857, 6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)}dx$$

$$\Big\downarrow \text{ 6857}$$

$$-\int \frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 6374}$$

$$-\int \left(\frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{d}-\frac{e\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{e}\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2(-d)^{3/2}}-\frac{a}{dx}-\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{b\operatorname{arccosh}\left(\frac{1}{cx}\right)}{dx}+\frac{bc\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}{d}$$

input
```
Int[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
(b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/d - a/(d*x) - (b*ArcCosh[1/(c*x
)])/(d*x) + (Sqrt[e]*(a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcCos
h[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b
*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt
[c^2*d + e])])/(2*(-d)^(3/2)) + (Sqrt[e]*(a + b*ArcCosh[1/(c*x)])*Log[1 -
(c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2
)) - (Sqrt[e]*(a + b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*
x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[2,
-((c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*(-d)^(
3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - S
qrt[c^2*d + e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^A
rcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e
]*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])
/(2*(-d)^(3/2))
```

3.114. $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2(d+ex^2)}\,dx$

3.114.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.114.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 52.02 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.71

method	result
parts	$-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\sqrt{de}}-\frac{a}{dx}+\frac{bc\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{d}-\frac{b\,\mathrm{arcsech}(cx)}{dx}+\frac{bce\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c\right.}\right.}{}$
derivativedivides	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}+\frac{b\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{d}-\frac{b\,\mathrm{arcsech}(cx)}{dcx}+\frac{be\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z\right.}\right.}{}\right.$
default	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}+\frac{b\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{d}-\frac{b\,\mathrm{arcsech}(cx)}{dcx}+\frac{be\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z\right.}\right.}{}\right.$

input
```
int((a+b*arcsech(c*x))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a*e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-a/d/x+b*c/d*(-(c*x-1)/c/x)^(1/2
)*((c*x+1)/c/x)^(1/2)-b*arcsech(c*x)/d/x+1/2*b*c*e/d*sum(_R1/(_R1^2*c^2*d+
c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_
R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^
2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/2*b*c*e/d*sum(1/_R1/(_R1^2*c^2*d+c^2
*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)
+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d
*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

3.114.5 Fricas [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)}\,dx = \int \frac{b\,\text{arsech}\,(cx)+a}{(ex^2+d)x^2}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)/(e*x^4 + d*x^2), x)
```

3.114.6 Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)}\,dx = \int \frac{a+b\,\text{asech}\,(cx)}{x^2(d+ex^2)}\,dx$$

input
```
integrate((a+b*asech(c*x))/x**2/(e*x**2+d),x)
```

output
```
Integral((a + b*asech(c*x))/(x**2*(d + e*x**2)), x)
```

3.114.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2 (d + ex^2)} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.114.8 Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2 (d + ex^2)} dx = \int \frac{b \operatorname{arsech}(cx) + a}{(ex^2 + d)x^2} dx$$

input
```
integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/((e*x^2 + d)*x^2), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2 (d + ex^2)} dx = \int \frac{a + b \operatorname{acosh}\left(\frac{1}{c x}\right)}{x^2 (e x^2 + d)} dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)), x)
```

3.115 $\int \frac{x^5\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.115.1 Optimal result

Integrand size = 21, antiderivative size = 631

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{b\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2ce^2}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e^2}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)^2}{be^3}-\frac{bd\sqrt{-1+\frac{1}{c^2x^2}}\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{e^3}-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}-\frac{bd\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)}{e^3}-\frac{bd\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{bd\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{bd\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}-\frac{bd\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$

output
```
1/2*d*(a+b*arcsech(c*x))/e^2/(e+d/x^2)+1/2*x^2*(a+b*arcsech(c*x))/e^2+2*d*
(a+b*arcsech(c*x))^2/b/e^3+2*d*(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)
^(1/2)*(1+1/c/x)^(1/2))^2)/e^3-d*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-d*(a+b
*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/
(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-d*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-d*(a+b
*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/
(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,-1/(1/c/x+(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2))^2)/e^3-b*d*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,c*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-b
*d*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*b*d*arctanh((c^2*d+e
)^(1/2)/c/x/e^(1/2)/(-1+1/c^2/x^2)^(1/2))*(-1+1/c^2/x^2)^(1/2)/e^(5/2)/(c^
2*d+e)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-1/2*b*x*(-1+1/c/x)^(1/2)*(1+
1/c/x)^(1/2)/c/e^2
```

3.115.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.54 (sec) , antiderivative size = 1278, normalized size of antiderivative = 2.03

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\text{Too large to display}$$

input
```
Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*((2*e*
Sqrt[(1 - c*x)/(1 + c*x)])/c^2 + (2*e*x*Sqrt[(1 - c*x)/(1 + c*x)])/c - 2*e
*x^2*ArcSech[c*x] + (d^(3/2)*ArcSech[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (d^(3
/2)*ArcSech[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (16*I)*d*ArcSin[Sqrt[1 - (I*Sq
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcS
ech[c*x]/2])/Sqrt[c^2*d + e]] + (16*I)*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sq
rt[c^2*d + e]] - 8*d*ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] + 4*d*ArcSe
ch[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])
] - (8*I)*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(
Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 4*d*ArcSech[c*x]
*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (8
*I)*d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[
e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 4*d*ArcSech[c*x]*Log[
1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (8*I)*d*
ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sq
rt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 4*d*ArcSech[c*x]*Log[1 + (I*
(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (8*I)*d*ArcSin[
Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*
d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*d*Log[x] - 2*d*Log[1 + Sqrt[(1...
```

3.115.3 Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6857, 6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^2} \, dx$$

$$\Big\downarrow \ 6857$$

$$-\int \frac{x^3 \left(a + b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2} + e\right)^2} d\frac{1}{x}$$

$$\Big\downarrow \ 6374$$

$$-\int \left(\frac{\left(a + b\text{arccosh}\left(\frac{1}{cx}\right)\right) x^3}{e^2} - \frac{2d\left(a + b\text{arccosh}\left(\frac{1}{cx}\right)\right) x}{e^3} + \frac{2d^2\left(a + b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e^3 \left(\frac{d}{x^2} + e\right) x} + \frac{d^2\left(a + b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e^2 \left(\frac{d}{x^2} + e\right)^2 x}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{e^3}-\frac{d\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}-\frac{d\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{e^3}+\frac{2d\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)^2}{be^3}+\frac{2d\log\left(e^{-2\operatorname{arccosh}\left(\frac{1}{cx}\right)}+1\right)\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{e^3}+\frac{d\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{2e^2\left(\frac{d}{x^2}+e\right)}+\frac{x^2\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{2e^2}-\frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3}-\frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3}-\frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3}-\frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3}-\frac{bd\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arccosh}\left(\frac{1}{cx}\right)}\right)}{e^3}-\frac{bd\sqrt{\frac{1}{c^2x^2}-1}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}-1}}\right)}{2e^{5/2}\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}\sqrt{c^2d+e}}-\frac{bx\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}{2ce^2}$$

input
```
Int[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

3.115. $\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$

output
```
-1/2*(b*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)/(c*e^2) + (d*(a + b*ArcCos
h[1/(c*x)]))/(2*e^2*(e + d/x^2)) + (x^2*(a + b*ArcCosh[1/(c*x)]))/(2*e^2)
+ (2*d*(a + b*ArcCosh[1/(c*x)])^2)/(b*e^3) - (b*d*Sqrt[-1 + 1/(c^2*x^2)]*A
rcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(2*e^(5/2)*S
qrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (2*d*(a + b*ArcCosh
[1/(c*x)])*Log[1 + E^(-2*ArcCosh[1/(c*x)])])/e^3 - (d*(a + b*ArcCosh[1/(c*
x)])*Log[1 - (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])
/e^3 - (d*(a + b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])
/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcCosh[1/(c*x)])*Log[1 - (
c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (d*(a +
 b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sq
rt[c^2*d + e])])/e^3 - (b*d*PolyLog[2, -E^(-2*ArcCosh[1/(c*x)])])/e^3 - (b
*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e]
))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt
[c^2*d + e])])/e^3 - (b*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sq
rt[e] + Sqrt[c^2*d + e]))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/
(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3
```

3.115.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.115. $\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$

3.115.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.18 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.25

method	result
parts	$\frac{a x^2}{2e^2} - \frac{ad\ln(e x^2+d)}{e^3} - \frac{a d^2}{2e^3(e x^2+d)} + \frac{b\left(\frac{c^4\left(2\operatorname{arcsech}(cx)c^4 d x^2+e\operatorname{arcsech}(cx)c^4x^4-\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}c^3dx-\sqrt{-\frac{cx-}{cx}}\right.}{2\left(e c^2x^2+c^2d\right)e^2}\right.}{}$
derivativedivides	$\frac{\frac{a c^6x^2}{2e^2}-\frac{a c^6 d\ln\left(e c^2x^2+c^2d\right)}{e^3}-\frac{a c^8d^2}{2e^3\left(e c^2x^2+c^2d\right)}+b c^4\left(\frac{2\operatorname{arcsech}(cx)c^4dx^2+e\operatorname{arcsech}(cx)c^4x^4-\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}c^3dx-\sqrt{-\frac{c}{}}}{2\left(e c^2x^2+c^2d\right)e^2}\right.}{}$
default	$\frac{\frac{a c^6x^2}{2e^2}-\frac{a c^6 d\ln\left(e c^2x^2+c^2d\right)}{e^3}-\frac{a c^8d^2}{2e^3\left(e c^2x^2+c^2d\right)}+b c^4\left(\frac{2\operatorname{arcsech}(cx)c^4dx^2+e\operatorname{arcsech}(cx)c^4x^4-\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}c^3dx-\sqrt{-\frac{c}{}}}{2\left(e c^2x^2+c^2d\right)e^2}\right.}{}$

input
```
int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2/e^2-a*d/e^3*ln(e*x^2+d)-1/2*a*d^2/e^3/(e*x^2+d)+b/c^6*(1/2*c^4*(
2*arcsech(c*x)*c^4*d*x^2+e*arcsech(c*x)*c^4*x^4-(-(c*x-1)/c/x)^(1/2)*((c*x
+1)/c/x)^(1/2)*c^3*d*x-(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*e*c^3*x^3+
c^2*d+e*c^2*x^2)/(c^2*e*x^2+c^2*d)/e^2+1/2*(e*(c^2*d+e))^(1/2)/e^3/(c^2*d+
e)*d*c^6*arctanh(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2+2
*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))+2/e^3*d*c^6*arcsech(c*x)*ln(1+I*(1/c/x+(-
1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+2/e^3*d*c^6*arcsech(c*x)*ln(1-I*(1/c/x+(-
1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+2/e^3*d*c^6*dilog(1+I*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2)))+2/e^3*d*c^6*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/
c/x)^(1/2)))-1/2/e^3*d*c^6*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+
2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+di
log((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z
^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/2/e^3*d^2*c^8*sum((_R1^2+1)/(_R1^2*c^2*d+c
^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R
1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2
*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)))
```

3.115.5 Fricas [F]

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^5}{\left(ex^2+d\right)^2}\,dx$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^5*arcsech(c*x) + a*x^5)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.115.6 Sympy [F]

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{x^5(a+b\,\text{asech}\,(cx))}{\left(d+ex^2\right)^2}\,dx$$

input
```
integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**5*(a + b*asech(c*x))/(d + e*x**2)**2, x)
```

3.115.7 Maxima [F]

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^5}{\left(ex^2+d\right)^2}\,dx$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(d^2/(e^4*x^2 + d*e^3) - x^2/e^2 + 2*d*log(e*x^2 + d)/e^3) + b*inte
grate(x^5*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e^2*x^4 + 2*
d*e*x^2 + d^2), x)
```

3.115.8 Giac [F]

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^5}{\left(ex^2+d\right)^2}\,dx$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^2, x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{x^5\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.116 $\int \frac{x^3 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx$

3.116.1 Optimal result

Integrand size = 21, antiderivative size = 580

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be^2}$$
$$+\frac{b\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$
$$-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e^2}$$
$$+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+\frac{b\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(cx)}\right)}{2e^2}$$
$$+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$

output
```
1/2*(-a-b*arcsech(c*x))/e/(e+d/x^2)-(a+b*arcsech(c*x))^2/b/e^2-(a+b*arcsec
h(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/e^2+1/2*(a+b*ar
csech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+
b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)
/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2
*b*polylog(2,-1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/e^2+1/2*b*poly
log(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))/e^2+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2
))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*b*polylog(2,-c*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1
/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1
/2)+(c^2*d+e)^(1/2)))/e^2+1/2*b*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/
c^2/x^2)^(1/2))*(-1+1/c^2/x^2)^(1/2)/e^(3/2)/(c^2*d+e)^(1/2)/(-1+1/c/x)^(1
/2)/(1+1/c/x)^(1/2)
```

3.116.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 1208, normalized size of antiderivative = 2.08

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

output
```
((2*a*d)/(d + e*x^2) + (b*Sqrt[d]*ArcSech[c*x])/(Sqrt[d] - I*Sqrt[e]*x) +
(b*Sqrt[d]*ArcSech[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (8*I)*b*ArcSin[Sqrt[1 -
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan
h[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + (8*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/
(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2
])/Sqrt[c^2*d + e]] - 4*b*ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] + 2*b*
ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[
c*x])] - (4*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 +
 (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSech
[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]
 - (4*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-
Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSech[c*x]
*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (4*
I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e]
 + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSech[c*x]*Log[1
+ (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (4*I)*b*Ar
cSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt
[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*Log[x] + 2*a*Log[d + e*x^2
] - 2*b*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]
 + (b*Sqrt[e]*Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 ...
```

3.116.3 Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6857, 6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{6857}$$

$$-\int \frac{x\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{6374}$$

$$-\int \left(\frac{x\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e^2}-\frac{d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)x}-\frac{d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)^2x}\right)d\frac{1}{x}$$

$\downarrow$ 2009

$$\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2e^2}+\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2}+\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2e^2}-\frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{2e\left(\frac{d}{x^2}+e\right)}-\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)^2}{be^2}-\frac{\log\left(e^{-2\text{arccosh}\left(\frac{1}{cx}\right)}+1\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e^2}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2}+\frac{b\,\text{PolyLog}\left(2,-e^{-2\text{arccosh}\left(\frac{1}{cx}\right)}\right)}{2e^2}+\frac{b\sqrt{\frac{1}{c^2x^2}-1}\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}-1}}\right)}{2e^{3/2}\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}\sqrt{c^2d+e}}$$

input
```
Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcCosh[1/(c*x)])/(e*(e + d/x^2)) - (a + b*ArcCosh[1/(c*x)])^2
/(b*e^2) + (b*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sq
rt[-1 + 1/(c^2*x^2)]*x)])/(2*e^(3/2)*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sq
rt[1 + 1/(c*x)]) - ((a + b*ArcCosh[1/(c*x)])*Log[1 + E^(-2*ArcCosh[1/(c*x)
])])/e^2 + ((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcCosh[1/(c*x)
])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcCosh[1/(c*x)])*Log[1
 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) +
 ((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e
] + Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt
[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^2) + (b*PolyLo
g[2, -E^(-2*ArcCosh[1/(c*x)])])/(2*e^2) + (b*PolyLog[2, -((c*Sqrt[-d]*E^Ar
cCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*e^2) + (b*PolyLog[2, (c*
Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + (b*Po
lyLog[2, -((c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e]))])/
(2*e^2) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^
2*d + e])])/(2*e^2)
```

3.116. $\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$

3.116.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.116.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.71 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.11

method	result
parts	$\frac{a\ln(e\,x^2+d)}{2e^2}+\frac{ad}{2e^2(e\,x^2+d)}-\frac{b\,c^2x^2\,\mathrm{arcsech}(cx)}{2(e\,c^2x^2+c^2d)e}-\frac{b\sqrt{e(c^2d+e)}\,\mathrm{arctanh}\left(\frac{2c^2d\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2+2c^2d+4e\cdots}{4\sqrt{c^2de+e^2}}\cdots\right.}{2e^2(c^2d+e)}\cdots$
derivativedivides	$\frac{\frac{a\,c^6d}{2e^2\left(e\,c^2x^2+c^2d\right)}+\frac{a\,c^4\ln\left(e\,c^2x^2+c^2d\right)}{2e^2}+b\,c^4\left(-\frac{c^2x^2\,\mathrm{arcsech}(cx)}{2\left(e\,c^2x^2+c^2d\right)e}-\frac{\sqrt{e\left(c^2d+e\right)}\,\mathrm{arctanh}\left(\frac{2c^2d\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2+2c^2d\cdots}{4\sqrt{c^2de+e^2}}\cdots\right.}{2e^2\left(c^2d+e\right)}\cdots\right.}{\cdots}$
default	$\frac{\frac{a\,c^6d}{2e^2\left(e\,c^2x^2+c^2d\right)}+\frac{a\,c^4\ln\left(e\,c^2x^2+c^2d\right)}{2e^2}+b\,c^4\left(-\frac{c^2x^2\,\mathrm{arcsech}(cx)}{2\left(e\,c^2x^2+c^2d\right)e}-\frac{\sqrt{e\left(c^2d+e\right)}\,\mathrm{arctanh}\left(\frac{2c^2d\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2+2c^2d\cdots}{4\sqrt{c^2de+e^2}}\cdots\right.}{2e^2\left(c^2d+e\right)}\cdots\right.}{\cdots}$

input
```
int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a/e^2*ln(e*x^2+d)+1/2*a*d/e^2/(e*x^2+d)-1/2*b*c^2*x^2*arcsech(c*x)/(c^
2*e*x^2+c^2*d)/e-1/2*b*(e*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)*arctanh(1/4*(2*c^
2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^
(1/2))-b/e^2*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))
-b/e^2*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e^2
*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e^2*dilog(1-I*(1/c/
x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+1/4*b/e^2*sum((_R1^2*c^2*d+c^2*d+4*e)
/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1
/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1))
,_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/4*b*c^2/e^2*d*sum((_R1
^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/
_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

3.116.5 Fricas [F]

$$\int \frac{x^3(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\operatorname{arsech}(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*arcsech(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.116.6 Sympy [F]

$$\int \frac{x^3(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{x^3(a+b\operatorname{asech}(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**3*(a + b*asech(c*x))/(d + e*x**2)**2, x)
```

3.116.7 Maxima [F]

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^3}{\left(ex^2+d\right)^2}\,dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*log(sqr
t(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e^2*x^4 + 2*d*e*x^2 + d^2), x
)
```

3.116.8 Giac [F]

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^3}{\left(ex^2+d\right)^2}\,dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^2, x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{x^3\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.117 $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$

3.117.1 Optimal result

Integrand size = 19, antiderivative size = 147

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx = -\frac{a+b\text{sech}^{-1}(cx)}{2e(d+ex^2)} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{2de} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}}$$

output
```
1/2*(-a-b*arcsech(c*x))/e/(e*x^2+d)+1/2*b*arctanh((-c^2*x^2+1)^(1/2))*(1/(
c*x+1))^(1/2)*(c*x+1)^(1/2)/d/e-1/2*b*arctanh(e^(1/2)*(-c^2*x^2+1)^(1/2)/(
c^2*d+e)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d/e^(1/2)/(c^2*d+e)^(1/2)
```

3.117.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.35

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx = -\frac{\frac{2a}{d+ex^2} + \frac{2b\text{sech}^{-1}(cx)}{d+ex^2} + \frac{2b\log(x)}{d} - \frac{2b\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{d} + \frac{b\sqrt{e}\log\left(\frac{4\left(\frac{ide+c^2d^{3/2}\sqrt{e}x}{\sqrt{c^2d+e}\left(\sqrt{d}+i\sqrt{e}x\right)}+\frac{de\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{-i\sqrt{d}\sqrt{e}+ex}\right)}{b}\right)}{d\sqrt{c^2d+e}} + \frac{b\sqrt{\cdots}}{\cdots}}{4e}$$

input
```
Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/4*((2*a)/(d + e*x^2) + (2*b*ArcSech[c*x])/(d + e*x^2) + (2*b*Log[x])/d
- (2*b*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])
/d + (b*Sqrt[e]*Log[(4*((I*d*e + c^2*d^(3/2)*Sqrt[e]*x)/(Sqrt[c^2*d + e]*(
Sqrt[d] + I*Sqrt[e]*x)) + (d*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/((-I)*
Sqrt[d]*Sqrt[e] + e*x)))/b])/(d*Sqrt[c^2*d + e]) + (b*Sqrt[e]*Log[(4*((d*e
 + I*c^2*d^(3/2)*Sqrt[e]*x)/(Sqrt[c^2*d + e]*(I*Sqrt[d] + Sqrt[e]*x)) + (d
*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(I*Sqrt[d]*Sqrt[e] + e*x)))/b])/(d
*Sqrt[c^2*d + e]))/e
```

3.117.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6853, 2036, 354, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \; 6853$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\left(ex^2+d\right)}dx}{2e}-\frac{a+b\text{sech}^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \; 2036$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x\sqrt{1-c^2x^2}\left(ex^2+d\right)}dx}{2e}-\frac{a+b\text{sech}^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \; 354$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(ex^2+d\right)}dx^2}{4e}-\frac{a+b\text{sech}^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \; 97$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2}{d}-\frac{e\int \frac{1}{\sqrt{1-c^2x^2}\left(ex^2+d\right)}dx^2}{d}\right)}{4e}-\frac{a+b\text{sech}^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\downarrow \textbf{73}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2e\int \frac{1}{-\frac{ex^4}{c^2}+d+\frac{e}{c^2}}d\sqrt{1-c^2x^2}}{c^2d}-\frac{2\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2d}\right)}{4e}-\frac{a+b\text{sech}^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\downarrow \textbf{221}$$

$$-\frac{a+b\text{sech}^{-1}(cx)}{2e\left(d+ex^2\right)}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{d\sqrt{c^2d+e}}-\frac{2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d}\right)}{4e}$$

input
```
Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcSech[c*x])/(e*(d + e*x^2)) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
 + c*x]*((-2*ArcTanh[Sqrt[1 - c^2*x^2]])/d + (2*Sqrt[e]*ArcTanh[(Sqrt[e]*S
qrt[1 - c^2*x^2])/Sqrt[c^2*d + e]])/(d*Sqrt[c^2*d + e])))/(4*e)
```

3.117.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.117. $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2036
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

rule 6853
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Simp[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x
^2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e
, p}, x] && NeQ[p, -1]
```

3.117.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(124) = 248$.

Time = 5.52 (sec) , antiderivative size = 462, normalized size of antiderivative = 3.14

method	result
parts	$-\frac{a}{2e(e\,x^2+d)}+\frac{b\left(-\frac{c^4\,\mathrm{arcsech}(cx)}{2e\left(e\,c^2x^2+c^2d\right)}+\frac{c^3\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(-2\sqrt{\frac{c^2d+e}{e}}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^2d+\ln\left(-\frac{2\left(\sqrt{\frac{c^2d+e}{e}}\sqrt{-\cdots}\right.}{-\cdots}\right.\right.}{\cdots}\right)}{\cdots}$
derivativedivides	$\frac{-\frac{a\,c^4}{2e\left(e\,c^2x^2+c^2d\right)}+b\,c^4\left(-\frac{\mathrm{arcsech}(cx)}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(2\sqrt{\frac{c^2d+e}{e}}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^2d-\ln\left(\frac{2\sqrt{\frac{c^2d+e}{e}}\sqrt{-c^2x\cdots}}{cex\cdots}\right.\right.}{\cdots}\right)}{\cdots}$
default	$\frac{-\frac{a\,c^4}{2e\left(e\,c^2x^2+c^2d\right)}+b\,c^4\left(-\frac{\mathrm{arcsech}(cx)}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(2\sqrt{\frac{c^2d+e}{e}}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^2d-\ln\left(\frac{2\sqrt{\frac{c^2d+e}{e}}\sqrt{-c^2x\cdots}}{cex\cdots}\right.\right.}{\cdots}\right)}{\cdots}$

input
```
int(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/e/(e*x^2+d)+b/c^2*(-1/2*c^4/e/(c^2*e*x^2+c^2*d)*arcsech(c*x)+1/4*c^
3*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(-2*((c^2*d+e)/e)^(1/2)*arcta
nh(1/(-c^2*x^2+1)^(1/2))*c^2*d+ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/
2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^2*d+ln(2*(((c^2*
d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*
e)^(1/2)))*c^2*d-2*((c^2*d+e)/e)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))*e+ln(
-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e
*x+(-c^2*d*e)^(1/2)))*e+ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c
^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*e)/(-c^2*x^2+1)^(1/2)/d/(e+
(-c^2*d*e)^(1/2))/(-e+(-c^2*d*e)^(1/2))/((c^2*d+e)/e)^(1/2))
```

3.117. $\int \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$

3.117.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(89) = 178$.

Time = 0.31 (sec) , antiderivative size = 602, normalized size of antiderivative = 4.10

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

$$= \left[-\frac{2\,ac^2d^2+2\,ade-\sqrt{c^2de+e^2}(bex^2+bd)\log\left(\frac{c^4d^2+4\,c^2de-(c^4de+2\,c^2e^2)x^2+4\,(c^3de+ce^2)x\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+4\,e^2+2\,\left(c^2ex^2\right.}{ex^2+d}\right.}{4\,(}\right.$$

$$-\frac{ac^2d^2+ade+\sqrt{-c^2de-e^2}(bex^2+bd)\arctan\left(\frac{\sqrt{-c^2de-e^2}\,cdx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-\sqrt{-c^2de-e^2}\,(ex^2+d)}{(c^2de+e^2)x^2}\right)+(bc^2d^2+bde}{2\,(c^2d^3e+d^2e^2+(c^2d^2e^2+de^3}$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*c^2*d^2 + 2*a*d*e - sqrt(c^2*d*e + e^2)*(b*e*x^2 + b*d)*log((c^
4*d^2 + 4*c^2*d*e - (c^4*d*e + 2*c^2*e^2)*x^2 + 4*(c^3*d*e + c*e^2)*x*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) + 4*e^2 + 2*(c^2*e*x^2 - c^2*d - (c^3*d + 2*c*e
)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*e)*sqrt(c^2*d*e + e^2))/(e*x^2 + d)
) + 2*(b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*e^2)*x^2)*log((c*x*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)) - 1)/x) + 2*(b*c^2*d^2 + b*d*e)*log((c*x*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 + d*e^3)*
x^2), -1/2*(a*c^2*d^2 + a*d*e + sqrt(-c^2*d*e - e^2)*(b*e*x^2 + b*d)*arcta
n((sqrt(-c^2*d*e - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(-c^2*d
*e - e^2)*(e*x^2 + d))/((c^2*d*e + e^2)*x^2)) + (b*c^2*d^2 + b*d*e + (b*c^
2*d*e + b*e^2)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c
^2*d^2 + b*d*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*
d^3*e + d^2*e^2 + (c^2*d^2*e^2 + d*e^3)*x^2)]
```

3.117.6 Sympy [F]

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{x(a+b\,\text{asech}\,(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x*(a+b*asech(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x*(a + b*asech(c*x))/(d + e*x**2)**2, x)
```

3.117.7 Maxima [F]

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x}{(ex^2+d)^2}\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*(2*c^2*integrate(1/2*x^3/(c^2*d^2*x^2 + (c^2*d*e*x^2 - d*e)*x^2 + (c^2
*d^2*x^2 + (c^2*d*e*x^2 - d*e)*x^2 - d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) - d
^2), x) + (x^2*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - x^2*log(c) - x^2*lo
g(x))/(d*e*x^2 + d^2) - 2*integrate(1/2*x/(c^2*d^2*x^2 + (c^2*d*e*x^2 - d*
e)*x^2 - d^2), x))*b - 1/2*a/(e^2*x^2 + d*e)
```

3.117.8 Giac [F]

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x}{(ex^2+d)^2}\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^2, x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

input
```
int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.118 $\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x(d+ex^2)^2}\,dx$

3.118.1 Optimal result

Integrand size = 21, antiderivative size = 542

$$\begin{aligned}\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x(d+ex^2)^2}\,dx = &-\frac{e\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{2bd^2}\\
&+\frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,\mathrm{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}\\
&-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}-\frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\end{aligned}$$

output
```
-1/2*e*(a+b*arcsech(c*x))/d^2/(e+d/x^2)+1/2*(a+b*arcsech(c*x))^2/b/d^2-1/2
*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(
1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(
-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2
-1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))
/d^2-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*b*polylog(
2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e
)^(1/2)))/d^2-1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^2+1/2*b*arctanh((c^2*d+e)^(1/2)/c/x
/e^(1/2)/(-1+1/c^2/x^2)^(1/2))*e^(1/2)*(-1+1/c^2/x^2)^(1/2)/d^2/(c^2*d+e)^
(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)
```

3.118.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 1189, normalized size of antiderivative = 2.19

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

$$= \frac{\frac{2ad}{d+ex^2} + \frac{b\sqrt{d}\operatorname{sech}^{-1}(cx)}{\sqrt{d}-i\sqrt{e}x} + \frac{b\sqrt{d}\operatorname{sech}^{-1}(cx)}{\sqrt{d}+i\sqrt{e}x} - 2b\operatorname{sech}^{-1}(cx)^2 - 8ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\operatorname{arctanh}\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tanh\left(\frac{1}{2}\right.}{\sqrt{c^2d+e}}\right.}{}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
((2*a*d)/(d + e*x^2) + (b*Sqrt[d]*ArcSech[c*x])/(Sqrt[d] - I*Sqrt[e]*x) +
(b*Sqrt[d]*ArcSech[c*x])/(Sqrt[d] + I*Sqrt[e]*x) - 2*b*ArcSech[c*x]^2 - (8
*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*S
qrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] - (8*I)*b*ArcSin[
Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e]
)*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] - 2*b*ArcSech[c*x]*Log[1 + (I*(Sq
rt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (4*I)*b*ArcSin[Sqr
t[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d +
 e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*b*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e]
+ Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (4*I)*b*ArcSin[Sqrt[1 -
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))
/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*b*ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt
[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (4*I)*b*ArcSin[Sqrt[1 - (I*Sqr
t[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqr
t[d]*E^ArcSech[c*x])] - 2*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d
+ e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (4*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(
c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^
ArcSech[c*x])] + 4*a*Log[x] + 2*b*Log[x] - 2*a*Log[d + e*x^2] - 2*b*Log[1
+ Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]] + (b*Sqrt[e]*
Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt...
```

3.118.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6857, 6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{6857}$$

$$-\int \frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{6374}$$

$$-\int \left(\frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2}+e\right)x}-\frac{e\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)^2x}\right)d\frac{1}{x}$$

$\downarrow$ 2009

$$-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2d^2}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2d^2}-\frac{e\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{2d^2\left(\frac{d}{x^2}+e\right)}+\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)^2}{2bd^2}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}+\frac{b\sqrt{e}\sqrt{\frac{1}{c^2x^2}-1}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}-1}}\right)}{2d^2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}\sqrt{c^2d+e}}$$

input
```
Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
-1/2*(e*(a + b*ArcCosh[1/(c*x)]))/(d^2*(e + d/x^2)) + (a + b*ArcCosh[1/(c*
x)])^2/(2*b*d^2) + (b*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d +
e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(2*d^2*Sqrt[c^2*d + e]*Sqrt[-1 +
 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - ((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-
d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*Arc
Cosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2
*d + e])])/(2*d^2) - ((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcCo
sh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcCosh[1/(c*
x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])
/(2*d^2) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt
[c^2*d + e]))])/(2*d^2) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(S
qrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCos
h[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*d^2) - (b*PolyLog[2, (c*Sqrt
[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2)
```

3.118. $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)^2}\,dx$

3.118.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.118.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.00 (sec) , antiderivative size = 2226, normalized size of antiderivative = 4.11

method	result	size
parts	Expression too large to display	2226
derivativedivides	Expression too large to display	2275
default	Expression too large to display	2275

input
```
int((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d^2*ln(e*x^2+d)+1/2*a/d/(e*x^2+d)+a/d^2*ln(x)+b*(-(-c^2*d*(e*(c^2*d
+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/(c^2*d+e)/d^3/c^2*arcs
ech(c*x)^2+1/8*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))
^(1/2)*e)/d^2/e/(c^2*d+e)*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-1/2*(c^2*d-2*(e*(c^2*d+e))^
(1/2)+2*e)/d^4/c^4*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)
)^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*e-1/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)
+2*e)/d^3/c^2*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*
d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)+(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*
e)/d^4/c^4*arcsech(c*x)^2*e-1/4*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^
2-2*(e*(c^2*d+e))^(1/2)*e)/d^2/e/(c^2*d+e)*arcsech(c*x)^2+1/2*(e*(c^2*d+e)
)^(1/2)/d^2/(c^2*d+e)*arcsech(c*x)*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1
/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+1/2*(-c^2*d*(e*(c^2*d+e
))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/(c^2*d+e)/d^3/c^2*polylo
g(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e
))^(1/2)-2*e))-(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))
^(1/2)*e)*e/d^4/(c^2*d+e)/c^4*arcsech(c*x)^2+1/2*(-c^2*d*(e*(c^2*d+e))^(1/
2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*e/d^4/(c^2*d+e)/c^4*polylog(2,
d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(
1/2)-2*e))+1/8*(e*(c^2*d+e))^(1/2)/d/e/(c^2*d+e)*c^2*polylog(2,d*c^2*(1...
```

3.118.5 Fricas [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{\left(ex^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

3.118.6 Sympy [F]

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)^2}\,dx = \int \frac{a+b\operatorname{asech}(cx)}{x(d+ex^2)^2}\,dx$$

input
```
integrate((a+b*asech(c*x))/x/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*asech(c*x))/(x*(d + e*x**2)**2), x)
```

3.118.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.118.8 Giac [F]

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)^2}\,dx = \int \frac{b\operatorname{arsech}(cx)+a}{(ex^2+d)^2x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^2*x), x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^2),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^2), x)
```

3.119 $\int \frac{x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.119.1 Optimal result

Integrand size = 21, antiderivative size = 840

$$
\begin{aligned}
\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx =& -\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+\frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2}+\frac{bd\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2} \\
&+\frac{bd\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2} \\
&-\frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce^2} \\
&+\frac{3\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&-\frac{3\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+\frac{3\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&-\frac{3\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&-\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&-\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}
\end{aligned}
$$

output
```
x*(a+b*arcsech(c*x))/e^2-b*arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/c/e^2+
3/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arcsech(c*x
))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^
2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^
(1/2)/e^(5/2)-3/4*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*b*
polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(
5/2)-3/4*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*b*polylog(2,c*(1/c/x+(
-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d
)^(1/2)/e^(5/2)-1/4*d*(a+b*arcsech(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))+1/4
*d*(a+b*arcsech(c*x))/e^2/(d/x+(-d)^(1/2)*e^(1/2))+1/2*b*d*arctan((1+1/c/x
)^(1/2)*(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^
(1/2))^(1/2))/e^2/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^
(1/2)+1/2*b*d*arctan((1+1/c/x)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/
c/x)^(1/2)/(c*d-(-d)^(1/2)*e^(1/2))^(1/2))/e^2/(c*d-(-d)^(1/2)*e^(1/2))...
```

3.119.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 1270, normalized size of antiderivative = 1.51

$$\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\text{Too large to display}$$

input
```
Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

output
```
(4*a*Sqrt[e]*x + (2*a*d*Sqrt[e]*x)/(d + e*x^2) + 4*b*Sqrt[e]*x*ArcSech[c*x
] + (b*d*ArcSech[c*x])/((-I)*Sqrt[d] + Sqrt[e]*x) + (b*d*ArcSech[c*x])/(I*
Sqrt[d] + Sqrt[e]*x) - 6*a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - (8*b*Sqrt
[e]*ArcTan[Tanh[ArcSech[c*x]/2]])/c + 12*b*Sqrt[d]*ArcSin[Sqrt[1 - (I*Sqrt
[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSec
h[c*x]/2])/Sqrt[c^2*d + e]] - 12*b*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/
Sqrt[c^2*d + e]] + (3*I)*b*Sqrt[d]*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt
[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 6*b*Sqrt[d]*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c
*Sqrt[d]*E^ArcSech[c*x])] - (3*I)*b*Sqrt[d]*ArcSech[c*x]*Log[1 + (I*(-Sqrt
[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 6*b*Sqrt[d]*ArcSin[S
qrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*
d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (3*I)*b*Sqrt[d]*ArcSech[c*x]*Log[1
- (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 6*b*Sqrt[d
]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] +
Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (3*I)*b*Sqrt[d]*ArcSech[c*
x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 6
*b*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(S
qrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (I*b*Sqrt[d]*S...
```

3.119.3 Rubi [A] (verified)

Time = 2.86 (sec) , antiderivative size = 900, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6857, 6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^2} \, dx$$

$$\Big\downarrow \; 6857$$

$$-\int \frac{x^2 \left(a + b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2} + e\right)^2} d\frac{1}{x}$$

$$\Big\downarrow \; 6374$$

$$-\int \left(\frac{\left(a + b\text{arccosh}\left(\frac{1}{cx}\right)\right) x^2}{e^2} - \frac{d\left(a + b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e^2 \left(\frac{d}{x^2} + e\right)} - \frac{d\left(a + b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e \left(\frac{d}{x^2} + e\right)^2}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{e^2}+\frac{3\sqrt{-d}\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}-\frac{3\sqrt{-d}\log\left(\frac{\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}+\frac{3\sqrt{-d}\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}-\frac{3\sqrt{-d}\log\left(\frac{\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}-\frac{d\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{4e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{bd\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}+\frac{bd\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}-\frac{b\arctan\left(\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}\right)}{ce^2}-\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4e^{5/2}}+\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4e^{5/2}}-\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4e^{5/2}}+\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4e^{5/2}}$$

input
```
Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

3.119. $\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

output
```
-1/4*(d*(a + b*ArcCosh[1/(c*x)]))/(e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a +
 b*ArcCosh[1/(c*x)]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcCosh
[1/(c*x)]))/e^2 + (b*d*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*
x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*Sqrt[c*d - Sqr
t[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2) + (b*d*ArcTan[(Sqrt[c*d +
 Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-
1 + 1/(c*x)])])/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e
]]*e^2) - (b*ArcTan[Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]])/(c*e^2) + (3*Sq
rt[-d]*(a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(S
qrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcCosh[1/(c*
x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])
/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^
ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*
(a + b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e]
+ Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*
E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(4*e^(5/2)) + (3*b*Sqrt
[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e]
)])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCosh[1/(c*x)
])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (
c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)...
```

3.119.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.119. $\int \frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$

3.119.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 111.63 (sec) , antiderivative size = 1006, normalized size of antiderivative = 1.20

method	result	size
parts	Expression too large to display	1006
derivativedivides	Expression too large to display	1031
default	Expression too large to display	1031

input
```
int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/e^2*x-1/e^2*d*(-1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2
))))+b/c^5*(1/2*x*c^5*arcsech(c*x)*(2*c^2*e*x^2+3*c^2*d)/e^2/(c^2*e*x^2+c^
2*d)-2/e^2*c^4*arctan(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))-3/16/e^3*c^6
*d*sum((_R1^2*c^2*d+c^2*d+4*e)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*l
n((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^
2+c^2*d))+3/16/e^3*c^6*d*sum((_R1^2*c^2*d+4*_R1^2*e+c^2*d)/_R1/(_R1^2*c^2*
d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(
c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*(c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2+2*(e*(c^2*d+e))^(1
/2)*e)*c*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(
e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e^2/(c^2*d+e)/d^2+1/2*((c^2*d+2*(e*(c^2*
d+e))^(1/2)+2*e)*d)^(1/2)*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e
*(c^2*d+e))^(1/2)*e)*c*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e^2/(c^2*d+e)/d^2-1/2*(-(c^2
*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c
*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d
+e))^(1/2)-2*e)*d)^(1/2))/d^2/e^2-1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d
)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctan(c*d*(1/c/x+(-1+1/c/x...
```

3.119.5 Fricas [F]

$$\int \frac{x^4 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx=\int \frac{\left(b \operatorname{arsech}\left(cx\right)+a\right)x^4}{\left(ex^2+d\right)^2} \, dx$$

input
```
integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsech(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.119.6 Sympy [F]

$$\int \frac{x^4 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx=\int \frac{x^4(a+b \operatorname{asech}\left(cx\right))}{\left(d+ex^2\right)^2} \, dx$$

input
```
integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**4*(a + b*asech(c*x))/(d + e*x**2)**2, x)
```

3.119.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.119.8 Giac [F]

$$\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\operatorname{arsech}(cx)+a)x^4}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^2, x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^4\left(a+b\operatorname{acosh}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.120 $\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.120.1 Optimal result

Integrand size = 21, antiderivative size = 786

$$\int \frac{x^2 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^2} \, dx = \frac{a + b\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} - \frac{a + b\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)}$$
$$- \frac{b \arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd - \sqrt{-d}\sqrt{e}}\sqrt{cd + \sqrt{-d}\sqrt{e}}e}$$
$$- \frac{b \arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd - \sqrt{-d}\sqrt{e}}\sqrt{cd + \sqrt{-d}\sqrt{e}}e}$$
$$+ \frac{(a + b\text{sech}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$- \frac{(a + b\text{sech}^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$+ \frac{(a + b\text{sech}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$- \frac{(a + b\text{sech}^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$- \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$+ \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$- \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$+ \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

output
```
1/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arcsech(c*x
))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^
2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/
2)/(-d)^(1/2)-1/4*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*b*
polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(
1/2)-1/4*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*b*polylog(2,c*(1/c/x+(
-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(
3/2)/(-d)^(1/2)+1/4*(a+b*arcsech(c*x))/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(-a
-b*arcsech(c*x))/e/(d/x+(-d)^(1/2)*e^(1/2))-1/2*b*arctan((1+1/c/x)^(1/2)*(
c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1
/2))/e/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2)-1/2*b
*arctan((1+1/c/x)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c
*d-(-d)^(1/2)*e^(1/2))^(1/2))/e/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(
1/2)*e^(1/2))^(1/2)
```

3.120.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 1226, normalized size of antiderivative = 1.56

$$\int \frac{x^2(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

output
```
((-2*a*Sqrt[e]*x)/(d + e*x^2) + (b*ArcSech[c*x])/(I*Sqrt[d] - Sqrt[e]*x) -
 (b*ArcSech[c*x])/(I*Sqrt[d] + Sqrt[e]*x) + (2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/Sqrt[d] - (4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTa
nh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]])/Sqr
t[d] + (4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*
c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]])/Sqrt[d] - (I*
b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSec
h[c*x])])/Sqrt[d] - (2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt
[d] + (I*b*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d
]*E^ArcSech[c*x])])/Sqrt[d] + (2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c
*x])])/Sqrt[d] + (I*b*ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))
/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[d] - (2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(
c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^
ArcSech[c*x])])/Sqrt[d] - (I*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2
*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[d] + (2*b*ArcSin[Sqrt[1 + (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*S
qrt[d]*E^ArcSech[c*x])])/Sqrt[d] + (I*b*Sqrt[e]*Log[((2*I)*Sqrt[e]*(Sqrt[d
]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/S...
```

3.120.3 Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 842, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6857, 6324, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx$$

$$\Big\downarrow \text{ 6857}$$

$$-\int \frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 6324}$$

$$-\int \left(-\frac{d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2e\left(-\frac{d^2}{x^2}-ed\right)}-\frac{d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{4e\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}-\frac{\log\left(\frac{\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}+\frac{\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}-\frac{\log\left(\frac{\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}+\frac{a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)}{4e\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e}-\frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

3.120. $\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

output
```
(a + b*ArcCosh[1/(c*x)])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCosh[1
/(c*x)])/(4*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTan[(Sqrt[c*d - Sqrt[-d]*S
qrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)
])])/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e) - (b*
ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-
d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d
 + Sqrt[-d]*Sqrt[e]]*e) + ((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^
ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a
 + b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] -
Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcCosh[1/(c*x)])*Log[1
- (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d
]*e^(3/2)) - ((a + b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*
x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -(
(c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(4*Sqrt[-d]
*e^(3/2)) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[
c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCosh
[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLo
g[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt
[-d]*e^(3/2))
```

3.120.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6324
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.120. $\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.120.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 46.70 (sec) , antiderivative size = 910, normalized size of antiderivative = 1.16

method	result
parts	$-\frac{ax}{2e(e\,x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+\frac{b\left(-\frac{c^5\,\mathrm{arcsech}(cx)x}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{c^4\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(\mathrm{ar}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{-\frac{a\,c^5x}{2e\left(e\,c^2x^2+c^2d\right)}+\frac{a\,c^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b\,c^4\left(-\frac{\mathrm{arcsech}(cx)cx}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(\mathrm{arcs}\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}$
default	$\frac{-\frac{a\,c^5x}{2e\left(e\,c^2x^2+c^2d\right)}+\frac{a\,c^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b\,c^4\left(-\frac{\mathrm{arcsech}(cx)cx}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(\mathrm{arcs}\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}$

input
```
int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/e*x/(e*x^2+d)+1/2*a/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c^3*(-1
/2*c^5*arcsech(c*x)/e*x/(c^2*e*x^2+c^2*d)-1/4/e*c^4*sum(_R1/(_R1^2*c^2*d+c
^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R
1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2
*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)
*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arctanh(c*d*(1/c/x+(-1+1/c/x)^
(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/c/d^3
/e-1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d*(e*(c^2*d+e))^(
1/2)+2*c^2*d*e+2*e^2+2*(e*(c^2*d+e))^(1/2)*e)*arctanh(c*d*(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(
c^2*d+e)/d^3/c+1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e
*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/c/d^3/e-1/2*((c^2*d+2*(e*(c^
2*d+e))^(1/2)+2*e)*d)^(1/2)*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*
(e*(c^2*d+e))^(1/2)*e)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^3/c+1/4/e*c^4*
sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/
2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)))
```

3.120.5 Fricas [F]

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(b\operatorname{arsech}(cx)+a)x^2}{\left(ex^2+d\right)^2}\,dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsech(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.120.6 Sympy [F]

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{x^2(a+b\operatorname{asech}(cx))}{\left(d+ex^2\right)^2}\,dx$$

input
```
integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**2*(a + b*asech(c*x))/(d + e*x**2)**2, x)
```

3.120.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.120.8 Giac [F]

$$\int \frac{x^2(a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} \, dx = \int \frac{(b \operatorname{arsech}(cx) + a)x^2}{(ex^2 + d)^2} \, dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} \, dx = \int \frac{x^2 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)}{(e x^2 + d)^2} \, dx$$

input
```
int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.121 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^2} \, dx$

3.121.1 Optimal result

Integrand size = 18, antiderivative size = 786

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^2}\,dx = -\frac{a+b\operatorname{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\operatorname{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}+\frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

output
```
-1/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsech(c*
x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c
^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-
1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)
^(3/2)/e^(1/2)+1/4*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/
c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b
*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)
-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^
(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^
(1/2)+1/4*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,c*(1/c/x+
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-
d)^(3/2)/e^(1/2)+1/4*(-a-b*arcsech(c*x))/d/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(
a+b*arcsech(c*x))/d/(d/x+(-d)^(1/2)*e^(1/2))+1/2*b*arctan((1+1/c/x)^(1/2)*
(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(
1/2))/d/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2)+1/2*
b*arctan((1+1/c/x)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(
c*d-(-d)^(1/2)*e^(1/2))^(1/2))/d/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^
(1/2)*e^(1/2))^(1/2)
```

3.121.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 1216, normalized size of antiderivative = 1.55

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^2,x]
```

output
```
((2*a*Sqrt[d]*x)/(d + e*x^2) + (b*Sqrt[d]*ArcSech[c*x])/((-I)*Sqrt[d]*Sqrt
[e] + e*x) + (b*Sqrt[d]*ArcSech[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) + (2*a*Arc
Tan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])
/Sqrt[c^2*d + e]])/Sqrt[e] + (4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]
/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*
d + e]])/Sqrt[e] - (I*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]
))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] - (2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*
E^ArcSech[c*x])])/Sqrt[e] + (I*b*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[
c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] + (2*b*ArcSin[Sqrt[1 - (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] + (I*b*ArcSech[c*x]*Log[1 - (I*(Sqrt[
e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] - (2*b*ArcSin[
Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*
d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] - (I*b*ArcSech[c*x]*Log[1 +
(I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] + (2*
b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] +
Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] - (I*b*Log[((2*I)*S
qrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] ...
```

3.121.3 Rubi [A] (verified)

Time = 2.55 (sec) , antiderivative size = 842, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6847, 6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{6847}$$

$$-\int \frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{6374}$$

$$-\int \left(\frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2}+e\right)}-\frac{e\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)^2}\right)d\frac{1}{x}$$

$\downarrow$ 2009

$$-\frac{\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(\frac{\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(\frac{\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)}{4d\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}+\frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

input
```
Int[(a + b*ArcSech[c*x])/(d + e*x^2)^2,x]
```

3.121. $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^2}\,dx$

output
```
-1/4*(a + b*ArcCosh[1/(c*x)])/(d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcCos
h[1/(c*x)])/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTan[(Sqrt[c*d - Sqrt[-d
]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c
*x)])])/(2*d*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) +
(b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqr
t[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*d*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqr
t[c*d + Sqrt[-d]*Sqrt[e]]) - ((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]
*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e])
+ ((a + b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[
e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcCosh[1/(c*x)])
*Log[1 - (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*
(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcC
osh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*Po
lyLog[2, -((c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e]))])/
(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sq
rt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((c*Sqr
t[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*(-d)^(3/2)*Sqr
t[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*
d + e])])/(4*(-d)^(3/2)*Sqrt[e])
```

3.121.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6847
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

3.121.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 55.84 (sec) , antiderivative size = 898, normalized size of antiderivative = 1.14

method	result
parts	$\frac{ax}{2d(e\,x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+\frac{b\left(\frac{c^3\,\mathrm{arcsech}(cx)x}{2d\left(e\,c^2x^2+c^2d\right)}-\frac{\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\,\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\mathrm{arctanh}\left(\cdots\right.}{2d^4c^3}\cdots\right.}{\cdots}$
derivativedivides	$\frac{\frac{a\,c^3x}{2d\left(e\,c^2x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\mathrm{arcsech}(cx)x}{2cd\left(e\,c^2x^2+c^2d\right)}-\frac{\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\,\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\mathrm{arctanh}\left(\cdots\right.}{2c^7d^4}\cdots\right.}{\cdots}$
default	$\frac{\frac{a\,c^3x}{2d\left(e\,c^2x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\mathrm{arcsech}(cx)x}{2cd\left(e\,c^2x^2+c^2d\right)}-\frac{\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\,\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\mathrm{arctanh}\left(\cdots\right.}{2c^7d^4}\cdots\right.}{\cdots}$

input
```
int((a+b*arcsech(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x/d/(e*x^2+d)+1/2*a/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c*(1/2*c
^3*arcsech(c*x)*x/d/(c^2*e*x^2+c^2*d)-1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arctanh(c*d*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^
4/c^3+1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d*(e*(c^2*d+e)
)^(1/2)+2*c^2*d*e+2*e^2+2*(e*(c^2*d+e))^(1/2)*e)*arctanh(c*d*(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/
d^4/(c^2*d+e)/c^3-1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2
*(e*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/
2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/c^3+1/2*((c^2*d+2*(e*
(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2
-2*(e*(c^2*d+e))^(1/2)*e)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/
2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)/c^3-1/4/d*c
^2*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^
(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/4/d*c^2*sum(
1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/
_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

3.121.5 Fricas [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^2}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.121.6 Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^2}\,dx=\int \frac{a+b\,\text{asech}\,(cx)}{(d+ex^2)^2}\,dx$$

input
```
integrate((a+b*asech(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*asech(c*x))/(d + e*x**2)**2, x)
```

3.121.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.121.8 Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^2}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex^2 + d)^2}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^2, x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^2}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{(e\,x^2 + d)^2}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((a + b*acosh(1/(c*x)))/(d + e*x^2)^2, x)
```

3.122 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$

3.122.1 Optimal result

Integrand size = 21, antiderivative size = 844

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2}-\frac{a}{d^2x}-\frac{b\operatorname{sech}^{-1}(cx)}{d^2x}+\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{be\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}-\frac{be\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}-\frac{3\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

output
```
-a/d^2/x-b*arcsech(c*x)/d^2/x-3/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(
-d)^(5/2)+3/4*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*(a+b*a
rcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*(a+b*arcsech(c*x))*ln(1+c*
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/
2)))*e^(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*b*p
olylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c
^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5
/2)-3/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/
(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+1/4*e*(a+b*arcsech(c*x))/d^2
/(-d/x+(-d)^(1/2)*e^(1/2))-1/4*e*(a+b*arcsech(c*x))/d^2/(d/x+(-d)^(1/2)*e^
(1/2))+b*c*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/d^2-1/2*b*e*arctan((1+1/c/x)^(
1/2)*(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/
2))^(1/2))/d^2/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/
2)-1/2*b*e*arctan((1+1/c/x)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x
)^(1/2)/(c*d-(-d)^(1/2)*e^(1/2))^(1/2))/d^2/(c*d-(-d)^(1/2)*e^(1/2))^(1...
```

3.122.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 1305, normalized size of antiderivative = 1.55

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^2),x]
```

output
```
((-4*a*Sqrt[d])/x + 4*b*c*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)] + (4*b*Sqrt[d]
*Sqrt[(1 - c*x)/(1 + c*x)])/x - (2*a*Sqrt[d]*e*x)/(d + e*x^2) - (4*b*Sqrt[
d]*ArcSech[c*x])/x - (b*Sqrt[d]*e*ArcSech[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*
x) - (b*Sqrt[d]*e*ArcSech[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) - 6*a*Sqrt[e]*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]] + 12*b*Sqrt[e]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])
/Sqrt[c^2*d + e]] - 12*b*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/
Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d
 + e]] + (3*I)*b*Sqrt[e]*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e
]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 6*b*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/
(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E
^ArcSech[c*x])] - (3*I)*b*Sqrt[e]*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt
[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 6*b*Sqrt[e]*ArcSin[Sqrt[1 - (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(
c*Sqrt[d]*E^ArcSech[c*x])] - (3*I)*b*Sqrt[e]*ArcSech[c*x]*Log[1 - (I*(Sqrt
[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 6*b*Sqrt[e]*ArcSin[S
qrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d
 + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (3*I)*b*Sqrt[e]*ArcSech[c*x]*Log[1 +
 (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 6*b*Sqrt[e]
*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] ...
```

3.122.3 Rubi [A] (verified)

Time = 2.82 (sec) , antiderivative size = 904, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6857, 6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)^2}dx$$

$$\Big\downarrow \text{6857}$$

$$-\int \frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2x^4}d\frac{1}{x}$$

$$\Big\downarrow \text{6374}$$

$$-\int \left(\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)e^2}{d^2\left(\frac{d}{x^2}+e\right)^2}-\frac{2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)e}{d^2\left(\frac{d}{x^2}+e\right)}+\frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{d^2}\right)d\frac{1}{x}$$

$$\downarrow \text{2009}$$

$$-\frac{a}{d^2x}-\frac{b\text{arccosh}\left(\frac{1}{cx}\right)}{d^2x}+\frac{e\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{4d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{be\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}-\frac{be\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}-\frac{3\sqrt{e}\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{4(-d)^{5/2}}+\frac{3b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+\frac{3b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+\frac{bc\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}{d^2}$$

input
```
Int[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^2),x]
```

3.122. $\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$

output
```
(b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/d^2 - a/(d^2*x) - (b*ArcCosh[1/
(c*x)])/(d^2*x) + (e*(a + b*ArcCosh[1/(c*x)]))/(4*d^2*(Sqrt[-d]*Sqrt[e] -
d/x)) - (e*(a + b*ArcCosh[1/(c*x)]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b
*e*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqr
t[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*d^2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*S
qrt[c*d + Sqrt[-d]*Sqrt[e]]) - (b*e*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*S
qrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*d
^2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (3*Sqrt[e]
*(a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e]
 - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCosh[1/(c*x)])
*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*
(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^Ar
cCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*
(a + b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e]
+ Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d
]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(4*(-d)^(5/2)) - (3*b
*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d
+ e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCosh[1
/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*Poly
Log[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4...
```

3.122.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.122. $\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$

3.122.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 109.90 (sec) , antiderivative size = 1007, normalized size of antiderivative = 1.19

method	result	size
parts	Expression too large to display	1007
derivativedivides	Expression too large to display	1034
default	Expression too large to display	1034

input
```
int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/d^2*e*(1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-1/d^
2/x)+b*c*(-1/2*(-1+arcsech(c*x))/d^2*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/
x)^(1/2)+1)/x/c+1/2*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)-1)*(1+ar
csech(c*x))/d^2/x/c-1/2*arcsech(c*x)/d^2*e*x*c/(c^2*e*x^2+c^2*d)+1/2*(-(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*
e*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*
d+e))^(1/2)-2*e)*d)^(1/2))/d^5/c^5-1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)
*d)^(1/2)*(c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2+2*(e*(c^2*d+e))^(1/2)
*e)*e*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(
c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/c^5/(c^2*d+e)+1/2*((c^2*d+2*(e*(c^2*d+e
))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*e*arctan(c*d*(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)
^(1/2))/d^5/c^5-1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-c^2*d*(e
*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*e*arctan(c*d*(1
/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*
d)^(1/2))/d^5/c^5/(c^2*d+e)-3/4/d^2*e*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(a
rcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R
1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c
^2*d+4*e)*_Z^2+c^2*d))+3/4/d^2*e*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(
c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/...
```

3.122.5 Fricas [F]

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{b\operatorname{arsech}\left(cx\right)+a}{\left(ex^2+d\right)^2x^2}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

3.122.6 Sympy [F]

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\operatorname{asech}\left(cx\right)}{x^2\left(d+ex^2\right)^2}\,dx$$

input
```
integrate((a+b*asech(c*x))/x**2/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*asech(c*x))/(x**2*(d + e*x**2)**2), x)
```

3.122.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.122.8 Giac [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x^2 (d + ex^2)^2} \, dx = \int \frac{b \operatorname{arsech}(cx) + a}{(ex^2 + d)^2 x^2} \, dx$$

input
```
integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^2*x^2), x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x^2 (d + ex^2)^2} \, dx = \int \frac{a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)}{x^2 \left(e\,x^2 + d\right)^2} \, dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^2),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^2), x)
```

3.123 $\int \frac{x^5\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

3.123.1 Optimal result

Integrand size = 21, antiderivative size = 778

$$\int \frac{x^5 (a + b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{bd\left(c^2-\frac{1}{x^2}\right)}{8ce^2(c^2d+e)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{a+b\text{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} - \frac{(a+b\text{sech}^{-1}(cx))^2}{be^3} + \frac{b\sqrt{-1+\frac{1}{c^2x^2}}\,\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{b(c^2d+2e)\sqrt{-1+\frac{1}{c^2x^2}}\,\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}(c^2d+e)^{3/2}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - \frac{(a+b\text{sech}^{-1}(cx))\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{e^3} + \frac{(a+b\text{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\text{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\text{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\text{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)}{2e^3} + \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$

output
```
1/4*(-a-b*arcsech(c*x))/e/(e+d/x^2)^2+1/2*(-a-b*arcsech(c*x))/e^2/(e+d/x^2
)-(a+b*arcsech(c*x))^2/b/e^3-(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))^2)/e^3+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a
+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/
2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^
(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,-1/(1/c/x+(-1+1/c/x)^
(1/2)*(1+1/c/x)^(1/2))^2)/e^3+1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(
1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,
c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(
1/2)))/e^3+1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d
)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)
^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/8*b*d*
(c^2-1/x^2)/c/e^2/(c^2*d+e)/(e+d/x^2)/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1
/8*b*(c^2*d+2*e)*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/c^2/x^2)^(1/2))
*(-1+1/c^2/x^2)^(1/2)/e^(5/2)/(c^2*d+e)^(3/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(
1/2)+1/2*b*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/c^2/x^2)^(1/2))*(-1+1
/c^2/x^2)^(1/2)/e^(5/2)/(c^2*d+e)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2...
```

3.123.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.93 (sec) , antiderivative size = 2000, normalized size of antiderivative = 2.57

$$\int \frac{x^5(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^3} \, dx = \text{Too large to display}$$

input
```
Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*
x^2])/(2*e^3) + b*(-1/16*(d*(((-I)*Sqrt[e]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
c*x))/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcSech[c*x]/(Sqr
t[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + Sqrt[(1
- c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2*c^2*d
 + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[
c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/
(1 + c*x)]))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(
3/2))))/e^(5/2) - (d*((I*Sqrt[e]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(Sqr
t[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcSech[c*x]/(Sqrt[e]*(I*Sqrt[
d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)
] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2*c^2*d + e)*Log[(-4*d*
Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[
(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/((2
*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*e^(5/2)
) - (((7*I)/16)*Sqrt[d]*(-(ArcSech[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) + (I*(L
og[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1
+ c*x)]]/Sqrt[e] + Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(
1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sq
rt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d]))/e^(5/2) + (((7*I)/16)*Sqrt[d]*(-(...
```

3.123.3 Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 850, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6857, 6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{ 6857}$$

$$-\int \frac{x\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 6374}$$

$$-\int \left(\frac{x\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e^3}-\frac{d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e^3\left(\frac{d}{x^2}+e\right)x}-\frac{d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)^2x}-\frac{d\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)^3x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)^2}{be^3}-\frac{\log\left(1+e^{-2\text{arccosh}\left(\frac{1}{cx}\right)}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{e^3}+\frac{\log\left(1-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2e^3}+\frac{\log\left(\frac{\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2e^3}+\frac{\log\left(1-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2e^3}+\frac{\log\left(\frac{\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{2e^3}-\frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{2e^2\left(\frac{d}{x^2}+e\right)}-\frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{4e\left(\frac{d}{x^2}+e\right)^2}+\frac{b(dc^2+2e)\sqrt{\frac{1}{c^2x^2}-1}\text{arctanh}\left(\frac{\sqrt{dc^2+e}}{c\sqrt{e}\sqrt{\frac{1}{c^2x^2}-1}x}\right)}{8e^{5/2}\left(dc^2+e\right)^{3/2}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}+\frac{b\sqrt{\frac{1}{c^2x^2}-1}\text{arctanh}\left(\frac{\sqrt{dc^2+e}}{c\sqrt{e}\sqrt{\frac{1}{c^2x^2}-1}x}\right)}{2e^{5/2}\sqrt{dc^2+e}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}+\frac{b\,\text{PolyLog}\left(2,-e^{-2\text{arccosh}\left(\frac{1}{cx}\right)}\right)}{2e^3}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3}+\frac{bd\left(c^2-\frac{1}{x^2}\right)}{8ce^2\left(dc^2+e\right)\left(\frac{d}{x^2}+e\right)\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}x}$$

input
```
Int[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

3.123. $\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

output
```
(b*d*(c^2 - x^(-2)))/(8*c*e^2*(c^2*d + e)*(e + d/x^2)*Sqrt[-1 + 1/(c*x)]*S
qrt[1 + 1/(c*x)]*x) - (a + b*ArcCosh[1/(c*x)])/(4*e*(e + d/x^2)^2) - (a +
b*ArcCosh[1/(c*x)])/(2*e^2*(e + d/x^2)) - (a + b*ArcCosh[1/(c*x)])^2/(b*e^
3) + (b*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[-1
+ 1/(c^2*x^2)]*x)])/(2*e^(5/2)*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 +
 1/(c*x)]) + (b*(c^2*d + 2*e)*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d +
e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(8*e^(5/2)*(c^2*d + e)^(3/2)*Sqr
t[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - ((a + b*ArcCosh[1/(c*x)])*Log[1 + E^(
-2*ArcCosh[1/(c*x)])])/e^3 + ((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]
*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcCo
sh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d
 + e])])/(2*e^3) + ((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcCosh
[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcCosh[1/(c*x)
])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(
2*e^3) + (b*PolyLog[2, -E^(-2*ArcCosh[1/(c*x)])])/(2*e^3) + (b*PolyLog[2,
-((c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*e^3) +
 (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])
])/(2*e^3) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sq
rt[c^2*d + e]))])/(2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/
(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3)
```

3.123.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.123. $\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

3.123.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.87 (sec) , antiderivative size = 1549, normalized size of antiderivative = 1.99

method	result	size
parts	Expression too large to display	1549
derivativedivides	Expression too large to display	1562
default	Expression too large to display	1562

input
```
int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/2/e^3*ln(e*x^2+d)+d/e^3/(e*x^2+d)-1/4*d^2/e^3/(e*x^2+d)^2)+b/c^6*(-1/
8*c^6*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c^5*d^2*x+(-(c*x-1)/c/x)^(
1/2)*((c*x+1)/c/x)^(1/2)*c^5*d*e*x^3+4*arcsech(c*x)*c^6*d^2*x^2+6*c^6*d*e*
arcsech(c*x)*x^4+4*c^4*d*e*arcsech(c*x)*x^2+6*arcsech(c*x)*e^2*c^4*x^4-c^4
*d^2-2*c^4*d*e*x^2-c^4*e^2*x^4)/e^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2-3/4*(e*(
c^2*d+e))^(1/2)/(c^2*d+e)^2/e^2*c^6*arctanh(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)
^(1/2)*(1+1/c/x)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))-5/8*(e*(c^2*d+
e))^(1/2)/(c^2*d+e)^2/e^3*c^8*d*arctanh(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))-1/(c^2*d+e)/e^2*c^
6*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-1/(c^2*d+e
)/e^2*c^6*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-1/
(c^2*d+e)/e^2*c^6*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-1/(c
^2*d+e)/e^2*c^6*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+1/4/(c
^2*d+e)/e^2*c^6*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(arcse
ch(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/
c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d
+4*e)*_Z^2+c^2*d))+1/4/(c^2*d+e)/e^2*c^8*d*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*
d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+
dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*
_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/(c^2*d+e)/e^3*c^8*d*arcsech(c*x)*ln(1...
```

3.123.5 Fricas [F]

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\int \frac{(b\operatorname{arsech}\left(cx\right)+a)x^5}{\left(ex^2+d\right)^3}\,dx$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^5*arcsech(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
 + d^3), x)
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.123.7 Maxima [F]

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\int \frac{(b\operatorname{arsech}\left(cx\right)+a)x^5}{\left(ex^2+d\right)^3}\,dx$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
 + d)/e^3) + b*integrate(x^5*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(
c*x))/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

3.123.8 Giac [F]

$$\int \frac{x^5 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^3} \, dx = \int \frac{(b \operatorname{arsech}(cx) + a)x^5}{\left(ex^2 + d\right)^3} \, dx$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^3, x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^3} \, dx = \int \frac{x^5 \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^3} \, dx$$

input
```
int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.124 $\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3} \, dx$

3.124.1 Optimal result

Integrand size = 21, antiderivative size = 173

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3} \, dx=\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8e\left(c^2d+e\right)\left(d+ex^2\right)}+\frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\left(c^2d+e\right)^{3/2}}$$

output
```
1/4*x^4*(a+b*arcsech(c*x))/d/(e*x^2+d)^2-1/8*b*(c^2*d+2*e)*arctanh(e^(1/2)
*(-c^2*x^2+1)^(1/2)/(c^2*d+e)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d/e^(
3/2)/(c^2*d+e)^(3/2)+1/8*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1
/2)/e/(c^2*d+e)/(e*x^2+d)
```

3.124.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.81

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3} \, dx=$$

$$-\frac{-\frac{4ad}{(d+ex^2)^2}+\frac{8a}{d+ex^2}-\frac{2e\sqrt{\frac{1-cx}{1+cx}}(b+bcx)}{(c^2d+e)(d+ex^2)}+\frac{4b(d+2ex^2)\text{sech}^{-1}(cx)}{(d+ex^2)^2}+\frac{4b\log(x)}{d}-\frac{4b\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{d}+\frac{b\sqrt{e}(c^2d+2\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/16*((-4*a*d)/(d + e*x^2)^2 + (8*a)/(d + e*x^2) - (2*e*Sqrt[(1 - c*x)/(1
 + c*x)]*(b + b*c*x))/((c^2*d + e)*(d + e*x^2)) + (4*b*(d + 2*e*x^2)*ArcSe
ch[c*x])/(d + e*x^2)^2 + (4*b*Log[x])/d - (4*b*Log[1 + Sqrt[(1 - c*x)/(1 +
 c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/d + (b*Sqrt[e]*(c^2*d + 2*e)*Log[
(16*d*e^(3/2)*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]
*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]
))/(b*(c^2*d + 2*e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2)) +
(b*Sqrt[e]*(c^2*d + 2*e)*Log[(16*d*e^(3/2)*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^
2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e
]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/(b*(c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))]
)/(d*(c^2*d + e)^(3/2)))/e^2
```

3.124.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6855, 27, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \ 6855$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x^3}{4d\sqrt{1-c^2x^2}\left(ex^2+d\right)^2}dx+\frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x^3}{\sqrt{1-c^2x^2}\left(ex^2+d\right)^2}dx}{4d}+\frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}$$

$$\Big\downarrow \ 354$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x^2}{\sqrt{1-c^2x^2}\left(ex^2+d\right)^2}dx^2}{8d}+\frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}$$

$$\Big\downarrow \ 87$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{(c^2d+2e)\int\frac{1}{\sqrt{1-c^2x^2}(ex^2+d)}dx^2}{2e(c^2d+e)}+\frac{d\sqrt{1-c^2x^2}}{e(c^2d+e)(d+ex^2)}\right)}{8d}+\frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 73$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{d\sqrt{1-c^2x^2}}{e(c^2d+e)(d+ex^2)}-\frac{(c^2d+2e)\int\frac{1}{-\frac{ex^4}{c^2}+d+\frac{e}{c^2}}d\sqrt{1-c^2x^2}}{c^2e(c^2d+e)}\right)}{8d}+\frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{d\sqrt{1-c^2x^2}}{e(c^2d+e)(d+ex^2)}-\frac{(c^2d+2e)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{e^{3/2}(c^2d+e)^{3/2}}\right)}{8d}$$

input
```
Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

output
```
(x^4*(a + b*ArcSech[c*x]))/(4*d*(d + e*x^2)^2) + (b*Sqrt[(1 + c*x)^(-1)]*S
qrt[1 + c*x]*((d*Sqrt[1 - c^2*x^2])/(e*(c^2*d + e)*(d + e*x^2)) - ((c^2*d
+ 2*e)*ArcTanh[(Sqrt[e]*Sqrt[1 - c^2*x^2])/Sqrt[c^2*d + e]])/(e^(3/2)*(c^2
*d + e)^(3/2))))/(8*d)
```

3.124.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.124. $\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.124.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1361 vs. $2(147) = 294$.

Time = 5.09 (sec) , antiderivative size = 1362, normalized size of antiderivative = 7.87

method	result	size
parts	Expression too large to display	1362
derivativedivides	Expression too large to display	1395
default	Expression too large to display	1395

input
```
int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/e^2/(e*x^2+d)+1/4*d/e^2/(e*x^2+d)^2)+b/c^4*(-1/2*c^6*arcsech(c*x)/
e^2/(c^2*e*x^2+c^2*d)+1/4*c^8*arcsech(c*x)*d/e^2/(c^2*e*x^2+c^2*d)^2-1/16*
c^5*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e*(4*((c^2*d+e)/e)^(1/2)*ar
ctanh(1/(-c^2*x^2+1)^(1/2))*c^6*d^2*e*x^2+4*((c^2*d+e)/e)^(1/2)*arctanh(1/
(-c^2*x^2+1)^(1/2))*c^6*d^3-ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*
e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*x^2*c^6*d^2*e-ln(-2*(
((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(
-c^2*d*e)^(1/2)))*c^6*d^3-ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(
-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^6*d^2*e*x^2-ln(2*(((c^2
*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d
*e)^(1/2)))*c^6*d^3+8*((c^2*d+e)/e)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))*c^
4*d*e^2*x^2+8*((c^2*d+e)/e)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))*c^4*d^2*e+
2*(-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*c^4*d^2*e-3*ln(-2*(((c^2*d+e)/e)^
(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2
)))*x^2*c^4*d*e^2-3*ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c^2*
d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2*e-3*ln(2*(((c^2*d+e)/
e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1
/2)))*c^4*d*e^2*x^2-3*ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2
*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2*e+4*((c^2*d+e)/e)^(1/
2)*arctanh(1/(-c^2*x^2+1)^(1/2))*e^3*c^2*x^2+4*((c^2*d+e)/e)^(1/2)*arct...
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(115) = 230$.

Time = 0.38 (sec) , antiderivative size = 1346, normalized size of antiderivative = 7.78

$$\int \frac{x^3(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*a*c^4*d^4 + 2*(4*a - b)*c^2*d^3*e + 2*(2*a - b)*d^2*e^2 - 2*(b*c
^2*d*e^3 + b*e^4)*x^4 + 4*(2*a*c^4*d^3*e + (4*a - b)*c^2*d^2*e^2 + (2*a -
b)*d*e^3)*x^2 - (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(
b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(c^2*d*e + e^2)*log((c^4*d^2 + 4*c^2*d*e
 - (c^4*d*e + 2*c^2*e^2)*x^2 + 4*(c^3*d*e + c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)) + 4*e^2 + 2*(c^2*e*x^2 - c^2*d - (c^3*d + 2*c*e)*x*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)) - 2*e)*sqrt(c^2*d*e + e^2))/(e*x^2 + d)) + 4*(b*c^4*d^4
+ 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4
+ 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2
 - 1)/(c^2*x^2)) - 1)/x) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b
*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/
(c^2*x^2)) + 1)/(c*x)) - 2*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e
 + b*c*d^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^4*d^5*e^2 + 2*c^2*d^
4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e
^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 + (4*a - b)*c^2*d^3*
e + (2*a - b)*d^2*e^2 - (b*c^2*d*e^3 + b*e^4)*x^4 + 2*(2*a*c^4*d^3*e + (4*
a - b)*c^2*d^2*e^2 + (2*a - b)*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 + 2*
b*e^3)*x^4 + 2*b*d^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e -
e^2)*arctan((sqrt(-c^2*d*e - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - s
qrt(-c^2*d*e - e^2)*(e*x^2 + d))/((c^2*d*e + e^2)*x^2)) + 2*(b*c^4*d^4 ...
```

3.124.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.124.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^3} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.124.8 Giac [F]

$$\int \frac{x^3 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^3} dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^3}{(ex^2 + d)^3} dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^3, x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^3} dx = \int \frac{x^3 \left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^3} dx$$

input
```
int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.125 $\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

3.125.1 Optimal result

Integrand size = 19, antiderivative size = 217

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx = -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8d(c^2d+e)(d+ex^2)} - \frac{a+b\operatorname{sech}^{-1}(cx)}{4e(d+ex^2)^2} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{4d^2e} - \frac{b(3c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\,(c^2d+e)^{3/2}}$$

output
```
1/4*(-a-b*arcsech(c*x))/e/(e*x^2+d)^2+1/4*b*arctanh((-c^2*x^2+1)^(1/2))*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d^2/e-1/8*b*(3*c^2*d+2*e)*arctanh(e^(1/2)*(-
c^2*x^2+1)^(1/2)/(c^2*d+e)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d^2/(c^2
*d+e)^(3/2)/e^(1/2)-1/8*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/
2)/d/(c^2*d+e)/(e*x^2+d)
```

3.125.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.24

$$\int \frac{x(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{1}{16}\left(-\frac{4a}{e(d+ex^2)^2}-\frac{2\sqrt{\frac{1-cx}{1+cx}}(b+bcx)}{d(c^2d+e)(d+ex^2)}-\frac{4b\operatorname{sech}^{-1}(cx)}{e(d+ex^2)^2}-\frac{4b\log(x)}{d^2e}+\frac{4b\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{d^2e}-\frac{b(3c^2d+2e)\log\left(\frac{16d^2\sqrt{e}\sqrt{c^2d+e}\left(\sqrt{e}-ic^2\sqrt{d}x+\sqrt{c^2d+e}\sqrt{\frac{1-cx}{1+cx}}+c\sqrt{c^2d+e}x\sqrt{\frac{1-cx}{1+cx}}\right)}{b(3c^2d+2e)\left(-i\sqrt{d}+\sqrt{e}x\right)}\right)}{d^2\sqrt{e}(c^2d+e)^{3/2}}-\frac{b(3c^2d+2e)\log\left(\frac{16d^2\sqrt{e}\sqrt{c^2d+e}\left(\sqrt{e}+ic^2\sqrt{d}x+\sqrt{c^2d+e}\sqrt{\frac{1-cx}{1+cx}}+c\sqrt{c^2d+e}x\sqrt{\frac{1-cx}{1+cx}}\right)}{b(3c^2d+2e)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d^2\sqrt{e}(c^2d+e)^{3/2}}\right)$$

input
```
Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

output
```
((-4*a)/(e*(d + e*x^2)^2) - (2*Sqrt[(1 - c*x)/(1 + c*x)]*(b + b*c*x))/(d*(
c^2*d + e)*(d + e*x^2)) - (4*b*ArcSech[c*x])/(e*(d + e*x^2)^2) - (4*b*Log[
x])/(d^2*e) + (4*b*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/
(1 + c*x)]])/(d^2*e) - (b*(3*c^2*d + 2*e)*Log[(16*d^2*Sqrt[e]*Sqrt[c^2*d +
 e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)]
 + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/(b*(3*c^2*d + 2*e)*((-I
)*Sqrt[d] + Sqrt[e]*x))])/(d^2*Sqrt[e]*(c^2*d + e)^(3/2)) - (b*(3*c^2*d +
2*e)*Log[(16*d^2*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt
[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)
/(1 + c*x)]))/(b*(3*c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d^2*Sqrt[e]*(
c^2*d + e)^(3/2)))/16
```

3.125.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6853, 2036, 354, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{ 6853}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}(ex^2+d)^2}dx}{4e}-\frac{a+b\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 2036}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x\sqrt{1-c^2x^2}(ex^2+d)^2}dx}{4e}-\frac{a+b\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x^2\sqrt{1-c^2x^2}(ex^2+d)^2}dx^2}{8e}-\frac{a+b\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{-ex^2c^2+2dc^2+2e}{2x^2\sqrt{1-c^2x^2}\left(ex^2+d\right)}dx^2}{d(c^2d+e)}+\frac{e\sqrt{1-c^2x^2}}{d(c^2d+e)(d+ex^2)}\right)}{8e}-\frac{a+b\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{2\left(dc^2+e\right)-c^2ex^2}{x^2\sqrt{1-c^2x^2}\left(ex^2+d\right)}dx^2}{2d(c^2d+e)}+\frac{e\sqrt{1-c^2x^2}}{d(c^2d+e)(d+ex^2)}\right)}{8e}-\frac{a+b\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 174}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{2(c^2d+e)\int\frac{1}{x^2\sqrt{1-c^2x^2}}dx^2}{d}-\frac{e(3c^2d+2e)\int\frac{1}{\sqrt{1-c^2x^2}(ex^2+d)}dx^2}{d}}{2d(c^2d+e)}+\frac{e\sqrt{1-c^2x^2}}{d(c^2d+e)(d+ex^2)}\right)}{8e}-\frac{a+b\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$\Big\downarrow$ 73

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{2e(3c^2d+2e)\int\frac{1}{-\frac{ex^4}{c^2}+d+\frac{e}{c^2}}d\sqrt{1-c^2x^2}}{c^2d}-\frac{4(c^2d+e)\int\frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2d}}{2d(c^2d+e)}+\frac{e\sqrt{1-c^2x^2}}{d(c^2d+e)(d+ex^2)}\right)}{8e}-\frac{a+b\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$\Big\downarrow$ 221

$$-\frac{a+b\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{2\sqrt{e}(3c^2d+2e)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{d\sqrt{c^2d+e}}-\frac{4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)(c^2d+e)}{d}}{2d(c^2d+e)}+\frac{e\sqrt{1-c^2x^2}}{d(c^2d+e)(d+ex^2)}\right)}{8e}$$

input
```
Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a + b*ArcSech[c*x])/(e*(d + e*x^2)^2) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt
[1 + c*x]*((e*Sqrt[1 - c^2*x^2])/(d*(c^2*d + e)*(d + e*x^2)) + ((-4*(c^2*d
 + e)*ArcTanh[Sqrt[1 - c^2*x^2]])/d + (2*Sqrt[e]*(3*c^2*d + 2*e)*ArcTanh[(
Sqrt[e]*Sqrt[1 - c^2*x^2])/Sqrt[c^2*d + e]])/(d*Sqrt[c^2*d + e]))/(2*d*(c^
2*d + e))))/(8*e)
```

3.125.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.125. $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2036
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

3.125. $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

rule 6853
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Simp[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x
^2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e
, p}, x] && NeQ[p, -1]
```

3.125.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1317 vs. 2(186) = 372.

Time = 5.10 (sec) , antiderivative size = 1318, normalized size of antiderivative = 6.07

method	result	size
parts	Expression too large to display	1318
derivativedivides	Expression too large to display	1329
default	Expression too large to display	1329

input
```
int(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/e/(e*x^2+d)^2+b/c^2*(-1/4*c^6/e/(c^2*e*x^2+c^2*d)^2*arcsech(c*x)-1/
16*c^3*e^2*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(4*((c^2*d+e)/e)^(1/
2)*arctanh(1/(-c^2*x^2+1)^(1/2))*c^6*d^2*e*x^2+4*((c^2*d+e)/e)^(1/2)*arcta
nh(1/(-c^2*x^2+1)^(1/2))*c^6*d^3-3*ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)
^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*x^2*c^6*d^2*e-
3*ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/
(-c*e*x+(-c^2*d*e)^(1/2)))*c^6*d^3-3*ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1
)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^6*d^2*e*x^2-
3*ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(
c*e*x+(-c^2*d*e)^(1/2)))*c^6*d^3+8*((c^2*d+e)/e)^(1/2)*arctanh(1/(-c^2*x^2
+1)^(1/2))*c^4*d*e^2*x^2+8*((c^2*d+e)/e)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2
))*c^4*d^2*e-2*(-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*c^4*d^2*e-5*ln(-2*((
(c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-
c^2*d*e)^(1/2)))*x^2*c^4*d*e^2-5*ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(
1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2*e-5*ln(2
*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+
(-c^2*d*e)^(1/2)))*c^4*d*e^2*x^2-5*ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^
(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2*e+4*((c^
2*d+e)/e)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))*e^3*c^2*x^2+4*((c^2*d+e)/e)^
(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))*c^2*d*e^2-2*(-c^2*x^2+1)^(1/2)*((c^...
```

3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(135) = 270.

Time = 0.37 (sec) , antiderivative size = 1232, normalized size of antiderivative = 5.68

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*a*c^4*d^4 + 2*(4*a + b)*c^2*d^3*e + 2*(2*a + b)*d^2*e^2 + 2*(b*c
^2*d*e^3 + b*e^4)*x^4 + 4*(b*c^2*d^2*e^2 + b*d*e^3)*x^2 - (3*b*c^2*d^3 + (
3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x
^2)*sqrt(c^2*d*e + e^2)*log((c^4*d^2 + 4*c^2*d*e - (c^4*d*e + 2*c^2*e^2)*x
^2 + 4*(c^3*d*e + c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 4*e^2 + 2*(c^2
*e*x^2 - c^2*d - (c^3*d + 2*c*e)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*e)*s
qrt(c^2*d*e + e^2))/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^
2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2
*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) +
 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c
^2*x^2)) + 1)/(c*x)) + 2*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e +
 b*c*d^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^4*d^6*e + 2*c^2*d^5*e^
2 + d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2
 + 2*c^2*d^4*e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 + (4*a + b)*c^2*d^3*e
+ (2*a + b)*d^2*e^2 + (b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^2*d^2*e^2 + b*d*e
^3)*x^2 + (3*b*c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*
b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e - e^2)*arctan((sqrt(-c^2*d*e -
 e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(-c^2*d*e - e^2)*(e*x^2 +
 d))/((c^2*d*e + e^2)*x^2)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (
b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d...
```

3.125.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*asech(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.125.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.125.8 Giac [F]

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x}{(ex^2+d)^3}\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^3, x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.126 $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)^3}\,dx$

3.126.1 Optimal result

Integrand size = 21, antiderivative size = 741

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = -\frac{be\left(c^2-\frac{1}{x^2}\right)}{8cd^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} + \frac{e^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2bd^3} + \frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - \frac{b\sqrt{e}(c^2d+2e)\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{8d^3\left(c^2d+e\right)^{3/2}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

output
```
1/4*e^2*(a+b*arcsech(c*x))/d^3/(e+d/x^2)^2-e*(a+b*arcsech(c*x))/d^3/(e+d/x
^2)+1/2*(a+b*arcsech(c*x))^2/b/d^3-1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-
1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-
1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/
x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/
d^3-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/2*b*polylog(2,-c*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2
*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2
)-(c^2*d+e)^(1/2)))/d^3-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/2*b*polylog(2,c*(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))
/d^3-1/8*b*e*(c^2-1/x^2)/c/d^2/(c^2*d+e)/(e+d/x^2)/x/(-1+1/c/x)^(1/2)/(1+1
/c/x)^(1/2)-1/8*b*(c^2*d+2*e)*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/c^
2/x^2)^(1/2))*e^(1/2)*(-1+1/c^2/x^2)^(1/2)/d^3/(c^2*d+e)^(3/2)/(-1+1/c/x)^
(1/2)/(1+1/c/x)^(1/2)+b*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/c^2/x^2)
^(1/2))*e^(1/2)*(-1+1/c^2/x^2)^(1/2)/d^3/(c^2*d+e)^(1/2)/(-1+1/c/x)^(1/2)/
(1+1/c/x)^(1/2)
```

3.126.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.09 (sec) , antiderivative size = 2054, normalized size of antiderivative = 2.77

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\,(d + ex^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^3),x]
```

output
```
a/(4*d*(d + e*x^2)^2) + a/(2*d^2*(d + e*x^2)) + (a*Log[x])/d^3 - (a*Log[d
+ e*x^2])/(2*d^3) + b*((Sqrt[e]*(((-I)*Sqrt[e]*Sqrt[(1 - c*x)/(1 + c*x)]*(
1 + c*x))/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcSech[c*x]/
(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + Sqrt
[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2*c
^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + S
qrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c
*x)/(1 + c*x)]))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d +
e)^(3/2))))/(16*d^2) + (Sqrt[e]*((I*Sqrt[e]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
 c*x))/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcSech[c*x]/(Sqrt[
e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + Sqrt[(1 - c*x
)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2*c^2*d + e)
*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d
 + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 +
c*x)]))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/
(16*d^2) - (((5*I)/16)*Sqrt[e]*(-(ArcSech[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x))
+ (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c
*x)/(1 + c*x)]]/Sqrt[e] + Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 +
c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[
d] + Sqrt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d]))/d^(5/2) + (((5*I)/16)*Sqrt...
```

3.126.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 801, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6857, 6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{6857}$$

$$-\int \frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3x^5}d\frac{1}{x}$$

$$\Big\downarrow \text{6374}$$

$$-\int \left(\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)e^2}{d^2\left(\frac{d}{x^2}+e\right)^3x}-\frac{2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)e}{d^2\left(\frac{d}{x^2}+e\right)^2x}+\frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{d^2\left(\frac{d}{x^2}+e\right)x}\right)d\frac{1}{x}$$

$\downarrow$ 2009

$$\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)e^2}{4d^3\left(\frac{d}{x^2}+e\right)^2}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)e}{d^3\left(\frac{d}{x^2}+e\right)}-\frac{b\left(c^2-\frac{1}{x^2}\right)e}{8cd^2\left(dc^2+e\right)\left(\frac{d}{x^2}+e\right)\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}x}-\frac{b\left(dc^2+2e\right)\sqrt{\frac{1}{c^2x^2}-1}\text{arctanh}\left(\frac{\sqrt{dc^2+e}}{c\sqrt{e}\sqrt{\frac{1}{c^2x^2}-1}x}\right)\sqrt{e}}{8d^3\left(dc^2+e\right)^{3/2}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}+\frac{b\sqrt{\frac{1}{c^2x^2}-1}\text{arctanh}\left(\frac{\sqrt{dc^2+e}}{c\sqrt{e}\sqrt{\frac{1}{c^2x^2}-1}x}\right)\sqrt{e}}{d^3\sqrt{dc^2+e}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}+\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)^2}{2bd^3}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{2d^3}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{2d^3}-\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3}-\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3}$$

input
```
Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^3),x]
```

3.126. $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)^3}\,dx$

output
```
-1/8*(b*e*(c^2 - x^(-2)))/(c*d^2*(c^2*d + e)*(e + d/x^2)*Sqrt[-1 + 1/(c*x)
]*Sqrt[1 + 1/(c*x)]*x) + (e^2*(a + b*ArcCosh[1/(c*x)]))/(4*d^3*(e + d/x^2)
^2) - (e*(a + b*ArcCosh[1/(c*x)]))/(d^3*(e + d/x^2)) + (a + b*ArcCosh[1/(c
*x)])^2/(2*b*d^3) + (b*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d +
 e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(d^3*Sqrt[c^2*d + e]*Sqrt[-1 +
1/(c*x)]*Sqrt[1 + 1/(c*x)]) - (b*Sqrt[e]*(c^2*d + 2*e)*Sqrt[-1 + 1/(c^2*x^
2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(8*d^3*
(c^2*d + e)^(3/2)*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - ((a + b*ArcCosh[
1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d +
e])])/(2*d^3) - ((a + b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/
(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcCosh[1/(c*x)])*
Log[1 - (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d
^3) - ((a + b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(S
qrt[e] + Sqrt[c^2*d + e])])/(2*d^3) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCos
h[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*d^3) - (b*PolyLog[2, (c*Sqrt
[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - (b*PolyLo
g[2, -((c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*d
^3) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] + Sqrt[c^2*d
+ e])])/(2*d^3)
```

3.126.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.126. $\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^3}\,dx$

3.126.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.33 (sec) , antiderivative size = 3727, normalized size of antiderivative = 5.03

method	result	size
parts	Expression too large to display	3727
derivativedivides	Expression too large to display	3801
default	Expression too large to display	3801

input
```
int((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d^3*ln(e*x^2+d)+1/4*a/d/(e*x^2+d)^2+1/2*a/d^2/(e*x^2+d)+a/d^3*ln(x)
+b*(-2*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e
)*e/(c^4*d^2+2*c^2*d*e+e^2)/d^4/c^2*arcsech(c*x)^2+(-c^2*d*(e*(c^2*d+e))^(
1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*e/(c^4*d^2+2*c^2*d*e+e^2)/d^
4/c^2*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2
*(e*(c^2*d+e))^(1/2)-2*e))-3/4*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^4/(c^2*
d+e)/c^2*e*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^
2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^
2-2*(e*(c^2*d+e))^(1/2)*e)/(c^4*d^2+2*c^2*d*e+e^2)*e^2/d^5/c^4*arcsech(c*x
)^2-1/4*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d/e*c^4*arcsech(c*x)^2+1/8*(e*(c^2
*d+e))^(1/2)/(c^2*d+e)^2/d/e*c^4*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(
1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+1/2*(-c^2*d*(e*(c^2*
d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/(c^4*d^2+2*c^2*d*e+e^
2)*e^2/d^5/c^4*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/
(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))+1/2*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^
3*e*arcsech(c*x)*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c
^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^5/(c^
2*d+e)/c^4*e^2*arcsech(c*x)^2+3/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^4/(c
^2*d+e)/c^2*e*arcsech(c*x)^2+3/4*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^2*c^2*a
rcsech(c*x)*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2...
```

3.126.5 Fricas [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x
), x)
```

3.126.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asech(c*x))/x/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.126.7 Maxima [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d
)/d^3 + 4*log(x)/d^3) + b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1
) + 1/(c*x))/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

3.126.8 Giac [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\,\text{arsech}\left(cx\right)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^3*x), x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^3}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^3),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^3), x)
```

3.127 $\int \frac{x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

3.127.1 Optimal result

Integrand size = 21, antiderivative size = 1272

$$
\begin{aligned}
\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx =\ & \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
& + \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} \\
& + \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
& - \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2} \\
& - \frac{bd\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}e} \\
& - \frac{3b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2} \\
& - \frac{bd\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}e} \\
& + \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
& - \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
& + \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
& - \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
& - \frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
& + \frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
& - \frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}
\end{aligned}
$$

output
```
3/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arcsech(c
*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(
c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^
(5/2)/(-d)^(1/2)-3/16*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/
16*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*b*polylog(2,c*(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/
(-d)^(1/2)-3/16*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*b*polylog(2,c*
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/
2)))/e^(5/2)/(-d)^(1/2)-1/8*b*d*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(1/2)*e^(
1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/e/(c*d-(-d)^(
1/2)*e^(1/2))^(3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)-1/8*b*d*arctan((1+1/c/x
)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(1/2)*e^
(1/2))^(1/2))/e/(c*d-(-d)^(1/2)*e^(1/2))^(3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3
/2)+1/16*(a+b*arcsech(c*x))*(-d)^(1/2)/e^(3/2)/(-d/x+(-d)^(1/2)*e^(1/2))^2
+3/16*(a+b*arcsech(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arcsec...
```

3.127.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.11 (sec) , antiderivative size = 2022, normalized size of antiderivative = 1.59

$$\int \frac{x^4(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

output
```
(a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*(((I/16)*Sqrt[d]*(((-I)*Sqrt
[e]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d
] + Sqrt[e]*x)) - ArcSech[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + Lo
g[x]/(d*Sqrt[e]) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/
(1 + c*x)]]/(d*Sqrt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]
*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] +
c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*((-I)*Sqrt[
d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/e^2 - ((I/16)*Sqrt[d]*((I*Sqrt[
e]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] +
Sqrt[e]*x)) - ArcSech[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d
*Sqrt[e]) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*
x)]]/(d*Sqrt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[
e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[
c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[
e]*x))])/(d*(c^2*d + e)^(3/2))))/e^2 + (5*(-(ArcSech[c*x]/(I*Sqrt[d]*Sqrt[
e] + e*x)) + (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*
Sqrt[(1 - c*x)/(1 + c*x)]]/Sqrt[e] + Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 -
 c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]
))/(I*Sqrt[d] + Sqrt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d]))/(16*e^2) + (5*(...
```

3.127.3 Rubi [A] (verified)

Time = 2.52 (sec) , antiderivative size = 1336, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6857, 6324, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{ 6857}$$

$$-\int \frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 6324}$$

$$-\int \left(-\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)d^3}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^3}-\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)d^3}{8(-d)^{3/2}e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^3}-\frac{3\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)d}{8e^2\left(-\frac{d^2}{x^2}-ed\right)}-\frac{3\left(a+b\text{arccosh}\right.}{16e^2\left(\sqrt{-d}\sqrt{e}\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sqrt{-d}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16e^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{-d}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16e^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{3\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{3\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{\sqrt{-d}\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{\sqrt{-d}\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)}{16e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}-\frac{bd\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}e}-\frac{3b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}-\frac{bd\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}e}-\frac{3b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}+\frac{3\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}}+\frac{3\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}}-\frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}+\frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}+\frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

input
```
Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

3.127. $\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

output
```
(b*c*Sqrt[-d]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*e^(3/2)*(c^2*d + e
)*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[-d]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/
(c*x)])/(16*e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a +
 b*ArcCosh[1/(c*x)]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*
ArcCosh[1/(c*x)]))/(16*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[-d]*(a + b*Ar
cCosh[1/(c*x)]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (3*(a + b*ArcCo
sh[1/(c*x)]))/(16*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (3*b*ArcTan[(Sqrt[c*d -
Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1
 + 1/(c*x)])])/(8*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]
]*e^2) - (b*d*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqr
t[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*(c*d - Sqrt[-d]*Sqrt[e]
)^(3/2)*(c*d + Sqrt[-d]*Sqrt[e])^(3/2)*e) - (3*b*ArcTan[(Sqrt[c*d + Sqrt[-
d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(
c*x)])])/(8*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2)
 - (b*d*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d
- Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*(c*d - Sqrt[-d]*Sqrt[e])^(3/2
)*(c*d + Sqrt[-d]*Sqrt[e])^(3/2)*e) + (3*(a + b*ArcCosh[1/(c*x)])*Log[1 -
(c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]
*e^(5/2)) - (3*(a + b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c
*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*A...
```

3.127.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6324
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.127. $\int \frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

3.127.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 110.96 (sec) , antiderivative size = 1960, normalized size of antiderivative = 1.54

method	result	size
parts	Expression too large to display	1960
derivativedivides	Expression too large to display	1983
default	Expression too large to display	1983

input
```
int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*((-5/8/e*x^3-3/8*d/e^2*x)/(e*x^2+d)^2+3/8/e^2/(d*e)^(1/2)*arctan(e*x/(d*
e)^(1/2)))+b/c^5*(-1/8*x*c^7*(-(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c^
3*d*e*x-(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*e^2*c^3*x^3+3*d^2*c^4*arc
sech(c*x)+5*c^4*d*e*arcsech(c*x)*x^2+3*c^2*d*e*arcsech(c*x)+5*e^2*arcsech(
c*x)*c^2*x^2)/e^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2-3/8*(-(c^2*d-2*(e*(c^2*d+e
))^(1/2)+2*e)*d)^(1/2)*(c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2+2*(e*(c^
2*d+e))^(1/2)*e)*c^3*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/
((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e^2/d^2-3/8*((c^
2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*
d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*c^3*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/
e^2/d^2+1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*
d+e))^(1/2)+2*e)*c*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((
-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/e/d^3-1/2*(-(c^2*d-2
*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*
e^2+2*(e*(c^2*d+e))^(1/2)*e)*c*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^3+
1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/
2)+2*e)*c*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e
*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/e/d^3-1/2*((c^2*d+2*(e*(c^2*...
```

3.127.5 Fricas [F]

$$\int \frac{x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx=\int \frac{(b\,\mathrm{arsech}\,(cx)+a)x^4}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsech(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
 + d^3), x)
```

3.127.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.127.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.127.8 Giac [F]

$$\int \frac{x^4(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x^4}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^3, x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^4\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.128 $\int \frac{x^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

3.128.1 Optimal result

Integrand size = 21, antiderivative size = 1276

$$\int \frac{x^2(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{a+b\operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\operatorname{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{a+b\operatorname{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}} - \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e} + \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}} - \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e} - \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

output
```
-1/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(
-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arcsech(
c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x
+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(
-d)^(3/2)/e^(3/2)+1/16*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(
1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1
/16*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*b*polylog(2,c*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3
/2)/e^(3/2)+1/16*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*b*polylog(2,c
*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1
/2)))/(-d)^(3/2)/e^(3/2)+1/8*b*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(1/2)*e^(1
/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/(c*d-(-d)^(1/2
)*e^(1/2))^(3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)+1/8*b*arctan((1+1/c/x)^(1/
2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(1/2)*e^(1/2)
)^(1/2))/(c*d-(-d)^(1/2)*e^(1/2))^(3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)+1/1
6*(a+b*arcsech(c*x))/(-d)^(1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))^2+1/16*(
a+b*arcsech(c*x))/d/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*(-a-b*arcsech(c*x)...
```

3.128.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.11 (sec) , antiderivative size = 2030, normalized size of antiderivative = 1.59

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(((-1/16*I)*(((-I)*Sqrt[e]*Sqrt[(
1 - c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e
]*x)) - ArcSech[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sq
rt[e]) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]
]/(d*Sqrt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e]
- I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2
*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[
e]*x))])/(d*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) + ((I/16)*((I*Sqrt[e]*Sqrt[(1
 - c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)
) - ArcSech[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e])
- Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sq
rt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2
*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]
*x*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(
d*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) - (-(ArcSech[c*x]/(I*Sqrt[d]*Sqrt[e] +
e*x)) + (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[
(1 - c*x)/(1 + c*x)]]/Sqrt[e] + Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)
/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I
*Sqrt[d] + Sqrt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d])/(16*d*e) - (-(ArcSech...
```

3.128.3 Rubi [A] (verified)

Time = 3.68 (sec) , antiderivative size = 1340, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6857, 6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3} \, dx$$

$$\Big\downarrow \ 6857$$

$$-\int \frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3 x^2}d\frac{1}{x}$$

$$\Big\downarrow \ 6374$$

$$-\int \left(\frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2}+e\right)^2}-\frac{e\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)^3}\right)d\frac{1}{x}$$

↓ 2009

$$\frac{b\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16\sqrt{-d}\sqrt{e}\,(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16\sqrt{-d}\sqrt{e}\,(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)}{16\sqrt{-d}\sqrt{e}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}-\frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e}+\frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}}-\frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e}+\frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}}-\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{\left(a+b\operatorname{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

3.128. $\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

output
```
(b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e
)*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(
16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcCosh
[1/(c*x)])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcCo
sh[1/(c*x)])/(16*d*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCosh[1/(c*x)])/
(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (a + b*ArcCosh[1/(c*x)]
)/(16*d*e*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[
e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])
/(8*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sqrt[-d]*Sqrt[e])^(3/2)) - (b*Ar
cTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]
*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*d*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d
 + Sqrt[-d]*Sqrt[e]]*e) + (b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 +
 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*(c*d - S
qrt[-d]*Sqrt[e])^(3/2)*(c*d + Sqrt[-d]*Sqrt[e])^(3/2)) - (b*ArcTan[(Sqrt[c
*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sq
rt[-1 + 1/(c*x)])])/(8*d*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*
Sqrt[e]]*e) - ((a + b*ArcCosh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcCosh[1/(c
*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcC
osh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*
d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcCosh[1/(c*x)])*Log[1 - (...
```

3.128.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6857
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.128. $\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

3.128.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 150.99 (sec) , antiderivative size = 1398, normalized size of antiderivative = 1.10

method	result	size
parts	Expression too large to display	1398
derivativedivides	Expression too large to display	1417
default	Expression too large to display	1417

input
```
int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*((1/8/d*x^3-1/8/e*x)/(e*x^2+d)^2+1/8/e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1
/2)))+b/c^3*(1/8*x*c^5*(c^4*d*e*arcsech(c*x)*x^2-d^2*c^4*arcsech(c*x)-(-(c
*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*e^2*c^3*x^3-(-(c*x-1)/c/x)^(1/2)*((c*
x+1)/c/x)^(1/2)*c^3*d*e*x+e^2*arcsech(c*x)*c^2*x^2-c^2*d*e*arcsech(c*x))/d
/e/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2-1/16/d/(c^2*d+e)*c^4*sum(_R1/(_R1^2*c^2*d
+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/
_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c
^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/16/(c^2*d+e)/e*c^6*sum(_R1/(_R1^2*c
^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/
2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=Root
Of(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/16/d/(c^2*d+e)*c^4*sum(1/_R1/(_
R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R
1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/8*(-(c^2*d-2*(e*(c^2*d+e)
)^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctanh(c*d*(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d
)^(1/2))/(c^2*d+e)/e/d^3-1/8*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*
(c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2+2*(e*(c^2*d+e))^(1/2)*e)*c*arct
anh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^
(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^3+1/8*((c^2*d+2*(e*(c^2*d+e))^(1/2...
```

3.128.5 Fricas [F]

$$\int \frac{x^2 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{\left(b\,\text{arsech}\,(cx)+a\right)x^2}{\left(ex^2+d\right)^3}\,dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsech(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
 + d^3), x)
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.128.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.128.8 Giac [F]

$$\int \frac{x^2 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^2}{\left(ex^2+d\right)^3}\,dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^3, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^3}\,dx$$

input
```
int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.129 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx$

3.129.1 Optimal result

Integrand size = 18, antiderivative size = 1272

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx = \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

$$+\frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}$$

$$-\frac{5\left(a+b\text{sech}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$+\frac{5\left(a+b\text{sech}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{5b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$

$$-\frac{be\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}}$$

$$+\frac{5b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$

$$-\frac{be\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}}$$

$$+\frac{3\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

$$-\frac{3\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

$$+\frac{3\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

$$-\frac{3\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

$$-\frac{3b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

$$-\frac{3b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

output
```
3/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arcsech(c
*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(
c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-
d)^(5/2)/e^(1/2)-3/16*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/
16*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*polylog(2,c*(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/
2)/e^(1/2)-3/16*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*polylog(2,c*
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/
2)))/(-d)^(5/2)/e^(1/2)-1/8*b*e*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(1/2)*e^(
1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/d/(c*d-(-d)^(
1/2)*e^(1/2))^(3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)-1/8*b*e*arctan((1+1/c/x
)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(1/2)*e^
(1/2))^(1/2))/d/(c*d-(-d)^(1/2)*e^(1/2))^(3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3
/2)+1/16*(a+b*arcsech(c*x))*e^(1/2)/(-d)^(3/2)/(-d/x+(-d)^(1/2)*e^(1/2))^2
-5/16*(a+b*arcsech(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arcsec...
```

3.129.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.07 (sec) , antiderivative size = 2015, normalized size of antiderivative = 1.58

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^3,x]
```

output
```
(a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*(((I/16)*(((-I)*Sqrt[e]*Sqrt[(1
- c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*
x)) - ArcSech[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt
[e]) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/
(d*Sqrt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] -
I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d
 + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]
*x))])/(d*(c^2*d + e)^(3/2))))/d^(3/2) - ((I/16)*((I*Sqrt[e]*Sqrt[(1 - c*x
)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - Ar
cSech[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[
1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e])
 + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[
d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqr
t[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2
*d + e)^(3/2))))/d^(3/2) - (3*(-(ArcSech[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) +
 (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*
x)/(1 + c*x)]]/Sqrt[e] + Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c
*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d
] + Sqrt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d]))/(16*d^2) - (3*(-(ArcSech[c*...
```

3.129.3 Rubi [A] (verified)

Time = 4.45 (sec) , antiderivative size = 1336, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6847, 6374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{ 6847}$$

$$-\int \frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3x^4}d\frac{1}{x}$$

$$\Big\downarrow \text{ 6374}$$

$$-\int \left(\frac{\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)e^2}{d^2\left(\frac{d}{x^2}+e\right)^3}-\frac{2\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)e}{d^2\left(\frac{d}{x^2}+e\right)^2}+\frac{a+b\text{arccosh}\left(\frac{1}{cx}\right)}{d^2\left(\frac{d}{x^2}+e\right)}\right)d\frac{1}{x}$$

$\Big\downarrow$ 2009

$$\frac{b\sqrt{e}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16(-d)^{3/2}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{e}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16(-d)^{3/2}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{5\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{5\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{\sqrt{e}\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{\sqrt{e}\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)}{16(-d)^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}-\frac{be\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8d\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}}+\frac{5b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}-\frac{be\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8d\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}}+\frac{5b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}+\frac{3\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3\left(a+b\text{arccosh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arccosh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

input
```
Int[(a + b*ArcSech[c*x])/(d + e*x^2)^3,x]
```

3.129. $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx$

output
```
(b*c*Sqrt[e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*(-d)^(3/2)*(c^2*d +
 e)*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1
/(c*x)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(
a + b*ArcCosh[1/(c*x)]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(
a + b*ArcCosh[1/(c*x)]))/(16*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[e]*(a +
 b*ArcCosh[1/(c*x)]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) + (5*(a +
 b*ArcCosh[1/(c*x)]))/(16*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (5*b*ArcTan[(Sqr
t[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]
*Sqrt[-1 + 1/(c*x)])])/(8*d^2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt
[-d]*Sqrt[e]]) - (b*e*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x
)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*d*(c*d - Sqrt[-
d]*Sqrt[e])^(3/2)*(c*d + Sqrt[-d]*Sqrt[e])^(3/2)) + (5*b*ArcTan[(Sqrt[c*d
+ Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[
-1 + 1/(c*x)])])/(8*d^2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*S
qrt[e]]) - (b*e*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(S
qrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*d*(c*d - Sqrt[-d]*Sqr
t[e])^(3/2)*(c*d + Sqrt[-d]*Sqrt[e])^(3/2)) + (3*(a + b*ArcCosh[1/(c*x)])*
Log[1 - (c*Sqrt[-d]*E^ArcCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*
(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCosh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^Ar
cCosh[1/(c*x)])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) +...
```

3.129.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6374
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6847
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

3.129. $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx$

3.129.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 181.43 (sec) , antiderivative size = 1950, normalized size of antiderivative = 1.53

method	result	size
parts	Expression too large to display	1950
derivativedivides	Expression too large to display	1975
default	Expression too large to display	1975

input
```
int((a+b*arcsech(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x/d/(e*x^2+d)^2+3/8*a/d^2*x/(e*x^2+d)+3/8*a/d^2/(d*e)^(1/2)*arctan(e
*x/(d*e)^(1/2))+b/c*(1/8*x*c^3*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c
^3*d*e*x+(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*e^2*c^3*x^3+5*d^2*c^4*ar
csech(c*x)+3*c^4*d*e*arcsech(c*x)*x^2+5*c^2*d*e*arcsech(c*x)+3*e^2*arcsech
(c*x)*c^2*x^2)/d^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)+5/8*(-(c^2*d-2*(e*(c^2*d+
e))^(1/2)+2*e)*d)^(1/2)*(c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2+2*(e*(c
^2*d+e))^(1/2)*e)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-
c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)^2/c+5/8*((c^2*d+2
*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2
*e^2-2*(e*(c^2*d+e))^(1/2)*e)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)^2/c-1/2
*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)
+2*e)*e*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e
*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+e)/c^3+1/2*(-(c^2*d-2*(e*(c^2*
d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2+2*(e*
(c^2*d+e))^(1/2)*e)*e*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+e)^2/c^3-1/2*((c
^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)
*e*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d
+e))^(1/2)+2*e)*d)^(1/2))/d^5/(c^2*d+e)/c^3+1/2*((c^2*d+2*(e*(c^2*d+e))...
```

3.129.5 Fricas [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx = \int \frac{b\,\text{arsech}\,(cx)+a}{(ex^2+d)^3}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
 x)
```

3.129.6 Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx = \int \frac{a+b\,\text{asech}\,(cx)}{(d+ex^2)^3}\,dx$$

input
```
integrate((a+b*asech(c*x))/(e*x**2+d)**3,x)
```

output
```
Integral((a + b*asech(c*x))/(d + e*x**2)**3, x)
```

3.129.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.129.8 Giac [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx = \int \frac{b\,\text{arsech}\,(cx)+a}{(ex^2+d)^3}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^3, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx = \int \frac{a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((a + b*acosh(1/(c*x)))/(d + e*x^2)^3, x)
```

3.130 $\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.130.1 Optimal result

Integrand size = 23, antiderivative size = 447

$$\begin{aligned}
&\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx \\
&= \frac{b(23c^4d^2+12c^2de-75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} \\
&\quad+\frac{b(29c^2d-25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e^2} \\
&\quad-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e^2}+\frac{d^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3} \\
&\quad-\frac{2d(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}+\frac{(d+ex^2)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3} \\
&\quad-\frac{b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{1680c^7e^{5/2}} \\
&\quad-\frac{8bd^{7/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{105e^3}
\end{aligned}$$

output
```
1/3*d^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^3-2/5*d*(e*x^2+d)^(5/2)*(a+b*
arcsech(c*x))/e^3+1/7*(e*x^2+d)^(7/2)*(a+b*arcsech(c*x))/e^3-1/1680*b*(105
*c^6*d^3-35*c^4*d^2*e+63*c^2*d*e^2+75*e^3)*arctan(e^(1/2)*(-c^2*x^2+1)^(1/
2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^7/e^(5/2)-8/105*b*
d^(7/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1
/2)*(c*x+1)^(1/2)/e^3+1/840*b*(29*c^2*d-25*e)*(e*x^2+d)^(3/2)*(1/(c*x+1))^
(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^2-1/42*b*(e*x^2+d)^(5/2)*(1/(
c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2+1/1680*b*(23*c^4*d^
2+12*c^2*d*e-75*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e
*x^2+d)^(1/2)/c^6/e^2
```

3.130.2 Mathematica [A] (verified)

Time = 37.75 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int x^5\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{\sqrt{d+ex^2}\left(16ac^6(8d^3-4d^2ex^2+3de^2x^4+15e^3x^6)-be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(75e^2+2c^2e(19d+25ex^2)+c^4(-\right.\right.}{1680c^6e^3}$$
$$-\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{-1+c^2x^2}\left(128c^7d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\arctan\right.}{1680c^7e^3(-1+cx)}$$

input
```
Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

output
```
(Sqrt[d + e*x^2]*(16*a*c^6*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6
) - b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(75*e^2 + 2*c^2*e*(19*d + 25*e
*x^2) + c^4*(-41*d^2 + 22*d*e*x^2 + 40*e^2*x^4)) + 16*b*c^6*(8*d^3 - 4*d^2
*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6)*ArcSech[c*x]))/(1680*c^6*e^3) - (b*Sqrt
[(1 - c*x)/(1 + c*x)]*Sqrt[-1 + c^2*x^2]*(128*c^7*d^(7/2)*ArcTan[(Sqrt[d]*
Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e]*(105*c^6*d^3 - 35*c^4*d^2*e
 + 63*c^2*d*e^2 + 75*e^3)*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d +
 e*x^2])]))/(1680*c^7*e^3*(-1 + c*x))
```

3.130.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.85, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {6855, 27, 7282, 2118, 27, 171, 27, 171, 27, 175, 66, 104, 218, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)dx$$

$$\Big\downarrow \text{6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\left(ex^2+d\right)^{3/2}\left(15e^2x^4-12dex^2+8d^2\right)}{105e^3x\sqrt{1-c^2x^2}}dx+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\left(ex^2+d\right)^{3/2}\left(15e^2x^4-12dex^2+8d^2\right)}{x\sqrt{1-c^2x^2}}dx}{105e^3}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \text{7282}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\left(ex^2+d\right)^{3/2}\left(15e^2x^4-12dex^2+8d^2\right)}{x^2\sqrt{1-c^2x^2}}dx^2}{210e^3}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \text{2118}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{3e\left(ex^2+d\right)^{3/2}\left(16c^2d^2-\left(29c^2d-25e\right)ex^2\right)}{2x^2\sqrt{1-c^2x^2}}dx^2}{3c^2e}-\frac{5e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int\frac{\left(ex^2+d\right)^{3/2}\left(16c^2d^2-\left(29c^2d-25e\right)ex^2\right)}{x^2\sqrt{1-c^2x^2}}dx^2}{2c^2}-\frac{5e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 171$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{e\sqrt{1-c^2x^2}\left(29c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}-\frac{\int-\frac{\sqrt{ex^2+d}\left(64c^4d^3-e\left(23d^2c^4+12dec^2-75e^2\right)x^2\right)}{2x^2\sqrt{1-c^2x^2}}dx^2}{2c^2}}{2c^2}-\frac{5e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 27$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\int\frac{\sqrt{ex^2+d}\left(64c^4d^3-e\left(23d^2c^4+12dec^2-75e^2\right)x^2\right)}{x^2\sqrt{1-c^2x^2}}dx^2}{4c^2}+\frac{e\sqrt{1-c^2x^2}\left(29c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{2c^2}-\frac{5e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 171$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\frac{e\sqrt{1-c^2x^2}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}-\frac{\int-\frac{128d^4c^6+e\left(105d^3c^6-35d^2ec^4+63de^2c^2+75e^3\right)x^2}{2x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{c^2}}{4c^2}+\frac{e\sqrt{1-c^2x^2}\left(29c^2d-25e\right)\left(d \cdots\right.}{2c^2}}{2c^2} \cdots\right.}{210e^3}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}$$

3.130. $\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

$$\Big\downarrow \; 27$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\frac{\int \frac{128d^4c^6+e\left(105d^3c^6-35d^2ec^4+63de^2c^2+75e^3\right)x^2}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{1-c^2x^2}\left(29c^2d-25e\right)\left(d+\cdots\right.}{2c^2}}{2c^2}\cdots\right.}{210e^3}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 175$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\frac{128c^6d^4\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(23c^4d^2+12c^2de-75\cdots\right.}{c^2}}{4c^2}\cdots}{2c^2}\cdots\right.}{210e^3}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 66$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\frac{128c^6d^4\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+2e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(23c^4d^2+12c^2de-75\cdots\right.}{c^2}}{4c^2}\cdots}{2c^2}\cdots\right.}{210e^3}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 104$$

3.130. $\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\frac{256c^6d^4\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}+2e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d}}{c^2}}{4c^2}}{2c^2}\right.}{210e^3}$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 218$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\frac{256c^6d^4\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}-\frac{2\sqrt{e}\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(23c^4d^2+12c^2de-75e^2\right)}{c^2}}{4c^2}}{2c^2}\right.}{210e^3}$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 220$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}+$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\frac{-\frac{2\sqrt{e}\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}-256c^6d^{7/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(23c^4d^2+12c^2\right.}{c^2}}{4c^2}}{2c^2}\right.}{210e^3}$$

input
```
Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

3.130. $\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

output
```
(d^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5
/2)*(a + b*ArcSech[c*x]))/(5*e^3) + ((d + e*x^2)^(7/2)*(a + b*ArcSech[c*x]
))/(7*e^3) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-5*e*Sqrt[1 - c^2*x^2
]*(d + e*x^2)^(5/2))/c^2 + (((29*c^2*d - 25*e)*e*Sqrt[1 - c^2*x^2]*(d + e*
x^2)^(3/2))/(2*c^2) + ((e*(23*c^4*d^2 + 12*c^2*d*e - 75*e^2)*Sqrt[1 - c^2*
x^2]*Sqrt[d + e*x^2])/c^2 + ((-2*Sqrt[e]*(105*c^6*d^3 - 35*c^4*d^2*e + 63*
c^2*d*e^2 + 75*e^3)*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])
])/c - 256*c^6*d^(7/2)*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])
])/(2*c^2))/(4*c^2))/(2*c^2)))/(210*e^3)
```

3.130.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.130. $\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.130.4 Maple [F]

$$\int x^5(a+b\operatorname{arcsech}(cx))\sqrt{e\,x^2+d}dx$$

input
```
int(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

3.130.5 Fricas [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 1995, normalized size of antiderivative = 4.46

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/6720*(128*b*c^7*d^(7/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d
^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sq
rt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (105*b*c^6*d^3 - 35*b*c^4*d^2
*e + 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c
^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*s
qrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 64*(15*b*c
^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x
^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7
*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 - (40*b
*c^6*e^3*x^5 + 2*(11*b*c^6*d*e^2 + 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e - 3
8*b*c^4*d*e^2 - 75*b*c^2*e^3)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^
2 + d))/(c^7*e^3), 1/3360*(64*b*c^7*d^(7/2)*log(((c^4*d^2 - 6*c^2*d*e + e^
2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^
2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (105*b*c^6*d
^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(e)*arctan(1/2*(2*c^2
*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) + 32*(15*b*c^7*e^3*x^6 + 3
*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c
*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 48*
a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 - (40*b*c^6*e^3*x^...
```

3.130.6 Sympy [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^5(a+b\,\text{asech}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**5*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**5*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

3.130.7 Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.130.8 Giac [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)x^5\,dx$$

input
```
integrate(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x^5, x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^5\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^5*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^5*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

3.131 $\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.131.1 Optimal result

Integrand size = 23, antiderivative size = 329

$$\begin{aligned}
&\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx \\
&= -\frac{b(c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e} \\
&\quad -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c^2e} \\
&\quad -\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2} \\
&\quad +\frac{b(15c^4d^2-10c^2de-9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^5e^{3/2}} \\
&\quad +\frac{2bd^{5/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{15e^2}
\end{aligned}$$

output
```
-1/3*d*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*arc
sech(c*x))/e^2+1/120*b*(15*c^4*d^2-10*c^2*d*e-9*e^2)*arctan(e^(1/2)*(-c^2*
x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^5/e^(3/2
)+2/15*b*d^(5/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c
*x+1))^(1/2)*(c*x+1)^(1/2)/e^2-1/20*b*(e*x^2+d)^(3/2)*(1/(c*x+1))^(1/2)*(c
*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e-1/120*b*(c^2*d+9*e)*(1/(c*x+1))^(1/2)
*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c^4/e
```

3.131.2 Mathematica [A] (verified)

Time = 23.45 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.11

$$\int x^3\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx =$$
$$-\frac{\sqrt{d+ex^2}\Big(8ac^4(2d^2-dex^2-3e^2x^4)+be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(9e+c^2(7d+6ex^2)\right)+8bc^4(2d^2-dex^2-3e^2}{120c^4e^2}$$
$$-\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{-c^2d-e}\sqrt{e}(15c^4d^2-10c^2de-9e^2)\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+16c\right.}{120c^7e^2(-1+cx)\sqrt{d+ex^2}}$$

input
```
Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

output
```
-1/120*(Sqrt[d + e*x^2]*(8*a*c^4*(2*d^2 - d*e*x^2 - 3*e^2*x^4) + b*e*Sqrt[
(1 - c*x)/(1 + c*x)]*(1 + c*x)*(9*e + c^2*(7*d + 6*e*x^2)) + 8*b*c^4*(2*d^
2 - d*e*x^2 - 3*e^2*x^4)*ArcSech[c*x]))/(c^4*e^2) - (b*Sqrt[(1 - c*x)/(1 +
 c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*(15*c^4*d^
2 - 10*c^2*d*e - 9*e^2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt
[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])] + 16*c^7*d^(5/2)*S
qrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(12
0*c^7*e^2*(-1 + c*x)*Sqrt[d + e*x^2])
```

3.131.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.85, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {6855, 27, 435, 171, 27, 171, 27, 175, 66, 104, 218, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$\Big\downarrow \text{ 6855}$$
$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{(2d-3ex^2)\left(ex^2+d\right)^{3/2}}{15e^2x\sqrt{1-c^2x^2}}dx+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(2d-3ex^2)(ex^2+d)^{3/2}}{x\sqrt{1-c^2x^2}}dx}{15e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 435$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(2d-3ex^2)(ex^2+d)^{3/2}}{x^2\sqrt{1-c^2x^2}}dx^2}{30e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 171$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{3e\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{2c^2}-\frac{\int -\frac{\sqrt{ex^2+d}\left(8c^2d^2-e\left(dc^2+9e\right)x^2\right)}{2x^2\sqrt{1-c^2x^2}}dx^2}{2c^2}\right)}{30e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{\sqrt{ex^2+d}\left(8c^2d^2-e\left(dc^2+9e\right)x^2\right)}{x^2\sqrt{1-c^2x^2}}dx^2}{4c^2}+\frac{3e\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{2c^2}\right)}{30e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 171$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{e\sqrt{1-c^2x^2}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}-\frac{\int -\frac{16d^3c^4+e\left(15d^2c^4-10dec^2-9e^2\right)x^2}{2x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{c^2}}{4c^2}+\frac{3e\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{2c^2}\right)}{30e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 27$$

3.131. $\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\int \frac{16d^3c^4+e\left(15d^2c^4-10dec^2-9e^2\right)x^2}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 175$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{16c^4d^3\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+e\left(15c^4d^2-10c^2de-9e^2\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 66$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{16c^4d^3\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+2e\left(15c^4d^2-10c^2de-9e^2\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 104$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{32c^4d^3\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}+2e\left(15c^4d^2-10c^2de-9e^2\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 218$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{32c^4d^3\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}-\frac{2\sqrt{e}\left(15c^4d^2-10c^2de-9e^2\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}$$

3.131. $\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

$$\Big\downarrow \; 220$$

$$\frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}-\frac{d(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{-\frac{2\sqrt{e}\left(15c^4d^2-10c^2de-9e^2\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}-32c^4d^{5/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{}}{}\right.}{30e^2}$$

input
```
Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

output
```
-1/3*(d*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/e^2 + ((d + e*x^2)^(5/2)*(
a + b*ArcSech[c*x]))/(5*e^2) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((3*e
*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(2*c^2) + ((e*(c^2*d + 9*e)*Sqrt[1 -
 c^2*x^2]*Sqrt[d + e*x^2])/c^2 + ((-2*Sqrt[e]*(15*c^4*d^2 - 10*c^2*d*e - 9
*e^2)*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/c - 32*c^4*
d^(5/2)*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(2*c^2))/(4*
c^2)))/(30*e^2)
```

3.131.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

3.131. $\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.131.4 Maple [F]

$$\int x^{3}(a+b\,\mathrm{arcsech}\,(cx))\sqrt{e\,x^{2}+d}dx$$

input
```
int(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 1669, normalized size of antiderivative = 5.07

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/480*(16*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + (15*b*c^4*d^2 - 10*b*c^2*d*e -
9*b*e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c
^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x
^2 - 2*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
+ 1)/(c*x)) + 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 - (6*b*
c^4*e^2*x^3 + (7*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
))*sqrt(e*x^2 + d))/(c^5*e^2), 1/240*(8*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^
2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)
*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + (1
5*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c
^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e
^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) + 16*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2
 - 2*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) +
1)/(c*x)) + 2*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 - (6*b*c^
4*e^2*x^3 + (7*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))
*sqrt(e*x^2 + d))/(c^5*e^2), 1/480*(32*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^
3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/...
```

3.131.6 Sympy [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^3(a+b\,\text{asech}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**3*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

3.131.7 Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.131.8 Giac [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x^3, x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^3\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

3.132 $\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.132.1 Optimal result

Integrand size = 21, antiderivative size = 221

$$\begin{aligned}\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx = &-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2}\\&+\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}\\&-\frac{b(3c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^3\sqrt{e}}\\&-\frac{bd^{3/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e}\end{aligned}$$

output
```
1/3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e-1/3*b*d^(3/2)*arctanh((e*x^2+d)^(
1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e-1/6*b*(
3*c^2*d+e)*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1)
)^(1/2)*(c*x+1)^(1/2)/c^3/e^(1/2)-1/6*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-
c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c^2
```

3.132.2 Mathematica [A] (verified)

Time = 22.76 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.39

$$\int x\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{\sqrt{d+ex^2}\left(-be\sqrt{\frac{1-cx}{1+cx}}(1+cx)+2ac^2(d+ex^2)+2bc^2(d+ex^2)\operatorname{sech}^{-1}(cx)\right)}{6c^2e}$$
$$+\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{-c^2d-e}\sqrt{e}(3c^2d+e)\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+2c^5d^{3/2}\sqrt{-d-e\ldots}\right.}{6c^5e(-1+cx)\sqrt{d+ex^2}}$$

input
```
Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

output
```
(Sqrt[d + e*x^2]*(-(b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)) + 2*a*c^2*(d
+ e*x^2) + 2*b*c^2*(d + e*x^2)*ArcSech[c*x]))/(6*c^2*e) + (b*Sqrt[(1 - c*x
)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*(3*c
^2*d + e)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c
^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])] + 2*c^5*d^(3/2)*Sqrt[-d - e*x^2]
*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(6*c^5*e*(-1 + c*x
)*Sqrt[d + e*x^2])
```

3.132.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.86, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6853, 2036, 354, 113, 27, 175, 66, 104, 218, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6853}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{(ex^2+d)^{3/2}}{x\sqrt{1-cx}\sqrt{cx+1}}dx}{3e}+\frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{2036}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(ex^2+d)^{3/2}}{x\sqrt{1-c^2x^2}}dx}{3e}+\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{354}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(ex^2+d)^{3/2}}{x^2\sqrt{1-c^2x^2}}dx^2}{6e}+\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{113}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{2c^2d^2+e(3dc^2+e)x^2}{2x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e}+\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{2c^2d^2+e(3dc^2+e)x^2}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e}+\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{175}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2c^2d^2\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+e(3c^2d+e)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e}+\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{66}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2c^2d^2\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+2e(3c^2d+e)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e}+\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{104}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{4c^2d^2\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}+2e(3c^2d+e)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e}+\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{218}$$

3.132. $\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{4c^2d^2\int\frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}-\frac{2\sqrt{e}\left(3c^2d+e\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}-\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \; 220$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{-\frac{2\sqrt{e}\left(3c^2d+e\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}-4c^2d^{3/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2c^2}-\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e}$$

input
```
Int[x*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

output
```
((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3*e) + (b*Sqrt[(1 + c*x)^(-1)]*S
qrt[1 + c*x]*(-((e*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/c^2) + ((-2*Sqrt[e]*
(3*c^2*d + e)*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/c -
 4*c^2*d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(2*c^
2)))/(6*e)
```

3.132.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2036
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

3.132. $\int x\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$

rule 6853
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Simp[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x
^2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e
, p}, x] && NeQ[p, -1]
```

3.132.4 Maple [F]

$$\int x(a+b\,\mathrm{arcsech}\,(cx))\,\sqrt{e\,x^2+d}dx$$

input
```
int(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

3.132.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(131) = 262$.

Time = 0.45 (sec) , antiderivative size = 1382, normalized size of antiderivative = 6.25

$$\int x\sqrt{d+ex^2}\left(a+b\mathrm{sech}^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(2*b*c^3*d^(3/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
 d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (3*b*c^2*d + b*e)*sqrt(-e)*log(8*
c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e
*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)) + e^2) + 8*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e), 1/12*(b*c^
3*d^(3/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4
*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)
/(c^2*x^2)) + 8*d^2)/x^4) - (3*b*c^2*d + b*e)*sqrt(e)*arctan(1/2*(2*c^2*e*
x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
)/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) + 4*(b*c^3*e*x^2 + b*c^3*d)*s
qrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*
a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e
*x^2 + d))/(c^3*e), -1/24*(4*b*c^3*sqrt(-d)*d*arctan(-1/2*((c^3*d - c*e)*x
^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2
*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d + b*e)*sqrt(-e)*log(8*
c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e
*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/...
```

3.132.6 Sympy [F]

$$\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x(a+b\,\text{asech}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate(x*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

3.132.7 Maxima [F]

$$\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
1/3*((e*x^2 + d)^(3/2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/e - 3*integra
te(1/3*sqrt(e*x^2 + d)*(6*(c^2*e*x^2 - e)*x*log(sqrt(x)) + 3*(c^2*e*x^2*lo
g(c) - e*log(c))*x + (6*(c^2*e*x^2 - e)*x*log(sqrt(x)) + ((3*e*log(c) + e)
*c^2*x^2 + c^2*d - 3*e*log(c))*x)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))
)/(c^2*e*x^2 + (c^2*e*x^2 - e)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)) -
e), x))*b + 1/3*(e*x^2 + d)^(3/2)*a/e
```

3.132.8 Giac [F]

$$\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x, x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x\,\sqrt{e\,x^2+d}\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

3.133 $\int \frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}\,dx$

3.133.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x},x\right)$$

output
```
Unintegrable((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x)
```

3.133.2 Mathematica [N/A]

Not integrable

Time = 8.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}\,dx = \int \frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x, x]
```

3.133.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x} \, dx$$
$$\Big\downarrow \text{ 6865}$$
$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x}dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x,x]
```

output
```
$Aborted
```

3.133.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.133.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b \operatorname{arcsech}(cx)) \sqrt{e\,x^2+d}}{x}dx$$

input
```
int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x)
```

output
```
int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x)
```

3.133.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{\sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x, x)
```

3.133.6 Sympy [N/A]

Not integrable

Time = 4.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{(a+b\,\text{asech}\,(cx))\,\sqrt{d+ex^2}}{x}\,dx$$

input
```
integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x,x)
```

output
```
Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x, x)
```

3.133.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.133.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x, x)
```

3.133.9 Mupad [N/A]

Not integrable

Time = 4.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x, x)
```

3.134 $\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx$

3.134.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^3}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^3},x\right)$$

output
```
Unintegrable((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

3.134.2 Mathematica [N/A]

Not integrable

Time = 9.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^3}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3, x]
```

3.134.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^3} dx$$
$$\Big\downarrow \text{ 6865}$$
$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^3} dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3,x]
```

output
```
$Aborted
```

3.134.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.134.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\operatorname{arcsech}(cx))\sqrt{e\,x^2+d}}{x^3}dx$$

input
```
int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

output
```
int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

3.134.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{\sqrt{ex^2+d}(b\operatorname{arsech}(cx)+a)}{x^3}\,dx$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^3, x)
```

3.134.6 Sympy [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{(a+b\operatorname{asech}(cx))\sqrt{d+ex^2}}{x^3}\,dx$$

input
```
integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**3,x)
```

output
```
Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**3, x)
```

3.134.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.134.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^3}\,dx=\int \frac{\sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^3, x)
```

3.134.9 Mupad [N/A]

Not integrable

Time = 4.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^3}\,dx=\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^3, x)
```

3.135 $\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.135.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Int}\left(x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right),x\right)$$

output
```
Unintegrable(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

3.135.2 Mathematica [N/A]

Not integrable

Time = 19.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

input
```
Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

output
```
Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

3.135.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6865}$$

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)dx$$

input
```
Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

output
```
$Aborted
```

3.135.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.135.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2(a+b\,\text{arcsech}\,(cx))\sqrt{e\,x^2+d}dx$$

input
```
int(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

3.135.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsech(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

3.135.6 Sympy [N/A]

Not integrable

Time = 7.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^2(a+b\,\text{asech}\,(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**2*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

3.135.7 Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.135.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x^2, x)
```

3.135.9 Mupad [N/A]

Not integrable

Time = 4.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^2\,\sqrt{e\,x^2+d}\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

3.136 $\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.136.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Int}\left(\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right),x\right)$$

output
```
Unintegrable((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

3.136.2 Mathematica [N/A]

Not integrable

Time = 6.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

3.136.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6865}$$

$$\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

output
```
$Aborted
```

3.136.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.136.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\,\operatorname{arcsech}\left(c x\right)\right)\sqrt{e\,x^{2}+d}dx$$

input
```
int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

3.136.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)\,dx$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a), x)
```

3.136.6 Sympy [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \int (a+b\,\text{asech}\,(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

3.136.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.136.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\operatorname{arsech}(cx)+a)\,dx$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a), x)
```

3.136.9 Mupad [N/A]

Not integrable

Time = 4.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int \sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

3.137 $\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx$

3.137.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx=\text{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2},x\right)$$

output
```
Unintegrable((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

3.137.2 Mathematica [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2, x]
```

3.137.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^2} dx$$

$$\Big\downarrow \text{ 6865}$$

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^2}dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2,x]
```

output
```
$Aborted
```

3.137.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.137.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\ \text{arcsech}\,(cx))\,\sqrt{e\,x^2+d}}{x^2}dx$$

input
```
int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

output
```
int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

3.137.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^2, x)
```

3.137.6 Sympy [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{(a+b\,\text{asech}\,(cx))\,\sqrt{d+ex^2}}{x^2}\,dx$$

input
```
integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**2,x)
```

output
```
Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**2, x)
```

3.137.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.137.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^2, x)
```

3.137.9 Mupad [N/A]

Not integrable

Time = 4.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^2, x)
```

3.138 $\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx$

3.138.1 Optimal result

Integrand size = 23, antiderivative size = 312

$$\begin{aligned}\int \frac{\sqrt{d+ex^2}\,(a+b\,\text{sech}^{-1}(cx))}{x^4}\,dx &= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3}\\ &\quad+\frac{2b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx}-\frac{(d+ex^2)^{3/2}\,(a+b\,\text{sech}^{-1}(cx))}{3dx^3}\\ &\quad+\frac{2bc(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{9d\sqrt{1+\frac{ex^2}{d}}}\\ &\quad-\frac{b(c^2d+e)\,(2c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9cd\sqrt{d+ex^2}}\end{aligned}$$

output
```
-1/3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/d/x^3+1/9*b*(1/(c*x+1))^(1/2)*(c*x
+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/x^3+2/9*b*(c^2*d+2*e)*(1/(c*x
+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/x+2/9*b*c*(c
^2*d+2*e)*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*
(e*x^2+d)^(1/2)/d/(1+e*x^2/d)^(1/2)-1/9*b*(c^2*d+e)*(2*c^2*d+3*e)*Elliptic
F(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/
c/d/(e*x^2+d)^(1/2)
```

3.138.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.18 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^4}\,dx$$

$$= \frac{\frac{b\sqrt{\frac{1-cx}{1+cx}}(d+ex^2)}{x^3}+\frac{bc\sqrt{\frac{1-cx}{1+cx}}(d+ex^2)}{x^2}+\frac{2b(c^2d+2e)\sqrt{\frac{1-cx}{1+cx}}(d+ex^2)}{dx}-\frac{3a(d+ex^2)^2}{dx^3}-\frac{3b(d+ex^2)^2\operatorname{sech}^{-1}(cx)}{dx^3}-\frac{2ib\left(c\sqrt{d}-i\sqrt{e}\right)^2\sqrt{}}{}}{}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^4,x]
```

output
```
((b*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2))/x^3 + (b*c*Sqrt[(1 - c*x)/(1 +
c*x)]*(d + e*x^2))/x^2 + (2*b*(c^2*d + 2*e)*Sqrt[(1 - c*x)/(1 + c*x)]*(d +
 e*x^2))/(d*x) - (3*a*(d + e*x^2)^2)/(d*x^3) - (3*b*(d + e*x^2)^2*ArcSech[
c*x])/(d*x^3) - ((2*I)*b*(c*Sqrt[d] - I*Sqrt[e])^2*Sqrt[(1 - c*x)/(1 + c*x
)]*(1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1
+ c*x))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*
x))]*((c^2*d + 2*e)*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*S
qrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] -
 I*Sqrt[e])^2] + ((2*I)*c*Sqrt[d] - 3*Sqrt[e])*Sqrt[e]*EllipticF[I*ArcSinh
[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*
Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))/(c*d*Sqrt[-(((c*Sqrt[d
] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x)))]))/(9*Sqrt
[d + e*x^2])
```

3.138.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.82, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {6855, 27, 376, 445, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^4}\,dx$$

$$\Big\downarrow \text{6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{\left(ex^2+d\right)^{3/2}}{3dx^4\sqrt{1-c^2x^2}}dx-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\left(ex^2+d\right)^{3/2}}{x^4\sqrt{1-c^2x^2}}dx}{3d}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \text{376}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\int \frac{e\left(dc^2+3e\right)x^2+2d\left(dc^2+2e\right)}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx-\frac{d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3x^3}\right)}{3d}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \text{445}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(-\frac{\int -\frac{de\left(-2\left(dc^2+2e\right)x^2c^2+dc^2+3e\right)}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d}-\frac{2\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)-\frac{d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3x^3}\right)}{3d}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(\frac{\int \frac{de\left(-2\left(dc^2+2e\right)x^2c^2+dc^2+3e\right)}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d}-\frac{2\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)-\frac{d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3x^3}\right)}{3d}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(e\int \frac{-2\left(dc^2+2e\right)x^2c^2+dc^2+3e}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx-\frac{2\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)-\frac{d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3x^3}\right)}{3d}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \text{399}$$

3.138. $\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(2c^2d+3e)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{e}-\frac{2c^2(c^2d+2e)\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}\right)-\frac{2\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{d+ex^2}}{x}\right)\right.}{3d}$$

$$\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3dx^3}$$

$\downarrow$ 323

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{2c^2(c^2d+2e)\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}\right)-\frac{2\sqrt{1-c^2x^2}(c^2d+2e)}{x}\right.\right.}{3d}$$

$$\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3dx^3}$$

$\downarrow$ 321

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{2c^2(c^2d+2e)\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}\right)-\frac{2\sqrt{1-c^2x^2}(c^2d}{x}\right.\right.}{3d}$$

$$\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3dx^3}$$

$\downarrow$ 330

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{2c^2(c^2d+2e)\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{\frac{ex^2}{d}+1}}\right)-2\sqrt{1-}\right.\right.}{3d}$$

$$\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3dx^3}$$

$\downarrow$ 327

$$-\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3dx^3}-$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{2c(c^2d+2e)\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{\frac{ex^2}{d}+1}}\right)-2\sqrt{1}\right.\right.}{3d}$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^4,x]
```

3.138. $\int\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}dx$

output
```
-1/3*((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(d*x^3) - (b*Sqrt[(1 + c*x)^
(-1)]*Sqrt[1 + c*x]*(-1/3*(d*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/x^3 + ((-2
*(c^2*d + 2*e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/x + e*((-2*c*(c^2*d + 2*
e)*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[1 + (e*x^
2)/d]) + ((c^2*d + e)*(2*c^2*d + 3*e)*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin
[c*x], -(e/(c^2*d))])/(c*e*Sqrt[d + e*x^2])))/3))/(3*d)
```

3.138.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

3.138. $\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx$

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.138.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsech}\,(cx))\,\sqrt{e\,x^2+d}}{x^4}dx$$

input
```
int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

output
```
int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx =$$

$$-\frac{3\left(bcdex^2+bcd^2\right)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+\left(3\,acdex^2+3\,acd^2-\left(bc^2d^2x+2\left(bc^4d^2+2\,bc^2de\right)x^3\right.\right.}{}$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/9*(3*(b*c*d*e*x^2 + b*c*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 -
1)/(c^2*x^2)) + 1)/(c*x)) + (3*a*c*d*e*x^2 + 3*a*c*d^2 - (b*c^2*d^2*x + 2*
(b*c^4*d^2 + 2*b*c^2*d*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2
+ d) - (2*(b*c^6*d^2 + 2*b*c^4*d*e)*x^3*elliptic_e(arcsin(c*x), -e/(c^2*d)
) - (2*b*c^6*d^2 + (4*b*c^4 + b*c^2)*d*e + 3*b*e^2)*x^3*elliptic_f(arcsin(
c*x), -e/(c^2*d)))*sqrt(d))/(c*d^2*x^3)
```

3.138.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{(a+b\,\text{asech}\,(cx))\sqrt{d+ex^2}}{x^4}\,dx$$

input
```
integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**4,x)
```

output
```
Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**4, x)
```

3.138.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.138.8 Giac [F]

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^4, x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^4, x)
```

3.139 $\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx$

3.139.1 Optimal result

Integrand size = 23, antiderivative size = 446

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx &= \frac{b(12c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3} \\
&+ \frac{b(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225d^2x} \\
&+ \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{25dx^5} \\
&- \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3} \\
&+ \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{225d^2\sqrt{1+\frac{ex^2}{d}}} \\
&- \frac{b(c^2d+e)\left(24c^4d^2+7c^2de-30e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{225cd^2\sqrt{d+ex^2}}
\end{aligned}$$

output
```
-1/5*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*
arcsech(c*x))/d^2/x^3+1/25*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^
(1/2)*(e*x^2+d)^(1/2)/x^5+1/45*b*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x
^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/x^3+1/75*b*(4*c^2*d+e)*(1/(c*x+1))^(1/2)*(c*
x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/x^3-2/15*b*e^2*(1/(c*x+1))
^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x+1/45*b*e*(2*
c^2*d+e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2
)/d^2/x+1/75*b*(8*c^4*d^2+3*c^2*d*e-2*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)
*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x-2/15*b*c*e^2*EllipticE(c*x,(-e/c
^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(1+e*x^2/
d)^(1/2)+1/45*b*c*e*(2*c^2*d+e)*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1)
)^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(1+e*x^2/d)^(1/2)+1/75*b*c*(8*c^
4*d^2+3*c^2*d*e-2*e^2)*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(
c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(1+e*x^2/d)^(1/2)-1/75*b*c*(8*c^2*d-e)*(c
^2*d+e)*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1
+e*x^2/d)^(1/2)/d/(e*x^2+d)^(1/2)-2/45*b*c*e*(c^2*d+e)*EllipticF(c*x,(-e/c
^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(e*x^2+d)
^(1/2)+2/15*b*e^2*(c^2*d+e)*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1
/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d^2/(e*x^2+d)^(1/2)
```

3.139.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.93 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx$$

$$= \frac{\frac{15a(d+ex^2)^2(-3d+2ex^2)}{x^5} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(d+ex^2)(-31e^2x^4+dex^2(8+19c^2x^2)+3d^2(3+4c^2x^2+8c^4x^4))}{x^5} + \frac{15b(d+ex^2)^2(-3d+2ex^2)\operatorname{sec}}{x^5}}{}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^6,x]
```

output
```
((15*a*(d + e*x^2)^2*(-3*d + 2*e*x^2))/x^5 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*
(1 + c*x)*(d + e*x^2)*(-31*e^2*x^4 + d*e*x^2*(8 + 19*c^2*x^2) + 3*d^2*(3 +
 4*c^2*x^2 + 8*c^4*x^4)))/x^5 + (15*b*(d + e*x^2)^2*(-3*d + 2*e*x^2)*ArcSe
ch[c*x])/x^5 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c^2*(24*c^4*d^2 + 19*c^2*d*
e - 31*e^2)*(d + e*x^2)) - (I*(c*Sqrt[d] - I*Sqrt[e])^2*(1 + c*x)*Sqrt[(c*
(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*(Sqr
t[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*((24*c^4*d^2 + 1
9*c^2*d*e - 31*e^2)*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*S
qrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] -
 I*Sqrt[e])^2] + 2*Sqrt[e]*((24*I)*c^3*d^(3/2) - 36*c^2*d*Sqrt[e] - (29*I)
*c*Sqrt[d]*e + 30*e^(3/2))*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x)
)/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sq
rt[d] - I*Sqrt[e])^2]))/Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sq
rt[d] + I*Sqrt[e])*(1 + c*x)))]))/c)/(225*d^2*Sqrt[d + e*x^2])
```

3.139.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.83, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {6855, 27, 442, 442, 445, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}dx$$

$$\Big\downarrow \text{6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{\left(3d-2ex^2\right)\left(ex^2+d\right)^{3/2}}{15d^2x^6\sqrt{1-c^2x^2}}dx+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\left(3d-2ex^2\right)\left(ex^2+d\right)^{3/2}}{x^6\sqrt{1-c^2x^2}}dx}{15d^2}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{442}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\int\frac{\sqrt{ex^2+d}\left(\left(3c^2d-10e\right)ex^2+d\left(12c^2d-e\right)\right)}{x^4\sqrt{1-c^2x^2}}dx-\frac{3d\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{5x^5}\right)}{15d^2}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}$$

$\downarrow$ 442

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{2e\left(6d^2c^4+4dec^2-15e^2\right)x^2+d\left(24d^2c^4+19dec^2-31e^2\right)}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx-\frac{d\sqrt{1-c^2x^2}\left(12c^2d-e\right)\sqrt{d+ex^2}}{3x^3}\right)-\frac{3d\sqrt{1-c^2x^2}(\cdots}{5x^5}\right.}{15d^2}\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}$$

$\downarrow$ 445

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\frac{\int-\frac{de\left(2\left(6d^2c^4+4dec^2-15e^2\right)-c^2\left(24d^2c^4+19dec^2-31e^2\right)x^2\right)}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d}-\frac{\sqrt{1-c^2x^2}\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{d+ex^2}}{x}\right.\right.\right.}{15d^2}\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}$$

$\downarrow$ 25

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int\frac{de\left(2\left(6d^2c^4+4dec^2-15e^2\right)-c^2\left(24d^2c^4+19dec^2-31e^2\right)x^2\right)}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d}-\frac{\sqrt{1-c^2x^2}\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{d+ex^2}}{x}\right)-\right.\right.}{15d^2}\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}$$

$\downarrow$ 27

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\int\frac{2\left(6d^2c^4+4dec^2-15e^2\right)-c^2\left(24d^2c^4+19dec^2-31e^2\right)x^2}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx-\frac{\sqrt{1-c^2x^2}\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{d+ex^2}}{x}\right)-\right.\right.}{15d^2}\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}$$

$\downarrow$ 399

3.139. $\int\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}dx$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(24c^4d^2+7c^2de-30e^2)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{e}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}\right)-\sqrt{}\right.\right.\right.}{15d^2}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{5dx^5}$$

$\downarrow$ 323

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}}{e}\right.\right.\right.\right.}{15d^2}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{5dx^5}$$

$\downarrow$ 321

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int\frac{\sqrt{ex}}{\sqrt{1-}}}{e}\right.\right.\right.\right.}{15d^2}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{5dx^5}$$

$\downarrow$ 330

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\sqrt{d+e}}{e\sqrt{\frac{ex^2}{d}+1}}\right.\right.\right.\right.}{15d^2}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{5dx^5}$$

$\downarrow$ 327

$$\frac{2e(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{5dx^5}-$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c(24c^4d^2+19c^2de-31e^2)\sqrt{d+ex^2}E(}{e\sqrt{\frac{ex^2}{d}+1}}\right.\right.\right.\right.}{15d^2}$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^6,x]
```

3.139. $\int\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx$

output
```
-1/5*((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(d*x^5) + (2*e*(d + e*x^2)^(
3/2)*(a + b*ArcSech[c*x]))/(15*d^2*x^3) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 +
 c*x]*((-3*d*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(5*x^5) + (-1/3*(d*(12*c
^2*d - e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/x^3 + (-(((24*c^4*d^2 + 19*c^
2*d*e - 31*e^2)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/x) + e*(-((c*(24*c^4*d^
2 + 19*c^2*d*e - 31*e^2)*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d
))])/(e*Sqrt[1 + (e*x^2)/d])) + ((c^2*d + e)*(24*c^4*d^2 + 7*c^2*d*e - 30*
e^2)*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*e*Sqrt[d
 + e*x^2])))/3)/5))/(15*d^2)
```

3.139.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.139.4 Maple [F]

$$\int \frac{(a + b\ \mathrm{arcsech}\,(cx))\,\sqrt{e\,x^2+d}}{x^6}dx$$

input
```
int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

output
```
int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx$$
$$= \frac{15\left(2\,bcde^2x^4 - bcd^2ex^2 - 3\,bcd^3\right)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) + \left(30\,acde^2x^4 - 15\,acd^2ex^2 - 45\,acd^3 + \right.}{}$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
1/225*(15*(2*b*c*d*e^2*x^4 - b*c*d^2*e*x^2 - 3*b*c*d^3)*sqrt(e*x^2 + d)*lo
g((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + (30*a*c*d*e^2*x^4 - 15
*a*c*d^2*e*x^2 - 45*a*c*d^3 + (9*b*c^2*d^3*x + (24*b*c^6*d^3 + 19*b*c^4*d^
2*e - 31*b*c^2*d*e^2)*x^5 + 4*(3*b*c^4*d^3 + 2*b*c^2*d^2*e)*x^3)*sqrt(-(c^
2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d) + ((24*b*c^8*d^3 + 19*b*c^6*d^2*e -
 31*b*c^4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (24*b*c^8*d^3 +
 (19*b*c^6 + 12*b*c^4)*d^2*e - (31*b*c^4 - 8*b*c^2)*d*e^2 - 30*b*e^3)*x^5*
elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(d))/(c*d^3*x^5)
```

3.139.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(a+b\operatorname{asech}(cx))\sqrt{d+ex^2}}{x^6}\,dx$$

input
```
integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**6,x)
```

output
```
Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**6, x)
```

3.139.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.139.8 Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^6}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)}{x^6}\,dx$$

input
```
integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^6, x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^6}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^6, x)
```

3.140 $\int x^3(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.140.1 Optimal result

Integrand size = 23, antiderivative size = 418

$$\begin{aligned}\int x^3(d+ex^2)^{3/2}&(a+b\text{sech}^{-1}(cx))\,dx = \frac{b(3c^4d^2-38c^2de-25e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{560c^6e}\\
&-\frac{b(13c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e}\\
&-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e}\\
&-\frac{d(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}+\frac{(d+ex^2)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}\\
&+\frac{b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{560c^7e^{3/2}}\\
&+\frac{2bd^{7/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{35e^2}\end{aligned}$$

output
```
-1/5*d*(e*x^2+d)^(5/2)*(a+b*arcsech(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*arc
sech(c*x))/e^2+1/560*b*(35*c^6*d^3-35*c^4*d^2*e-63*c^2*d*e^2-25*e^3)*arcta
n(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^
(1/2)/c^7/e^(3/2)+2/35*b*d^(7/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2
+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^2-1/840*b*(13*c^2*d+25*e)*(e*
x^2+d)^(3/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e-1/42
*b*(e*x^2+d)^(5/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/
e+1/560*b*(3*c^4*d^2-38*c^2*d*e-25*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-
c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c^6/e
```

3.140.2 Mathematica [A] (verified)

Time = 37.50 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.75

$$\int x^3(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx =$$

$$-\frac{\sqrt{d+ex^2}\left(48ac^6(2d-5ex^2)\left(d+ex^2\right)^2+be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(75e^2+2c^2e(82d+25ex^2)+c^4(57d^2+106de\right.\right.}{1680c^6e^2}$$

$$-\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{-1+c^2x^2}\left(-32c^7d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}(-35c^6d^3+35c^4d^2e+63c^2de^2+25e^3)\operatorname{arctanh}\right.}{560c^7e^2(-1+cx)}$$

input
```
Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

output
```
-1/1680*(Sqrt[d + e*x^2]*(48*a*c^6*(2*d - 5*e*x^2)*(d + e*x^2)^2 + b*e*Sqr
t[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(75*e^2 + 2*c^2*e*(82*d + 25*e*x^2) + c^4
*(57*d^2 + 106*d*e*x^2 + 40*e^2*x^4)) + 48*b*c^6*(2*d - 5*e*x^2)*(d + e*x^
2)^2*ArcSech[c*x]))/(c^6*e^2) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[-1 + c^2
*x^2]*(-32*c^7*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]
] + Sqrt[e]*(-35*c^6*d^3 + 35*c^4*d^2*e + 63*c^2*d*e^2 + 25*e^3)*ArcTanh[(
Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(560*c^7*e^2*(-1 + c*x)
)
```

3.140.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.85, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {6855, 27, 435, 171, 27, 171, 27, 171, 27, 175, 66, 104, 218, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)dx$$

$$\Big\downarrow \text{6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{\left(2d-5ex^2\right)\left(ex^2+d\right)^{5/2}}{35e^2x\sqrt{1-c^2x^2}}dx+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\left(2d-5ex^2\right)\left(ex^2+d\right)^{5/2}}{x\sqrt{1-c^2x^2}}dx}{35e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{435}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\left(2d-5ex^2\right)\left(ex^2+d\right)^{5/2}}{x^2\sqrt{1-c^2x^2}}dx^2}{70e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{171}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{5e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{5/2}}{3c^2}-\frac{\int -\frac{\left(ex^2+d\right)^{3/2}\left(12c^2d^2-e\left(13dc^2+25e\right)x^2\right)}{2x^2\sqrt{1-c^2x^2}}dx^2}{3c^2}\right)}{70e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{\left(ex^2+d\right)^{3/2}\left(12c^2d^2-e\left(13dc^2+25e\right)x^2\right)}{x^2\sqrt{1-c^2x^2}}dx^2}{6c^2}+\frac{5e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{5/2}}{3c^2}\right)}{70e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}$$

$\Big\downarrow$ 171

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{e\sqrt{1-c^2x^2}\left(13c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}-\frac{\int -\frac{3\sqrt{ex^2+d}\left(16d^3c^4+e\left(3d^2c^4-38dec^2-25e^2\right)x^2\right)}{2x^2\sqrt{1-c^2x^2}}dx^2}{2c^2}}{6c^2}+\frac{5e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{5/2}}{3c^2}\right)}{70e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}$$

$\Big\downarrow$ 27

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}\left(16d^3c^4+e\left(3d^2c^4-38dec^2-25e^2\right)x^2\right)}{x^2\sqrt{1-c^2x^2}}dx^2}{4c^2}+\frac{e\sqrt{1-c^2x^2}\left(13c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{6c^2}+\frac{5e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{5/2}}{3c^2}\right)}{70e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}$$

$\Big\downarrow$ 171

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{3\left(-\frac{\int -\frac{32d^4c^6+e\left(35d^3c^6-35d^2ec^4-63de^2c^2-25e^3\right)x^2}{2x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{1-c^2x^2}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}+\frac{e\sqrt{1-c^2x^2}\left(13c^2d+2\cdots\right.}{2c^2}}{6c^2}\cdots\right.}{70e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}$$

$\Big\downarrow$ 27

3.140. $\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{3\left(\frac{\int \frac{32d^4c^6+e\left(35d^3c^6-35d^2ec^4-63de^2c^2-25e^3\right)x^2}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{1-c^2x^2}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}+\frac{e\sqrt{1-c^2x^2}\left(13c^2d+25e\right)}{2c^2}}{6c^2}\right.}{70e^2}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}$$

$\downarrow$ 175

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{3\left(\frac{32c^6d^4\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{1-c^2x^2}\left(3c^4d^2-38c^2de-\right.}{c^2}\right.}{4c^2}}{6c^2}\right.}{70e^2}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}$$

$\downarrow$ 66

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{3\left(\frac{32c^6d^4\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+2e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{1-c^2x^2}\left(3c^4d^2-38c^2de-\right.}{c^2}\right.}{4c^2}}{6c^2}\right.}{70e^2}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}$$

$\downarrow$ 104

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{3\left(\frac{64c^6d^4\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}+2e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{1-c^2x^2}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{\right.}}{c^2}\right.}{4c^2}}{6c^2}\right.}{70e^2}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}$$

$\downarrow$ 218

3.140. $\int x^3(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{3\left(\frac{64c^6d^4\int\frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}-\frac{2\sqrt{e}\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}-\frac{e\sqrt{1-c^2x^2}\left(3c^4d^2-38c^2de-25e^2\right.}{c^2}}{4c^2}}{6c^2}\right.}{70e^2}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{220}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2}-$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{3\left(\frac{-\frac{2\sqrt{e}\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}-64c^6d^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2c^2}-\frac{e\sqrt{1-c^2x^2}\left(3c^4d^2-38c^2\right.}{c^2}}{4c^2}}{6c^2}\right.}{70e^2}$$

input
```
Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

output
```
-1/5*(d*(d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/e^2 + ((d + e*x^2)^(7/2)*(
a + b*ArcSech[c*x]))/(7*e^2) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((5*e
*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(5/2))/(3*c^2) + ((e*(13*c^2*d + 25*e)*Sqrt
[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(2*c^2) + (3*(-((e*(3*c^4*d^2 - 38*c^2*d*
e - 25*e^2)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/c^2) + ((-2*Sqrt[e]*(35*c^6
*d^3 - 35*c^4*d^2*e - 63*c^2*d*e^2 - 25*e^3)*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*
x^2])/(c*Sqrt[d + e*x^2])])/c - 64*c^6*d^(7/2)*ArcTanh[Sqrt[d + e*x^2]/(Sq
rt[d]*Sqrt[1 - c^2*x^2])])/(2*c^2)))/(4*c^2))/(6*c^2)))/(70*e^2)
```

3.140. $\int x^3(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$

3.140.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

3.140. $\int x^3(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

3.140.4 Maple [F]

$$\int x^{3} \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\operatorname{arcsech}\left(c x \right)\right)dx$$

input
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

output
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 1989, normalized size of antiderivative = 4.76

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx=\text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
[1/6720*(96*b*c^7*d^(7/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^
2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqr
t(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2
*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c
^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*s
qrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 192*(5*b*c
^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2
 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*e
^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - (40*b*c
^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 + 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e + 164
*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2
 + d))/(c^7*e^2), 1/3360*(48*b*c^7*d^(7/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2
)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2
 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 3*(35*b*c^6*d
^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(e)*arctan(1/2*(2*c^2
*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) + 96*(5*b*c^7*e^3*x^6 + 8*
b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*
sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 384*a*
c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - (40*b*c^6*e^3*x^5 +...
```

3.140.6 Sympy [F]

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int x^3 (a+b\,\text{asech}\,(cx)) \left(d+ex^2\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**3*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
```

output
```
Integral(x**3*(a + b*asech(c*x))*(d + e*x**2)**(3/2), x)
```

3.140.7 Maxima [F(-2)]

Exception generated.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.140.8 Giac [F]

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\,\text{arsech}\,(cx)+a)x^3 \, dx$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*x^3, x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int x^3 \left(e\,x^2+d\right)^{3/2} \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int(x^3*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

3.141 $\int x(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.141.1 Optimal result

Integrand size = 21, antiderivative size = 297

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx = -\frac{b(7c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2} + \frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e} - \frac{b(15c^4d^2+10c^2de+3e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^5\sqrt{e}} - \frac{bd^{5/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{5e}$$

output
```
1/5*(e*x^2+d)^(5/2)*(a+b*arcsech(c*x))/e-1/5*b*d^(5/2)*arctanh((e*x^2+d)^(
1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e-1/40*b*
(15*c^4*d^2+10*c^2*d*e+3*e^2)*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d
)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^5/e^(1/2)-1/20*b*(e*x^2+d)^(3/2
)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2-1/40*b*(7*c^2*d+3
*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c^4
```

3.141.2 Mathematica [A] (verified)

Time = 23.44 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.15

$$\int x(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))\,dx=\frac{\sqrt{d+ex^2}\left(8ac^4(d+ex^2)^2-be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(3e+c^2(9d+2ex^2)\right)+8bc^4(d+ex^2)^2\operatorname{sec}\right.}{40c^4e}+\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{-c^2d-e}\sqrt{e}(15c^4d^2+10c^2de+3e^2)\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+8c^7d^{5/}\right.}{40c^7e(-1+cx)\sqrt{d+ex^2}}$$

input
```
Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

output
```
(Sqrt[d + e*x^2]*(8*a*c^4*(d + e*x^2)^2 - b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1
 + c*x)*(3*e + c^2*(9*d + 2*e*x^2)) + 8*b*c^4*(d + e*x^2)^2*ArcSech[c*x]))
/(40*c^4*e) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*S
qrt[-(c^2*d) - e]*Sqrt[e]*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*Sqrt[(c^2*(d +
 e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqr
t[-(c^2*d) - e])] + 8*c^7*d^(5/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1
- c^2*x^2])/Sqrt[-d - e*x^2]]))/(40*c^7*e*(-1 + c*x)*Sqrt[d + e*x^2])
```

3.141.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.86, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6853, 2036, 354, 113, 27, 171, 27, 175, 66, 104, 218, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))\,dx$$

$$\Big\downarrow \ 6853$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{(ex^2+d)^{5/2}}{x\sqrt{1-cx}\sqrt{cx+1}}dx}{5e}+\frac{(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{5e}$$

$$\Big\downarrow \ 2036$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\left(ex^2+d\right)^{5/2}}{x\sqrt{1-c^2x^2}}dx}{5e}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \ 354$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\left(ex^2+d\right)^{5/2}}{x^2\sqrt{1-c^2x^2}}dx^2}{10e}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \ 113$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{\sqrt{ex^2+d}\left(4c^2d^2+e\left(7dc^2+3e\right)x^2\right)}{2x^2\sqrt{1-c^2x^2}}dx^2}{2c^2}-\frac{e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{10e}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \ 27$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{\sqrt{ex^2+d}\left(4c^2d^2+e\left(7dc^2+3e\right)x^2\right)}{x^2\sqrt{1-c^2x^2}}dx^2}{4c^2}-\frac{e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{10e}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \ 171$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{-\frac{\int -\frac{8d^3c^4+e\left(15d^2c^4+10dec^2+3e^2\right)x^2}{2x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{1-c^2x^2}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{10e}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \ 27$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\int \frac{8d^3c^4+e\left(15d^2c^4+10dec^2+3e^2\right)x^2}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{1-c^2x^2}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{10e}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \ 175$$

3.141. $\int x\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{8c^4d^3\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+e\left(15c^4d^2+10c^2de+3e^2\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{1-c^2x^2}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{1-c^2x^2}\cdots}{2\cdots}\cdots\right.}{10e}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$\Big\downarrow$ 66

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{8c^4d^3\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+2e\left(15c^4d^2+10c^2de+3e^2\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{1-c^2x^2}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{1-c^2x^2}\cdots}{2\cdots}\cdots\right.}{10e}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$\Big\downarrow$ 104

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{16c^4d^3\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}+2e\left(15c^4d^2+10c^2de+3e^2\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{1-c^2x^2}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{1-c^2x^2}\left(d+e\cdots\right.}{2c^2}\cdots\right.}{10e}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$\Big\downarrow$ 218

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{16c^4d^3\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}-\frac{2\sqrt{e}\left(15c^4d^2+10c^2de+3e^2\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}-\frac{e\sqrt{1-c^2x^2}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{1-c^2x^2}\left(d+\cdots\right.}{2c^2}\cdots\right.}{10e}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$\Big\downarrow$ 220

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}+$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{-\frac{2\sqrt{e}\left(15c^4d^2+10c^2de+3e^2\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}-16c^4d^{5/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2c^2}-\frac{e\sqrt{1-c^2x^2}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-e\sqrt{\cdots}\cdots\right.}{10e}$$

3.141. $\int x(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

input
```
Int[x*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

output
```
((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e) + (b*Sqrt[(1 + c*x)^(-1)]*S
qrt[1 + c*x]*(-1/2*(e*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/c^2 + (-((e*(7*
c^2*d + 3*e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/c^2) + ((-2*Sqrt[e]*(15*c^
4*d^2 + 10*c^2*d*e + 3*e^2)*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d +
 e*x^2])])/c - 16*c^4*d^(5/2)*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^
2*x^2])])/(2*c^2))/(4*c^2)))/(10*e)
```

3.141.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2036
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

3.141. $\int x(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

rule 6853
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Simp[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x
^2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e
, p}, x] && NeQ[p, -1]
```

3.141.4 Maple [F]

$$\int x\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsech}\left(cx\right)\right)dx$$

input
```
int(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

output
```
int(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

3.141.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(181) = 362$.

Time = 0.71 (sec) , antiderivative size = 1667, normalized size of antiderivative = 5.61

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
[1/160*(8*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
- d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(
-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4*d^2 + 10*b*c^2*d*e + 3
*b*e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^
2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*
sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 32*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^
2 + b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1
)/(c*x)) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 - (2*b*c^4*
e^2*x^3 + 3*(3*b*c^4*d*e + b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*s
qrt(e*x^2 + d))/(c^5*e), 1/80*(4*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e +
 e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e
*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4
*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d -
e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4
+ (c^2*d*e - e^2)*x^2 - d*e)) + 16*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^
5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))
 + 2*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 - (2*b*c^4*e^2*x^3
+ 3*(3*b*c^4*d*e + b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^
2 + d))/(c^5*e), -1/160*(16*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^3*d - c*e)*
x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/...
```

3.141.6 Sympy [F]

$$\int x(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x(a+b\,\text{asech}\,(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
```

output
```
Integral(x*(a + b*asech(c*x))*(d + e*x**2)**(3/2), x)
```

3.141.7 Maxima [F]

$$\int x(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int (ex^2+d)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
1/5*(e*x^2 + d)^(5/2)*a/e + 1/15*b*(((3*e^2*x^4 + d*e*x^2 - 2*d^2)*x^3 + 5
*(d*e*x^4 + d^2*x^2)*x)*sqrt(e*x^2 + d)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) +
 1)/(e*x^3) - 15*integrate(1/15*(15*(c^2*e^2*x^4*log(c) - e^2*x^2*log(c))*
x^3 + 15*(c^2*d*e*x^4*log(c) - d*e*x^2*log(c))*x + ((3*(5*e^2*log(c) + e^2
)*c^2*x^4 - 2*c^2*d^2 + (c^2*d*e - 15*e^2*log(c))*x^2)*x^3 + 5*((3*d*e*log
(c) + d*e)*c^2*x^4 + (c^2*d^2 - 3*d*e*log(c))*x^2)*x + 30*((c^2*e^2*x^4 -
e^2*x^2)*x^3 + (c^2*d*e*x^4 - d*e*x^2)*x)*log(sqrt(x)))*e^(1/2*log(c*x + 1
) + 1/2*log(-c*x + 1)) + 30*((c^2*e^2*x^4 - e^2*x^2)*x^3 + (c^2*d*e*x^4 -
d*e*x^2)*x)*log(sqrt(x)))*sqrt(e*x^2 + d)/(c^2*e*x^4 - e*x^2 + (c^2*e*x^4
- e*x^2)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))), x))
```

3.141.8 Giac [F]

$$\int x(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int (ex^2+d)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*x, x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int x\,(e\,x^2+d)^{3/2}\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

3.142 $\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx$

3.142.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx = \operatorname{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x},x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x)
```

3.142.2 Mathematica [N/A]

Not integrable

Time = 10.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx = \int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x, x]
```

3.142.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx$$

$$\Big\downarrow \ 6865$$

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x,x]
```

output
```
$Aborted
```

3.142.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.142.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\left(a+b\,\text{arcsech}\,(cx)\right)}{x}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x)
```

3.142.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\operatorname{sech}^{-1}(cx))}{x}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arsech}(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x,
 x)
```

3.142.6 Sympy [N/A]

Not integrable

Time = 23.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\operatorname{sech}^{-1}(cx))}{x}\,dx=\int \frac{(a+b\operatorname{asech}(cx))\,(d+ex^2)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x, x)
```

3.142.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\operatorname{sech}^{-1}(cx))}{x}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.142.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x, x)
```

3.142.9 Mupad [N/A]

Not integrable

Time = 4.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x, x)
```

3.143 $\int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^3}\,dx$

3.143.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx=\text{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3},x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x)
```

3.143.2 Mathematica [N/A]

Not integrable

Time = 8.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3, x]
```

3.143.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx))}{x^3} \, dx$$

$$\Big\downarrow \text{6865}$$

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx))}{x^3} dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3,x]
```

output
```
$Aborted
```

3.143.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.143.4 Maple [N/A] (verified)

Not integrable

Time = 0.72 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}} (a+b\,\text{arcsech}\,(cx))}{x^3} dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x)
```

3.143.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{sech}^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^
3, x)
```

3.143.6 Sympy [N/A]

Not integrable

Time = 19.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{sech}^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**3,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

3.143.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{sech}^{-1}(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.143.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^3, x)
```

3.143.9 Mupad [N/A]

Not integrable

Time = 4.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{(e\,x^2+d)^{3/2}\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^3, x)
```

3.144 $\int x^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

3.144.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Int}\left(x^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right),x\right)$$

output
```
Unintegrable(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

3.144.2 Mathematica [N/A]

Not integrable

Time = 19.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

input
```
Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

output
```
Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

3.144.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{6865}$$

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) dx$$

input
```
Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

output
```
$Aborted
```

3.144.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.144.4 Maple [N/A] (verified)

Not integrable

Time = 0.83 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^{2} \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\operatorname{arcsech}\left(c x\right)\right)dx$$

input
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

output
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

3.144.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int x^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int(ex^2+d)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arcsech(c*x))*sqrt(e*x^2
 + d), x)
```

3.144.6 Sympy [N/A]

Not integrable

Time = 69.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^2(a+b\,\text{asech}\,(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
```

output
```
Integral(x**2*(a + b*asech(c*x))*(d + e*x**2)**(3/2), x)
```

3.144.7 Maxima [F(-2)]

Exception generated.

$$\int x^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.144.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int\left(ex^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arsech}\left(cx\right)+a\right)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*x^2, x)
```

3.144.9 Mupad [N/A]

Not integrable

Time = 4.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int x^2\,\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

3.145 $\int (d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx)) \, dx$

3.145.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx)) \, dx = \text{Int}\Big((d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx)), x\Big)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

3.145.2 Mathematica [N/A]

Not integrable

Time = 8.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx)) \, dx = \int (d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx)) \, dx$$

input
```
Integrate[(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

output
```
Integrate[(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

3.145.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{ 6865}$$

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right)dx$$

input
```
Int[(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

output
```
$Aborted
```

3.145.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.145.4 Maple [N/A] (verified)

Not integrable

Time = 0.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\operatorname{arcsech}\left(c x\right)\right)dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

3.145.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\,\text{arsech}\,(cx)+a) \, dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d), x
)
```

3.145.6 Sympy [N/A]

Not integrable

Time = 20.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (a+b\,\text{asech}\,(cx)) \left(d+ex^2\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2), x)
```

3.145.7 Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.145.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\operatorname{arsech}\left(cx\right)+a\right)dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a), x)
```

3.145.9 Mupad [N/A]

Not integrable

Time = 4.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int \left(e\,x^2+d\right)^{3/2}\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

3.146 $\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx$

3.146.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2},x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x)
```

3.146.2 Mathematica [N/A]

Not integrable

Time = 14.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2, x]
```

3.146.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx))}{x^2} \, dx$$

$$\Big\downarrow \text{6865}$$

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx))}{x^2}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2,x]
```

output
```
$Aborted
```

3.146.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.146.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}} (a+b \operatorname{arcsech}(cx))}{x^2}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x)
```

3.146.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arsech}(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^
2, x)
```

3.146.6 Sympy [N/A]

Not integrable

Time = 18.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\operatorname{asech}(cx))(d+ex^2)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**2,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

3.146.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.146.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arsech}(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^2, x)
```

3.146.9 Mupad [N/A]

Not integrable

Time = 4.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(e\,x^2+d)^{3/2}\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^2, x)
```

3.147 $\int \frac{(d+ex^2)^{3/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^4}\,dx$

3.147.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^4},x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x)
```

3.147.2 Mathematica [N/A]

Not integrable

Time = 18.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{(d+ex^2)^{3/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^4}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4, x]
```

3.147.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^4}dx$$
$$\Big\downarrow \text{ 6865}$$
$$\int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^4}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4,x]
```

output
```
$Aborted
```

3.147.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.147.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}(a+b\,\text{arcsech}\,(cx))}{x^4}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x)
```

3.147.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arsech}(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^
4, x)
```

3.147.6 Sympy [N/A]

Not integrable

Time = 20.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x^4}\,dx = \int \frac{(a+b\operatorname{asech}(cx))(d+ex^2)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**4,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

3.147.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.147.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{sech}^{-1}(cx))}{x^4}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^4, x)
```

3.147.9 Mupad [N/A]

Not integrable

Time = 4.61 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{sech}^{-1}(cx))}{x^4}\,dx=\int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^4, x)
```

3.148 $\int \frac{(d+ex^2)^{3/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^6}\,dx$

3.148.1 Optimal result

Integrand size = 23, antiderivative size = 409

$$\begin{aligned}\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx &= \frac{4b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75x^3}\\
&+\frac{b(8c^4d^2+23c^2de+23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75dx}\\
&+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25x^5}-\frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}\\
&+\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{75d\sqrt{1+\frac{ex^2}{d}}}\\
&-\frac{b(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75cd\sqrt{d+ex^2}}\end{aligned}$$

output
```
-1/5*(e*x^2+d)^(5/2)*(a+b*arcsech(c*x))/d/x^5+1/25*b*(e*x^2+d)^(3/2)*(1/(c
*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^5+4/75*b*(c^2*d+2*e)*(1/(c
*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/x^3+1/75*b*(
8*c^4*d^2+23*c^2*d*e+23*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^
(1/2)*(e*x^2+d)^(1/2)/d/x+1/75*b*c*(8*c^4*d^2+23*c^2*d*e+23*e^2)*EllipticE
(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d/(
1+e*x^2/d)^(1/2)-1/75*b*(c^2*d+e)*(8*c^4*d^2+19*c^2*d*e+15*e^2)*EllipticF(
c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/
d/(e*x^2+d)^(1/2)
```

3.148.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.98 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx = \frac{-\frac{15a(d+ex^2)^3}{x^5} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(d+ex^2)\left(23e^2x^4+dex^2(11+23c^2x^2)+d^2(3+4c^2x^2+8c^4\cdots\right.}{x^5}\cdots}{\cdots}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^6,x]
```

output
```
((-15*a*(d + e*x^2)^3)/x^5 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e
*x^2)*(23*e^2*x^4 + d*e*x^2*(11 + 23*c^2*x^2) + d^2*(3 + 4*c^2*x^2 + 8*c^4
*x^4)))/x^5 - (15*b*(d + e*x^2)^3*ArcSech[c*x])/x^5 + (b*Sqrt[(1 - c*x)/(1
 + c*x)]*(-(c^2*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*(d + e*x^2)) - (I*(c*Sqr
t[d] - I*Sqrt[e])^2*(1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d]
 - I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I
*Sqrt[e])*(1 + c*x))]*((8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*EllipticE[I*ArcSi
nh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (
c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + 2*Sqrt[e]*((8*I)*c^3
*d^(3/2) - 12*c^2*d*Sqrt[e] + (7*I)*c*Sqrt[d]*e - 15*e^(3/2))*EllipticF[I*
ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))
]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))/Sqrt[-(((c*Sqrt
[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x)))]))/c)/(7
5*d*Sqrt[d + e*x^2])
```

3.148.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.83, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {6855, 27, 376, 442, 445, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx))}{x^6} \, dx$$

$$\Big\downarrow \text{6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{(ex^2+d)^{5/2}}{5dx^6\sqrt{1-c^2x^2}}dx - \frac{(d+ex^2)^{5/2} (a+b\text{sech}^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(ex^2+d)^{5/2}}{x^6\sqrt{1-c^2x^2}}dx}{5d} - \frac{(d+ex^2)^{5/2} (a+b\text{sech}^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \text{376}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\int \frac{\sqrt{ex^2+d}(e(dc^2+5e)x^2+4d(dc^2+2e))}{x^4\sqrt{1-c^2x^2}}dx - \frac{d\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{5x^5}\right)}{5d} - \frac{(d+ex^2)^{5/2} (a+b\text{sech}^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \text{442}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{e(4d^2c^4+11dec^2+15e^2)x^2+d(8d^2c^4+23dec^2+23e^2)}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx - \frac{4d\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{d+ex^2}}{3x^3}\right) - \frac{d\sqrt{1-c^2x^2}(d+}{5x^5}}{5d}$$
$$\frac{(d+ex^2)^{5/2} (a+b\text{sech}^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \text{445}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\frac{\int -\frac{de(4d^2c^4-(8d^2c^4+23dec^2+23e^2)x^2c^2+11dec^2+15e^2)}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d} - \frac{\sqrt{1-c^2x^2}(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}}{x}\right) -}{5d}$$
$$\frac{(d+ex^2)^{5/2} (a+b\text{sech}^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int \frac{de\left(4d^2c^4-\left(8d^2c^4+23dec^2+23e^2\right)x^2c^2+11dec^2+15e^2\right)}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d}-\frac{\sqrt{1-c^2x^2}\left(8c^4d^2+23c^2de+23e^2\right)\sqrt{d+ex^2}}{x}\right)-\frac{4d\cdots}{\cdots}\right.\right.}{5d}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}$$

$\Big\downarrow$ 27

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\int \frac{4d^2c^4-\left(8d^2c^4+23dec^2+23e^2\right)x^2c^2+11dec^2+15e^2}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx-\frac{\sqrt{1-c^2x^2}\left(8c^4d^2+23c^2de+23e^2\right)\sqrt{d+ex^2}}{x}\right)-\frac{4d\cdots}{\cdots}\right.\right.}{5d}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}$$

$\Big\downarrow$ 399

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\left(c^2d+e\right)\left(8c^4d^2+19c^2de+15e^2\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{e}-\frac{c^2\left(8c^4d^2+23c^2de+23e^2\right)\int \frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}\right)-\frac{\sqrt{1\cdots}}{\cdots}\right.\right.}{5d}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}$$

$\Big\downarrow$ 323

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\left(c^2d+e\right)\left(8c^4d^2+19c^2de+15e^2\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2\left(8c^4d^2+23c^2de+23e^2\right)\int \frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}d\cdots}{e}\right.\right.\right.}{5d}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}$$

$\Big\downarrow$ 321

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\left(c^2d+e\right)\left(8c^4d^2+19c^2de+15e^2\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c^2\left(8c^4d^2+23c^2de+23e^2\right)\int \frac{\sqrt{ex^2\cdots}}{\sqrt{1-c^{\cdots}}}}{e}\right.\right.\right.}{5d}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}$$

$\Big\downarrow$ 330

3.148. $\displaystyle\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c^2(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}\cdots}{e\sqrt{\frac{ex^2}{d}+1}}\cdots\right.\right.\right.\right.}{5d}$$

$$\frac{(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \; 327$$

$$-\frac{(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{5dx^5}-$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}E\left(\cdots\right.}{e\sqrt{\frac{ex^2}{d}+1}}\cdots\right.\right.\right.\right.}{5d}$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^6,x]
```

output
```
-1/5*((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(d*x^5) - (b*Sqrt[(1 + c*x)^
(-1)]*Sqrt[1 + c*x]*(-1/5*(d*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/x^5 + ((
-4*d*(c^2*d + 2*e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(3*x^3) + (-(((8*c^4
*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/x) + e*(-((
c*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x],
 -(e/(c^2*d))])/(e*Sqrt[1 + (e*x^2)/d])) + ((c^2*d + e)*(8*c^4*d^2 + 19*c^
2*d*e + 15*e^2)*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/
(c*e*Sqrt[d + e*x^2])))/3)/5))/(5*d)
```

3.148.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.148. $\int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^6}\,dx$

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

3.148. $\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.148.4 Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsech}\,(cx))}{x^6}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x)
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\mathrm{sech}^{-1}(cx))}{x^6}\,dx =$$

$$-\frac{15\,(bcde^2x^4+2\,bcd^2ex^2+bcd^3)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+\left(15\,acde^2x^4+30\,acd^2ex^2+15\,acd^3-(3\right.}{}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/75*(15*(b*c*d*e^2*x^4 + 2*b*c*d^2*e*x^2 + b*c*d^3)*sqrt(e*x^2 + d)*log(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + (15*a*c*d*e^2*x^4 + 30*a
*c*d^2*e*x^2 + 15*a*c*d^3 - (3*b*c^2*d^3*x + (8*b*c^6*d^3 + 23*b*c^4*d^2*e
 + 23*b*c^2*d*e^2)*x^5 + (4*b*c^4*d^3 + 11*b*c^2*d^2*e)*x^3)*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d) - ((8*b*c^8*d^3 + 23*b*c^6*d^2*e + 23*b
*c^4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (8*b*c^8*d^3 + (23*b
*c^6 + 4*b*c^4)*d^2*e + (23*b*c^4 + 11*b*c^2)*d*e^2 + 15*b*e^3)*x^5*ellipt
ic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(d))/(c*d^2*x^5)
```

3.148.6 Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^6}\,dx = \int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**6,x)
```

output
```
Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x**6, x)
```

3.148.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.148.8 Giac [F]

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\operatorname{sech}^{-1}(cx))}{x^6}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arsech}(cx)+a)}{x^6}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^6, x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\operatorname{sech}^{-1}(cx))}{x^6}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^6, x)
```

3.149 $\int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^8}\,dx$

3.149.1 Optimal result

Integrand size = 23, antiderivative size = 556

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^8}\,dx = \frac{b(120c^4d^2+159c^2de-37e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3}$$
$$+\frac{b(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675d^2x}$$
$$+\frac{b(30c^2d+11e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{1225dx^5}$$
$$+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{49dx^7}$$
$$-\frac{(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{7dx^7}+\frac{2e(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{35d^2x^5}$$
$$+\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{3675d^2\sqrt{1+\frac{ex^2}{d}}}$$
$$-\frac{2b(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3675cd^2\sqrt{d+ex^2}}$$

output
```
-1/7*(e*x^2+d)^(5/2)*(a+b*arcsech(c*x))/d/x^7+2/35*e*(e*x^2+d)^(5/2)*(a+b*
arcsech(c*x))/d^2/x^5+1/1225*b*(30*c^2*d+11*e)*(e*x^2+d)^(3/2)*(1/(c*x+1))
^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/x^5+1/49*b*(e*x^2+d)^(5/2)*(1/(c
*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/x^7+1/3675*b*(120*c^4*d^2+
159*c^2*d*e-37*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*
x^2+d)^(1/2)/d/x^3+1/3675*b*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e
^3)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2
/x+1/3675*b*c*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3)*EllipticE(
c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/
(1+e*x^2/d)^(1/2)-2/3675*b*(c^2*d+e)*(120*c^6*d^3+204*c^4*d^2*e+17*c^2*d*e
^2-105*e^3)*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2
)*(1+e*x^2/d)^(1/2)/c/d^2/(e*x^2+d)^(1/2)
```

3.149.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.47 (sec) , antiderivative size = 1187, normalized size of antiderivative = 2.13

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx = \left(-\frac{ad}{7x^7}-\frac{8ae}{35x^5}-\frac{ae^2}{35dx^3}+\frac{2ae^3}{35d^2x}\right)\sqrt{d+ex^2}$$
$$+\left(\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)}{3675d^2}+\frac{bd}{49x^7}+\frac{bcd}{49x^6}+\frac{b(30c^2d+61e)}{1225x^5}\right.$$
$$+\frac{bc(30c^2d+61e)}{1225x^4}+\frac{b(120c^4d^2+249c^2de+71e^2)}{3675dx^3}+\frac{bc(120c^4d^2+249c^2de+71e^2)}{3675dx^2}$$
$$\left.+\frac{b(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)}{3675d^2x}\right)\sqrt{\frac{1-cx}{1+cx}}\sqrt{d+ex^2}$$
$$-\frac{b(5d-2ex^2)\left(d+ex^2\right)^{5/2}\operatorname{sech}^{-1}(cx)}{35d^2x^7}$$
$$+\frac{-bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{\frac{1-cx}{1+cx}}\sqrt{\frac{e\left(-1+\frac{1-cx}{1+cx}\right)^2+c^2d\left(1+\frac{1-cx}{1+cx}\right)^2}{c^2\left(1+\frac{1-cx}{1+cx}\right)^2}}-\frac{ib\left(c\sqrt{d}-i\sqrt{e}\right)^2\sqrt{1+\frac{(c^2d+e)(}{(c\sqrt{d}-i\sqrt{e})}}}{}}{}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^8,x]
```

output
```
(-1/7*(a*d)/x^7 - (8*a*e)/(35*x^5) - (a*e^2)/(35*d*x^3) + (2*a*e^3)/(35*d^
2*x))*Sqrt[d + e*x^2] + ((b*c*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2
 - 247*e^3))/(3675*d^2) + (b*d)/(49*x^7) + (b*c*d)/(49*x^6) + (b*(30*c^2*d
 + 61*e))/(1225*x^5) + (b*c*(30*c^2*d + 61*e))/(1225*x^4) + (b*(120*c^4*d^
2 + 249*c^2*d*e + 71*e^2))/(3675*d*x^3) + (b*c*(120*c^4*d^2 + 249*c^2*d*e
+ 71*e^2))/(3675*d*x^2) + (b*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2
- 247*e^3))/(3675*d^2*x))*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[d + e*x^2] - (b*(
5*d - 2*e*x^2)*(d + e*x^2)^(5/2)*ArcSech[c*x])/(35*d^2*x^7) + (-(b*c*(240*
c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[(1 - c*x)/(1 + c*x
)]*Sqrt[(e*(-1 + (1 - c*x)/(1 + c*x))^2 + c^2*d*(1 + (1 - c*x)/(1 + c*x))^
2)/(c^2*(1 + (1 - c*x)/(1 + c*x))^2)]) - (I*b*(c*Sqrt[d] - I*Sqrt[e])^2*Sq
rt[1 + ((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] - I*Sqrt[e])^2*(1 + c*x))]*Sqrt
[1 + ((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]*((Sqrt
[(c^2*d + e)/(c*Sqrt[d] + I*Sqrt[e])^2]*(240*c^6*d^3 + 528*c^4*d^2*e + 193
*c^2*d*e^2 - 247*e^3)*Sqrt[(1 - c*x)/(1 + c*x)]*EllipticE[I*ArcSinh[Sqrt[(
(c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d]
 + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2])/Sqrt[((c^2*d + e)*(1 - c*x))/(
(c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))] + (2*Sqrt[e]*Sqrt[(c^2*d + e)/(c*Sqr
t[d] + I*Sqrt[e])^2]*((240*I)*c^5*d^(5/2) - 360*c^4*d^2*Sqrt[e] + (48*I)*c
^3*d^(3/2)*e - 207*c^2*d*e^(3/2) - (173*I)*c*Sqrt[d]*e^2 + 210*e^(5/2))...
```

3.149.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 462, normalized size of antiderivative = 0.83, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {6855, 27, 442, 442, 442, 445, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^8}dx$$

$$\Big\downarrow \text{ 6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{(5d-2ex^2)(ex^2+d)^{5/2}}{35d^2x^8\sqrt{1-c^2x^2}}dx+\frac{2e(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{7dx^7}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{(5d-2ex^2)(ex^2+d)^{5/2}}{x^8\sqrt{1-c^2x^2}}dx}{35d^2}+\frac{2e(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{7dx^7}$$

$\Big\downarrow$ 442

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\int\frac{(ex^2+d)^{3/2}((5c^2d-14e)ex^2+d(30dc^2+11e))}{x^6\sqrt{1-c^2x^2}}dx-\frac{5d\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{7x^7}\right)}{35d^2}+\frac{2e(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{7dx^7}$$

$\Big\downarrow$ 442

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(\frac{1}{5}\int\frac{\sqrt{ex^2+d}(2e(15d^2c^4+18dec^2-35e^2)x^2+d(120d^2c^4+159dec^2-37e^2))}{x^4\sqrt{1-c^2x^2}}dx-\frac{d\sqrt{1-c^2x^2}(30c^2d+11e)(d+ex^2)^{3/}}{5x^5}\right.\right.}{35d^2}$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{7dx^7}$$

$\Big\downarrow$ 442

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{e(120d^3c^6+249d^2ec^4+71de^2c^2-210e^3)x^2+d(240d^3c^6+528d^2ec^4+193de^2c^2-247e^3)}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx-\frac{d\sqrt{1-c^2x^2}(120}{}\right.\right.\right.}{35d^2}$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{7dx^7}$$

$\Big\downarrow$ 445

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\frac{\int-\frac{de\left(120d^3c^6+249d^2ec^4+71de^2c^2-\left(240d^3c^6+528d^2ec^4+193de^2c^2-247e^3\right)x^2c^2-210e^3\right)}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d}-\frac{\sqrt{1-c^2x^2}(24}{}\right.\right.\right.\right.}{}$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{7dx^7}$$

$\Big\downarrow$ 25

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int\frac{de\left(120d^3c^6+249d^2ec^4+71de^2c^2-\left(240d^3c^6+528d^2ec^4+193de^2c^2-247e^3\right)x^2c^2-210e^3\right)}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d}-\frac{\sqrt{1-c^2x^2}(240c^6d}{}\right.\right.\right.\right.}{}$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{7dx^7}$$

3.149. $\int\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}dx$

$$\Big\downarrow \ 27$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\int\frac{120d^3c^6+249d^2ec^4+71de^2c^2-(240d^3c^6+528d^2ec^4+193de^2c^2-247e^3)x^2c^2-210e^3}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx-\frac{\sqrt{1-c^2x^2}(240\cdots}{\cdots}\right.\right.\right.\right.}{\cdots}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7dx^7}$$

$$\Big\downarrow \ 399$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{e}-\frac{c^2(240c^6d^3+528c^4d^2e+193c\cdots}{e\cdots}\right.\right.\right.\right.\right.}{\cdots}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7dx^7}$$

$$\Big\downarrow \ 323$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2(240c^6d^3+528c\cdots}{\cdots}\right.\right.\right.\right.\right.}{\cdots}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7dx^7}$$

$$\Big\downarrow \ 321$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c^2(240c^6d^3+\cdots}{\cdots}\right.\right.\right.\right.\right.}{\cdots}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7dx^7}$$

$$\Big\downarrow \ 330$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c^2(240c^6d^3\cdots}{\cdots}\right.\right.\right.\right.\right.}{\cdots}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7dx^7}$$

$$\Big\downarrow \ 327$$

3.149. $\int\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}dx$

$$\frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7dx^7}-$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d+e)\left(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3\right)\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c(240c^6d^3+528\cdots}{}\right.\right.\right.\right.\right.}{}$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^8,x]
```

output
```
-1/7*((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(d*x^7) + (2*e*(d + e*x^2)^(
5/2)*(a + b*ArcSech[c*x]))/(35*d^2*x^5) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 +
 c*x]*((-5*d*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(5/2))/(7*x^7) + (-1/5*(d*(30*c
^2*d + 11*e)*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/x^5 + (-1/3*(d*(120*c^4*
d^2 + 159*c^2*d*e - 37*e^2)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/x^3 + (-(((
240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[1 - c^2*x^2]*S
qrt[d + e*x^2])/x) + e*(-((c*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2
- 247*e^3)*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[1
 + (e*x^2)/d])) + (2*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^2*e + 17*c^2*d*e
^2 - 105*e^3)*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c
*e*Sqrt[d + e*x^2])))/3)/5)/7))/(35*d^2)
```

3.149.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

3.149. $\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx$

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.149. $\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx$

3.149.4 Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsech}\,(cx))}{x^8}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x)
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\mathrm{sech}^{-1}(cx))}{x^8}\,dx = \frac{105\,(2\,bcde^3x^6 - bcd^2e^2x^4 - 8\,bcd^3ex^2 - 5\,bcd^4)\sqrt{ex^2+d}\log\Bigg(\frac{cx\sqrt{-}}{}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x, algorithm="fricas")
```

output
```
1/3675*(105*(2*b*c*d*e^3*x^6 - b*c*d^2*e^2*x^4 - 8*b*c*d^3*e*x^2 - 5*b*c*d
^4)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) +
(210*a*c*d*e^3*x^6 - 105*a*c*d^2*e^2*x^4 - 840*a*c*d^3*e*x^2 - 525*a*c*d^4
 + (75*b*c^2*d^4*x + (240*b*c^8*d^4 + 528*b*c^6*d^3*e + 193*b*c^4*d^2*e^2
- 247*b*c^2*d*e^3)*x^7 + (120*b*c^6*d^4 + 249*b*c^4*d^3*e + 71*b*c^2*d^2*e
^2)*x^5 + 3*(30*b*c^4*d^4 + 61*b*c^2*d^3*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*
x^2)))*sqrt(e*x^2 + d) + ((240*b*c^10*d^4 + 528*b*c^8*d^3*e + 193*b*c^6*d^
2*e^2 - 247*b*c^4*d*e^3)*x^7*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (240*b*
c^10*d^4 + 24*(22*b*c^8 + 5*b*c^6)*d^3*e + (193*b*c^6 + 249*b*c^4)*d^2*e^2
 - (247*b*c^4 - 71*b*c^2)*d*e^3 - 210*b*e^4)*x^7*elliptic_f(arcsin(c*x), -
e/(c^2*d)))*sqrt(d))/(c*d^3*x^7)
```

3.149.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx=\text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**8,x)
```

output
```
Timed out
```

3.149.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.149.8 Giac [F]

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arsech}(cx)+a)}{x^8}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^8, x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(e\,x^2+d)^{3/2}\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^8, x)
```

3.150 $\int \frac{x^5\left(a+b\,\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.150.1 Optimal result

Integrand size = 23, antiderivative size = 356

$$\begin{aligned}\int \frac{x^5\left(a+b\,\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx &= \frac{b(19c^2d-9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e^2} \\ &- \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c^2e^2} \\ &+ \frac{d^2\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{e^3} \\ &- \frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{3e^3} \\ &+ \frac{\left(d+ex^2\right)^{5/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{5e^3} \\ &- \frac{b(45c^4d^2-10c^2de+9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^5e^{5/2}} \\ &- \frac{8bd^{5/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{15e^3}\end{aligned}$$

output
```
-2/3*d*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^3+1/5*(e*x^2+d)^(5/2)*(a+b*arc
sech(c*x))/e^3-1/120*b*(45*c^4*d^2-10*c^2*d*e+9*e^2)*arctan(e^(1/2)*(-c^2*
x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^5/e^(5/2
)-8/15*b*d^(5/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c
*x+1))^(1/2)*(c*x+1)^(1/2)/e^3-1/20*b*(e*x^2+d)^(3/2)*(1/(c*x+1))^(1/2)*(c
*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2+d^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1
/2)/e^3+1/120*b*(19*c^2*d-9*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1
)^(1/2)*(e*x^2+d)^(1/2)/c^4/e^2
```

3.150.2 Mathematica [A] (verified)

Time = 23.56 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.03

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$
$$=\frac{\sqrt{d+ex^2}\left(8ac^4(8d^2-4dex^2+3e^2x^4)-be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(9e+c^2(-13d+6ex^2)\right)+8bc^4(8d^2-4dex^2+\right.}{120c^4e^3}$$
$$+\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{-c^2d-e}\sqrt{e}(45c^4d^2-10c^2de+9e^2)\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+64c\right.}{120c^7e^3(-1+cx)\sqrt{d+ex^2}}$$

input
```
Integrate[(x^5*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d + e*x^2]*(8*a*c^4*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) - b*e*Sqrt[(1 -
c*x)/(1 + c*x)]*(1 + c*x)*(9*e + c^2*(-13*d + 6*e*x^2)) + 8*b*c^4*(8*d^2 -
 4*d*e*x^2 + 3*e^2*x^4)*ArcSech[c*x]))/(120*c^4*e^3) + (b*Sqrt[(1 - c*x)/(
1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*(45*c^4
*d^2 - 10*c^2*d*e + 9*e^2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*S
qrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])] + 64*c^7*d^(5/2
)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/
(120*c^7*e^3*(-1 + c*x)*Sqrt[d + e*x^2])
```

3.150.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.86, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {6855, 27, 7282, 2118, 27, 171, 27, 175, 66, 104, 218, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \ 6855$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\sqrt{ex^2+d}\left(3e^2x^4-4dex^2+8d^2\right)}{15e^3x\sqrt{1-c^2x^2}}dx+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \ 27$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\sqrt{ex^2+d}\left(3e^2x^4-4dex^2+8d^2\right)}{x\sqrt{1-c^2x^2}}dx}{15e^3}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \ 7282$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\sqrt{ex^2+d}\left(3e^2x^4-4dex^2+8d^2\right)}{x^2\sqrt{1-c^2x^2}}dx^2}{30e^3}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \ 2118$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{e\sqrt{ex^2+d}\left(32c^2d^2-\left(19c^2d-9e\right)ex^2\right)}{2x^2\sqrt{1-c^2x^2}}dx^2}{2c^2e}-\frac{3e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \ 27$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int\frac{\sqrt{ex^2+d}\left(32c^2d^2-\left(19c^2d-9e\right)ex^2\right)}{x^2\sqrt{1-c^2x^2}}dx^2}{4c^2}-\frac{3e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 171$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{e\sqrt{1-c^2x^2}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}-\frac{\int-\frac{64d^3c^4+e\left(45d^2c^4-10dec^2+9e^2\right)x^2}{2x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{c^2}}{4c^2}-\frac{3e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 27$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\int\frac{64d^3c^4+e\left(45d^2c^4-10dec^2+9e^2\right)x^2}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 175$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{64c^4d^3\int\frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+e\left(45c^4d^2-10c^2de+9e^2\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-3e\sqrt{1-c}\right.}{30e^3}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 66$$

3.150. $\int\frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{64c^4d^3\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+2e\left(45c^4d^2-10c^2de+9e^2\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{1-c\ldots}}{\ldots}\right.$$
$$\overline{30e^3}$$
$$\frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 104$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{128c^4d^3\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}+2e\left(45c^4d^2-10c^2de+9e^2\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{1-c^2x^2}\left(\ldots\right.}{2c^2}\right.$$
$$\overline{30e^3}$$
$$\frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 218$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{128c^4d^3\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}-\frac{2\sqrt{e}\left(45c^4d^2-10c^2de+9e^2\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{1-c^2x^2}\ldots}{2c\ldots}\right.$$
$$\overline{30e^3}$$
$$\frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 220$$

$$\frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+$$
$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{-\frac{2\sqrt{e}\left(45c^4d^2-10c^2de+9e^2\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}-128c^4d^{5/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2c^2}+\frac{e\sqrt{1-c^2x^2}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3\ldots}{\ldots}\right.$$
$$\overline{30e^3}$$

3.150. $\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx$

input
```
Int[(x^5*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(d^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e^3 - (2*d*(d + e*x^2)^(3/2)*(a
 + b*ArcSech[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*
e^3) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-3*e*Sqrt[1 - c^2*x^2]*(d +
 e*x^2)^(3/2))/(2*c^2) + (((19*c^2*d - 9*e)*e*Sqrt[1 - c^2*x^2]*Sqrt[d + e
*x^2])/c^2 + ((-2*Sqrt[e]*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*ArcTan[(Sqrt[e
]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/c - 128*c^4*d^(5/2)*ArcTanh[Sqr
t[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(2*c^2))/(4*c^2)))/(30*e^3)
```

3.150.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.150. $\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.150.4 Maple [F]

$$\int \frac{x^5 (a + b \operatorname{arcsech}(cx))}{\sqrt{e\, x^2 + d}} dx$$

input
```
int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 1679, normalized size of antiderivative = 4.72

$$\int \frac{x^5 \left(a + b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d + ex^2}} \, dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/480*(64*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (45*b*c^4*d^2 - 10*b*c^2*d*e +
9*b*e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c
^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e
*x^2 + 8*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
) + 1)/(c*x)) + 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 - (6
*b*c^4*e^2*x^3 - (13*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*
x^2)))*sqrt(e*x^2 + d))/(c^5*e^3), 1/240*(32*b*c^5*d^(5/2)*log(((c^4*d^2 -
 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c
*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4)
 - (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3
 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(
c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) + 16*(3*b*c^5*e^2*x^4 - 4*b*c^5*
d*e*x^2 + 8*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2)) + 1)/(c*x)) + 2*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 -
 (6*b*c^4*e^2*x^3 - (13*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c
^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e^3), -1/480*(128*b*c^5*sqrt(-d)*d^2*arcta
n(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^...
```

3.150.6 Sympy [F]

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x^5(a+b\operatorname{asech}(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**5*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

3.150.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.150.8 Giac [F]

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\operatorname{arsech}(cx)+a)x^5}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^5/sqrt(e*x^2 + d), x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x^5\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.151 $\int \frac{x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.151.1 Optimal result

Integrand size = 23, antiderivative size = 251

$$\begin{aligned}\int \frac{x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx &= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e} \\ &\quad -\frac{d\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^2} \\ &\quad +\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3e^2} \\ &\quad +\frac{b(3c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^3e^{3/2}} \\ &\quad +\frac{2bd^{3/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^2}\end{aligned}$$

output
```
1/3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^2+1/6*b*(3*c^2*d-e)*arctan(e^(1/2
)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^
3/e^(3/2)+2/3*b*d^(3/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2)
)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^2-d*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)
/e^2-1/6*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1
/2)/c^2/e
```

3.151.2 Mathematica [A] (verified)

Time = 22.96 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.62

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$
$$= -\frac{\sqrt{d+ex^2}\left(be\sqrt{\frac{1-cx}{1+cx}}(1+cx)+2ac^2(2d-ex^2)+2bc^2(2d-ex^2)\operatorname{sech}^{-1}(cx)\right)}{6c^2e^2}$$
$$-\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(-3(-c^2)^{3/2}\,d\sqrt{-c^2d-e}\sqrt{e}\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+\sqrt{-c^2}\sqrt{-c^2d-e}\,ee^{3/2}}{6c^5e^2(-1+cx)\sqrt{d+ex^2}}$$

input
```
Integrate[(x^3*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
-1/6*(Sqrt[d + e*x^2]*(b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + 2*a*c^2*(
2*d - e*x^2) + 2*b*c^2*(2*d - e*x^2)*ArcSech[c*x]))/(c^2*e^2) - (b*Sqrt[(1
 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(-3*(-c^2)^(3/2)*d*Sqrt[-(c^2*d) - e]
*Sqrt[e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^
2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])] + Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*e
^(3/2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(Sqrt[-c^2]*Sqrt[e]*Sqrt
[1 - c^2*x^2])/(c*Sqrt[-(c^2*d) - e])] + 4*c^5*d^(3/2)*Sqrt[-d - e*x^2]*Ar
cTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(6*c^5*e^2*(-1 + c*x)
*Sqrt[d + e*x^2])
```

3.151.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.86, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6855, 27, 435, 171, 27, 175, 66, 104, 218, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \; 6855$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{(2d-ex^2)\sqrt{ex^2+d}}{3e^2x\sqrt{1-c^2x^2}}dx+\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^2}$$

$$\Big\downarrow 27$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(2d-ex^2)\sqrt{ex^2+d}}{x\sqrt{1-c^2x^2}}dx}{3e^2}+\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^2}$$

$$\Big\downarrow 435$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(2d-ex^2)\sqrt{ex^2+d}}{x^2\sqrt{1-c^2x^2}}dx^2}{6e^2}+\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^2}$$

$$\Big\downarrow 171$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}-\frac{\int -\frac{4c^2d^2+(3c^2d-e)ex^2}{2x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{c^2}\right)}{6e^2}+\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^2}$$

$$\Big\downarrow 27$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{4c^2d^2+(3c^2d-e)ex^2}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e^2}+\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^2}$$

$$\Big\downarrow 175$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{4c^2d^2\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+e(3c^2d-e)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e^2}+\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^2}$$

$$\Big\downarrow 66$$

3.151. $\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{4c^2d^2\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+2e\left(3c^2d-e\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \; 104$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{8c^2d^2\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}+2e\left(3c^2d-e\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \; 218$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{8c^2d^2\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}-\frac{2\sqrt{e}\left(3c^2d-e\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \; 220$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{-\frac{2\sqrt{e}\left(3c^2d-e\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}-8c^2d^{3/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2c^2}+\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e^2}$$

input
```
Int[(x^3*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
-((d*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e^2) + ((d + e*x^2)^(3/2)*(a +
b*ArcSech[c*x]))/(3*e^2) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((e*Sqrt[
1 - c^2*x^2]*Sqrt[d + e*x^2])/c^2 + ((-2*(3*c^2*d - e)*Sqrt[e]*ArcTan[(Sqr
t[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/c - 8*c^2*d^(3/2)*ArcTanh[Sq
rt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(2*c^2)))/(6*e^2)
```

3.151. $\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.151.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

3.151. $\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.151.4 Maple [F]

$$\int \frac{x^{3} \left(a +b \,\operatorname{arcsech}\left(c x \right)\right)}{\sqrt{e \,x^{2}+d}}d x$$

input
```
int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

3.151.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(159) = 318$.

Time = 0.48 (sec) , antiderivative size = 1389, normalized size of antiderivative = 5.53

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(4*b*c^3*d^(3/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
 d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + (3*b*c^2*d - b*e)*sqrt(-e)*log(8*
c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e
*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)) + e^2) + 8*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(
2*b*c^3*d^(3/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x
^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + (3*b*c^2*d - b*e)*sqrt(e)*arctan(1/2*(2*
c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) + 4*(b*c^3*e*x^2 - 2*b*
c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))
 + 2*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 4*a*c^3*d
)*sqrt(e*x^2 + d))/(c^3*e^2), 1/24*(8*b*c^3*sqrt(-d)*d*arctan(-1/2*((c^3*d
 - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d - b*e)*sqrt(-
e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4
*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*...
```

3.151.6 Sympy [F]

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x^3(a+b\operatorname{asech}(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

3.151.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\operatorname{sech}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.151.8 Giac [F]

$$\int \frac{x^3(a+b\operatorname{sech}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\operatorname{arsech}(cx)+a)x^3}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\operatorname{sech}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^3\left(a+b\,\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.152 $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.152.1 Optimal result

Integrand size = 21, antiderivative size = 153

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e}$$

output
```
-b*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*d^(1/2)*(1/(c*x+1))
^(1/2)*(c*x+1)^(1/2)/e-b*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/
2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c/e^(1/2)+(a+b*arcsech(c*x))*(e*x^2+d)
^(1/2)/e
```

3.152.2 Mathematica [A] (verified)

Time = 21.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.56

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e} + \frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{-c^2d-e}\sqrt{e}\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+c^3\sqrt{d}\sqrt{-d-ex^2}\arctan\left(\frac{\sqrt{\ }}{\sqrt{\ }}\right.\right.}{c^3e(-1+cx)\sqrt{d+ex^2}}$$

input
```
Integrate[(x*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sq
rt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*Sqrt[(c^2*(d + e*x^
2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c
^2*d) - e])] + c^3*Sqrt[d]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x
^2])/Sqrt[-d - e*x^2]]))/(c^3*e*(-1 + c*x)*Sqrt[d + e*x^2])
```

3.152.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6853, 2036, 354, 140, 27, 66, 104, 218, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{6853}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\sqrt{ex^2+d}}{x\sqrt{1-cx}\sqrt{cx+1}}dx}{e} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{2036}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\sqrt{ex^2+d}}{x\sqrt{1-c^2x^2}}dx}{e} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{354}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\sqrt{ex^2+d}}{x^2\sqrt{1-c^2x^2}}dx^2}{2e}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{ 140}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+\int \frac{d}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2\right)}{2e}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+d\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2\right)}{2e}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{ 66}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(d\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+2e\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}\right)}{2e}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(2d\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}+2e\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}\right)}{2e}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(2d\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}\right)}{2e}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)\right)}{2e}$$

input
```
Int[(x*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
+ c*x]*((-2*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])
])/c - 2*Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])]))/(2
*e)
```

3.152. $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.152.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.152. $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

rule 2036
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

rule 6853
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Simp[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x
^2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e
, p}, x] && NeQ[p, -1]
```

3.152.4 Maple [F]

$$\int \frac{x(a + b\,\mathrm{arcsech}\,(cx))}{\sqrt{e\,x^2 + d}}dx$$

input
```
int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

3.152.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(95) = 190$.

Time = 0.33 (sec) , antiderivative size = 1102, normalized size of antiderivative = 7.20

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

$$= \left[\frac{4\sqrt{ex^2+d}bc\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+bc\sqrt{d}\log\left(\frac{(c^4d^2-6c^2de+e^2)x^4-8(c^2d^2-de)x^2+4((c^3d-ce)x^3-2cdx)\sqrt{ex^2+d}\sqrt{d}\sqrt{-\cdots}}{x^4}\right.}{}\right.$$

$$-\frac{2bc\sqrt{-d}\arctan\left(-\frac{((c^3d-ce)x^3-2cdx)\sqrt{ex^2+d}\sqrt{-d}\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{2(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)-4\sqrt{ex^2+d}bc\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-4\sqrt{ex^2\cdots}}{}$$

$$-\frac{bc\sqrt{-d}\arctan\left(-\frac{((c^3d-ce)x^3-2cdx)\sqrt{ex^2+d}\sqrt{-d}\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{2(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)-2\sqrt{ex^2+d}bc\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-2\sqrt{ex^2+\cdots}}{2ce}$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(
c*x)) + b*c*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*
e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^
2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*a*c - b*sqrt(-e)*l
og(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*
c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) + e^2))/(c*e), 1/4*(4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt(-(c^2
*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + b*c*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e +
 e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e
*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*sqrt(e*
x^2 + d)*a*c - 2*b*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e
*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e -
 e^2)*x^2 - d*e)))/(c*e), -1/4*(2*b*c*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*
x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^
2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - 4*sqrt(e*x^2 + d)*b*c*log((c*x*s
qrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 4*sqrt(e*x^2 + d)*a*c + b*sqrt
(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 -
 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)) + e^2))/(c*e), -1/2*(b*c*sqrt(-d)*arctan(-1/2*((c^3*d -
 c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*...
```

3.152. $\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.152.6 Sympy [F]

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x(a+b\,\text{asech}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

3.152.7 Maxima [F]

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
b*(sqrt(e*x^2 + d)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/e - integrate((2*
(c^2*e*x^2 - e)*x*log(sqrt(x)) + (c^2*e*x^2*log(c) - e*log(c))*x + (2*(c^2
*e*x^2 - e)*x*log(sqrt(x)) + ((e*log(c) + e)*c^2*x^2 + c^2*d - e*log(c))*x
)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))/((c^2*e*x^2 + (c^2*e*x^2 - e)*
e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)) - e)*sqrt(e*x^2 + d)), x)) + sqrt
(e*x^2 + d)*a/e
```

3.152.8 Giac [F]

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.153 $\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$

3.153.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x)
```

3.153.2 Mathematica [N/A]

Not integrable

Time = 3.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]), x]
```

3.153.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{ 6865}$$
$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

3.153.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.153.4 Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\operatorname{arcsech}\left(c x\right)}{x\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x)
```

3.153.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{d + ex^2}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{ex^2 + dx}}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e*x^3 + d*x), x)
```

3.153.6 Sympy [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\text{asech}\,(cx)}{x\sqrt{d + ex^2}}\,dx$$

input
```
integrate((a+b*asech(c*x))/x/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asech(c*x))/(x*sqrt(d + e*x**2)), x)
```

3.153.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.153.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{\sqrt{ex^2+d}x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

3.153.9 Mupad [N/A]

Not integrable

Time = 4.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(1/2)), x)
```

3.154 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

3.154.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

3.154.2 Mathematica [N/A]

Not integrable

Time = 8.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

3.154.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}dx$$
$$\Big\downarrow \; 6865$$
$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

3.154.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.154.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\text{arcsech}\,(cx)}{x^3\sqrt{e\,x^2+d}}dx$$

input
```
int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

3.154.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^3\sqrt{d + ex^2}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{ex^2 + dx^3}}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e*x^5 + d*x^3), x)
```

3.154.6 Sympy [N/A]

Not integrable

Time = 7.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^3\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\text{asech}\,(cx)}{x^3\sqrt{d + ex^2}}\,dx$$

input
```
integrate((a+b*asech(c*x))/x**3/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asech(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

3.154.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^3\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.154.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arsech}\,(cx)+a}{\sqrt{ex^2+d}x^3}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

3.154.9 Mupad [N/A]

Not integrable

Time = 4.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^3\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)), x)
```

3.155 $\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.155.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

3.155.2 Mathematica [N/A]

Not integrable

Time = 13.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]
```

3.155.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}} \, dx$$

$$\Big\downarrow \text{6865}$$

$$\int \frac{x^2 \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx$$

input
```
Int[(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.155.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.155.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^{2} \left(a+b \,\operatorname{arcsech}\left(c x \right)\right)}{\sqrt{e \,x^{2}+d}}d x$$

input
```
int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

3.155.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^2}{\sqrt{ex^2 + d}}\,dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsech(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

3.155.6 Sympy [N/A]

Not integrable

Time = 3.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{x^2(a + b\,\text{asech}\,(cx))}{\sqrt{d + ex^2}}\,dx$$

input
```
integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

3.155.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.155.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\text{sech}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

3.155.9 Mupad [N/A]

Not integrable

Time = 4.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\text{sech}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.156 $\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

3.156.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

3.156.2 Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcSech[c*x])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcSech[c*x])/Sqrt[d + e*x^2], x]
```

3.156.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex^2}} \, dx$$
$$\Big\downarrow \text{ 6865}$$
$$\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcSech[c*x])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.156.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.156.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b \,\operatorname{arcsech}\left(c x \right)}{\sqrt{e \,x^{2}+d}}d x$$

input
```
int((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

3.156.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arsech}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)/sqrt(e*x^2 + d), x)
```

3.156.6 Sympy [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{asech}\,(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

3.156.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.156.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\operatorname{arsech}(cx)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/sqrt(e*x^2 + d), x)
```

3.156.9 Mupad [N/A]

Not integrable

Time = 4.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(1/2), x)
```

3.157 $\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

3.157.1 Optimal result

Integrand size = 23, antiderivative size = 221

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{dx} + \frac{bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{d\sqrt{1+\frac{ex^2}{d}}} - \frac{b(c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{cd\sqrt{d+ex^2}}$$

output
```
-(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/d/x+b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*
(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/x+b*c*EllipticE(c*x,(-e/c^2/d)^(1/2))
*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d/(1+e*x^2/d)^(1/2)-b*(c^
2*d+e)*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+
e*x^2/d)^(1/2)/c/d/(e*x^2+d)^(1/2)
```

3.157.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.41 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.27

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx =$$

$$-\frac{a\left(\frac{d}{x}+ex\right)+bc\sqrt{\frac{1-cx}{1+cx}}(d+ex^2)-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(d+ex^2)}{x}+\frac{b(d+ex^2)\text{sech}^{-1}(cx)}{x}+\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{\frac{c\left(\sqrt{d}+i\sqrt{e}x\right)}{\left(c\sqrt{d}+i\sqrt{e}\right)(1+cx)}}\left(i\sqrt{d}+\sqrt{\cdots}\right.}{\cdots}}{d\sqrt{\cdots}}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
-((a*(d/x + e*x) + b*c*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2) - (b*Sqrt[(1
- c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2))/x + (b*(d + e*x^2)*ArcSech[c*x])/
x + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt
[d] + I*Sqrt[e])*(1 + c*x))]*(I*Sqrt[d] + Sqrt[e]*x)*((c*Sqrt[d] - I*Sqrt[
e])*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[
e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] +
 (2*I)*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d
] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sq
rt[e])^2]))/(Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*S
qrt[e])*(1 + c*x)))]*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt
[e])*(1 + c*x))]))/(d*Sqrt[d + e*x^2]))
```

3.157.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.84, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6855, 25, 27, 377, 27, 326, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{\sqrt{ex^2+d}}{dx^2\sqrt{1-c^2x^2}}dx-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 25}$$

$$-b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\sqrt{ex^2+d}}{dx^2\sqrt{1-c^2x^2}}dx-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\sqrt{ex^2+d}}{x^2\sqrt{1-c^2x^2}}dx}{d}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 377}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\int \frac{e\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}dx-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\int \frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}dx-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 326}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\left(\frac{(c^2d+e)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{e}-\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}\right)-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 323}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\left(\frac{(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}\right)-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 321}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\left(\frac{(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}\right)-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 330}$$

3.157. $\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\left(\frac{(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c^2\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{\frac{ex^2}{d}+1}}\right)-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d}-\frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \; 327$$

$$-\frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{dx}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\left(\frac{(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{\frac{ex^2}{d}+1}}\right)-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d}$$

input
```
Int[(a + b*ArcSech[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
-((Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/(d*x)) - (b*Sqrt[(1 + c*x)^(-1)]*
Sqrt[1 + c*x]*(-((Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/x) + e*(-((c*Sqrt[d +
 e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[1 + (e*x^2)/d])) + (
(c^2*d + e)*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*e
*Sqrt[d + e*x^2]))))/d
```

3.157.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.157. $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

3.157.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arcsech}\left(cx\right)}{x^{2}\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.70

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{ex^2+d}bcd\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(bc^2dx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-acd\right)\sqrt{ex^2+d}-\left(bc^4dxE(\arcsin\left(cx\right)\mid-\frac{e}{c^2d}\right)}{cd^2x}$$

input
```
integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(e*x^2 + d)*b*c*d*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)
) - (b*c^2*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - a*c*d)*sqrt(e*x^2 + d) - (
b*c^4*d*x*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (b*c^4*d + b*e)*x*elliptic
_f(arcsin(c*x), -e/(c^2*d)))*sqrt(d))/(c*d^2*x)
```

3.157.6 Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{asech}\left(cx\right)}{x^{2}\sqrt{d+ex^{2}}}\,dx$$

input
```
integrate((a+b*asech(c*x))/x**2/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asech(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

3.157.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.157.8 Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{ex^2 + dx^2}}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{e\,x^2 + d}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

3.158 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

3.158.1 Optimal result

Integrand size = 23, antiderivative size = 346

$$\begin{aligned}
&\int \frac{a+b\text{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx \\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx^3}+\frac{b(2c^2d-5e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9d^2x} \\
&\quad-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x} \\
&\quad+\frac{bc(2c^2d-5e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d^2\sqrt{1+\frac{ex^2}{d}}} \\
&\quad-\frac{2b(c^2d-3e)(c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9cd^2\sqrt{d+ex^2}}
\end{aligned}$$

output
```
-1/3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/d/x^3+2/3*e*(a+b*arcsech(c*x))*(e*
x^2+d)^(1/2)/d^2/x+1/9*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2
)*(e*x^2+d)^(1/2)/d/x^3+1/9*b*(2*c^2*d-5*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2
)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x+1/9*b*c*(2*c^2*d-5*e)*EllipticE
(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2
/(1+e*x^2/d)^(1/2)-2/9*b*(c^2*d-3*e)*(c^2*d+e)*EllipticF(c*x,(-e/c^2/d)^(1
/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d^2/(e*x^2+d)^(1/
2)
```

3.158.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.04 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.77

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$

$$= \frac{\frac{bd\sqrt{\frac{1-cx}{1+cx}}(d+ex^2)}{x^3}+\frac{bcd\sqrt{\frac{1-cx}{1+cx}}(d+ex^2)}{x^2}+\frac{b(2c^2d-5e)\sqrt{\frac{1-cx}{1+cx}}(d+ex^2)}{x}-\frac{3a(d-2ex^2)(d+ex^2)}{x^3}-\frac{3b(d-2ex^2)(d+ex^2)\operatorname{sech}^{-1}(cx)}{x^3}-}{}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
((b*d*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2))/x^3 + (b*c*d*Sqrt[(1 - c*x)/(
1 + c*x)]*(d + e*x^2))/x^2 + (b*(2*c^2*d - 5*e)*Sqrt[(1 - c*x)/(1 + c*x)]*
(d + e*x^2))/x - (3*a*(d - 2*e*x^2)*(d + e*x^2))/x^3 - (3*b*(d - 2*e*x^2)*
(d + e*x^2)*ArcSech[c*x])/x^3 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[(c*(Sqrt
[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*(I*Sqrt[d] + Sqrt
[e]*x)*((2*c^3*d^(3/2) - (2*I)*c^2*d*Sqrt[e] - 5*c*Sqrt[d]*e + (5*I)*e^(3/
2))*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[
e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] +
 2*((2*I)*c^2*d - c*Sqrt[d]*Sqrt[e] - (6*I)*e)*Sqrt[e]*EllipticF[I*ArcSinh
[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*
Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))/(Sqrt[-(((c*Sqrt[d] -
I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x)))]*Sqrt[(c*(Sqrt
[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))]))/(9*d^2*Sqrt[d +
 e*x^2])
```

3.158.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.83, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {6855, 27, 442, 445, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{ 6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{\left(d-2ex^2\right)\sqrt{ex^2+d}}{3d^2x^4\sqrt{1-c^2x^2}}dx+\frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{\left(d-2ex^2\right)\sqrt{ex^2+d}}{x^4\sqrt{1-c^2x^2}}dx}{3d^2}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \text{ 442}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\int \frac{\left(c^2d-6e\right)ex^2+d\left(2c^2d-5e\right)}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx-\frac{d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \text{ 445}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(-\frac{\int -\frac{de\left(-\left(\left(2c^2d-5e\right)x^2c^2\right)+dc^2-6e\right)}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d}-\frac{\sqrt{1-c^2x^2}\left(2c^2d-5e\right)\sqrt{d+ex^2}}{x}\right)-\frac{d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(\frac{\int \frac{de\left(-\left(\left(2c^2d-5e\right)x^2c^2\right)+dc^2-6e\right)}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d}-\frac{\sqrt{1-c^2x^2}\left(2c^2d-5e\right)\sqrt{d+ex^2}}{x}\right)-\frac{d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(e\int \frac{-\left(\left(2c^2d-5e\right)x^2c^2\right)+dc^2-6e}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx-\frac{\sqrt{1-c^2x^2}\left(2c^2d-5e\right)\sqrt{d+ex^2}}{x}\right)-\frac{d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

3.158. $\int \frac{a+b\text{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

$\Big\downarrow$ 399

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d-3e)(c^2d+e)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{e}-\frac{c^2(2c^2d-5e)\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}\right)-\frac{\sqrt{1-c^2x^2}(2c^2d-5e)\sqrt{d+ex^2}}{x}\right)\right.}{3d^2}$$

$$\frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 323

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d-3e)(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2(2c^2d-5e)\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}\right)-\frac{\sqrt{1-c^2x^2}(2c^2d-5e)}{x}\right.\right.}{3d^2}$$

$$\frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 321

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d-3e)(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c^2(2c^2d-5e)\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}\right)-\frac{\sqrt{1-c^2x^2}(2c^2d}{x}\right.\right.}{3d^2}$$

$$\frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 330

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d-3e)(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c^2(2c^2d-5e)\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{\frac{ex^2}{d}+1}}\right)-\sqrt{1-c\right.\right.}{3d^2}$$

$$\frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 327

$$\frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}-$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d-3e)(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c(2c^2d-5e)\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{\frac{ex^2}{d}+1}}\right)-\sqrt{1-\right.\right.}{3d^2}$$

input
```
Int[(a + b*ArcSech[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

3.158. $\int\frac{a+b\text{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

output
```
-1/3*(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/(d*x^3) + (2*e*Sqrt[d + e*x^2]
*(a + b*ArcSech[c*x]))/(3*d^2*x) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(
-1/3*(d*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/x^3 + (-(((2*c^2*d - 5*e)*Sqrt[
1 - c^2*x^2]*Sqrt[d + e*x^2])/x) + e*(-((c*(2*c^2*d - 5*e)*Sqrt[d + e*x^2]
*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[1 + (e*x^2)/d])) + (2*(c^2*
d - 3*e)*(c^2*d + e)*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d
))])/(c*e*Sqrt[d + e*x^2])))/3))/(3*d^2)
```

3.158.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.158.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arcsech}\left(cx\right)}{x^{4}\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

3.158. $\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

3.158.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.71

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^4\sqrt{d + ex^2}}\, dx$$
$$= \frac{3\,(2\,bcdex^2 - bcd^2)\sqrt{ex^2 + d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) + \left(6\,acdex^2 - 3\,acd^2 + (bc^2d^2x + (2\,bc^4d^2 - 5\,bc^2de)x^3\right)}{}$$

input
```
integrate((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
1/9*(3*(2*b*c*d*e*x^2 - b*c*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 -
 1)/(c^2*x^2)) + 1)/(c*x)) + (6*a*c*d*e*x^2 - 3*a*c*d^2 + (b*c^2*d^2*x + (
2*b*c^4*d^2 - 5*b*c^2*d*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2
 + d) + ((2*b*c^6*d^2 - 5*b*c^4*d*e)*x^3*elliptic_e(arcsin(c*x), -e/(c^2*d
)) - (2*b*c^6*d^2 - (5*b*c^4 - b*c^2)*d*e - 6*b*e^2)*x^3*elliptic_f(arcsin
(c*x), -e/(c^2*d)))*sqrt(d))/(c*d^3*x^3)
```

3.158.6 Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^4\sqrt{d + ex^2}}\, dx = \int \frac{a + b\operatorname{asech}(cx)}{x^4\sqrt{d + ex^2}}\, dx$$

input
```
integrate((a+b*asech(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asech(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

3.158.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^4\sqrt{d + ex^2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.158.8 Giac [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{\sqrt{ex^2+dx^4}}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x^4\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)
```

3.159 $\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.159.1 Optimal result

Integrand size = 23, antiderivative size = 278

$$\int \frac{x^5(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2(a+b\text{sech}^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}\,(a+b\text{sech}^{-1}(cx))}{3e^3} + \frac{b(9c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^3e^{5/2}} + \frac{8bd^{3/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^3}$$

output
```
1/3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^3+1/6*b*(9*c^2*d-e)*arctan(e^(1/2
)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^
3/e^(5/2)+8/3*b*d^(3/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2)
)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^3-d^2*(a+b*arcsech(c*x))/e^3/(e*x^2+d)
^(1/2)-2*d*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/e^3-1/6*b*(1/(c*x+1))^(1/2)*
(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c^2/e^2
```

3.159.2 Mathematica [A] (verified)

Time = 23.34 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.57

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{-be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(d+ex^2\right)-2ac^2\left(8d^2+4dex^2-e^2x^4\right)-2bc^2\left(8d^2+4dex^2\right.}{6c^2e^3\sqrt{d+ex^2}}$$
$$-\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(-9\left(-c^2\right)^{3/2}d\sqrt{-c^2d-e}\sqrt{e}\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+\sqrt{-c^2}\sqrt{-c^2d-e}e^{3/2}\sqrt{\right.}{6c^5e^3(-1+cx)\sqrt{d+ex^2}}$$

input
```
Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(-(b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2)) - 2*a*c^2*(8*d^2 +
 4*d*e*x^2 - e^2*x^4) - 2*b*c^2*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*ArcSech[c*x]
)/(6*c^2*e^3*Sqrt[d + e*x^2]) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*
x^2]*(-9*(-c^2)^(3/2)*d*Sqrt[-(c^2*d) - e]*Sqrt[e]*Sqrt[(c^2*(d + e*x^2))/
(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d
) - e])] + Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*e^(3/2)*Sqrt[(c^2*(d + e*x^2))/(c
^2*d + e)]*ArcSin[(Sqrt[-c^2]*Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[-(c^2*d)
- e])] + 16*c^5*d^(3/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2]
)/Sqrt[-d - e*x^2]]))/(6*c^5*e^3*(-1 + c*x)*Sqrt[d + e*x^2])
```

3.159.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6855, 27, 7282, 2118, 27, 175, 66, 104, 218, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{-e^2x^4+4dex^2+8d^2}{3e^3x\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx-\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{-e^2x^4+4dex^2+8d^2}{x\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{3e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{7282}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int\frac{-e^2x^4+4dex^2+8d^2}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{6e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{2118}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}-\frac{\int-\frac{e\left(16c^2d^2+\left(9c^2d-e\right)ex^2\right)}{2x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{c^2e}\right)}{6e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int\frac{16c^2d^2+\left(9c^2d-e\right)ex^2}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{175}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{16c^2d^2\int\frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+e\left(9c^2d-e\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{66}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{16c^2d^2\int\frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+2e\left(9c^2d-e\right)\int\frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

3.159. $\int\frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}dx$

$$\Big\downarrow \; 104$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{32c^2d^2\int\frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}+2e(9c^2d-e)\int\frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 218$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{32c^2d^2\int\frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}-\frac{2\sqrt{e}\left(9c^2d-e\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}+\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 220$$

$$-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{-\frac{2\sqrt{e}\left(9c^2d-e\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}-32c^2d^{3/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2c^2}+\frac{e\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{c^2}\right)}{6e^3}$$

input
```
Int[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-((d^2*(a + b*ArcSech[c*x]))/(e^3*Sqrt[d + e*x^2])) - (2*d*Sqrt[d + e*x^2]
*(a + b*ArcSech[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3*e
^3) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((e*Sqrt[1 - c^2*x^2]*Sqrt[d +
 e*x^2])/c^2 + ((-2*(9*c^2*d - e)*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2
])/(c*Sqrt[d + e*x^2])])/c - 32*c^2*d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[
d]*Sqrt[1 - c^2*x^2])])/(2*c^2)))/(6*e^3)
```

3.159. $\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.159.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.159. $\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx$

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.159.4 Maple [F]

$$\int \frac{x^{5} \left(a +b \,\mathrm{arcsech}\left(c x \right)\right)}{\left(e \,x^{2}+d \right)^{\frac{3}{2}}}d x$$

input
```
int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

3.159.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(184) = 368$.

Time = 0.44 (sec) , antiderivative size = 1771, normalized size of antiderivative = 6.37

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/24*((9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*log(8*c^
4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x
^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*
x^2)) + e^2) + 8*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sqrt(e*x^
2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 16*(b*c^3*d*e
*x^2 + b*c^3*d^2)*sqrt(d)*log((((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^
2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqr
t(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d
*e*x^2 - 16*a*c^3*d^2 - (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt(-(c^2*x^2 - 1)/
(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/12*((9*b*c^2*d^2
 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c
^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e
^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) + 4*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2
- 8*b*c^3*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1
)/(c*x)) + 8*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(d)*log((((c^4*d^2 - 6*c^2*d*e
 + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt
(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 2*(2*a*
c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 - (b*c^2*e^2*x^3 + b*c^2*d*e*
x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e
^3), 1/24*(32*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*arctan(-1/2*((c^3*d ...
```

3.159.6 Sympy [F]

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^5(a+b\,\text{asech}\,(cx))}{\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**5*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

3.159.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.159.8 Giac [F]

$$\int \frac{x^5 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}} \, dx = \int \frac{\left(b \,\text{arsech}\,(cx) + a\right)x^5}{\left(ex^2 + d\right)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}} \, dx = \int \frac{x^5 \left(a + b \,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^{3/2}} \, dx$$

input
```
int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.160 $\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.160.1 Optimal result

Integrand size = 23, antiderivative size = 177

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{ce^{3/2}}-\frac{2b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e^2}$$

output
```
-b*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*
(c*x+1)^(1/2)/c/e^(3/2)-2*b*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(
1/2))*d^(1/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^2+d*(a+b*arcsech(c*x))/e^2
/(e*x^2+d)^(1/2)+(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/e^2
```

3.160.2 Mathematica [A] (verified)

Time = 23.17 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.41

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{\left(2d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{-c^2d-e}\sqrt{e}\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+2c^3\sqrt{d}\sqrt{-d-ex^2}\arctan\left(\frac{\sqrt{d}\sqrt{\cdots}}{\sqrt{-\cdots}}\right.\right.}{c^3e^2(-1+cx)\sqrt{d+ex^2}}$$

input
```
Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
((2*d + e*x^2)*(a + b*ArcSech[c*x]))/(e^2*Sqrt[d + e*x^2]) + (b*Sqrt[(1 -
c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*S
qrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(S
qrt[-c^2]*Sqrt[-(c^2*d) - e])] + 2*c^3*Sqrt[d]*Sqrt[-d - e*x^2]*ArcTan[(Sq
rt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(c^3*e^2*(-1 + c*x)*Sqrt[d +
e*x^2])
```

3.160.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6855, 27, 435, 175, 66, 104, 218, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{ex^2+2d}{e^2x\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{ex^2+2d}{x\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{e^2}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{435}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{ex^2+2d}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2e^2}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{175}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+2d\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2\right)}{2e^2}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(2d\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+2e\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}\right)}{2e^2}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(4d\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}+2e\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}\right)}{2e^2}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(4d\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}\right)}{2e^2}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}-4\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)\right)}{2e^2}$$

input
```
Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(d*(a + b*ArcSech[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*A
rcSech[c*x]))/e^2 + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-2*Sqrt[e]*Arc
Tan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/c - 4*Sqrt[d]*ArcTan
h[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])]))/(2*e^2)
```

3.160. $\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx$

3.160.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

3.160. $\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx$

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.160.4 Maple [F]

$$\int \frac{x^3(a+b\,\mathrm{arcsech}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

3.160.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(117) = 234$.

Time = 0.36 (sec) , antiderivative size = 1311, normalized size of antiderivative = 7.41

$$\int \frac{x^3\left(a+b\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((b*e*x^2 + b*d)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e +
8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2
 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) - 4*(b*c*e*x^2 + 2*b*
c*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) -
 2*(b*c*e*x^2 + b*c*d)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d
)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - 4*(a*c*e*x^2 + 2*a*c*d)*s
qrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), -1/2*((b*e*x^2 + b*d)*sqrt(e)*arcta
n(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2
 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) - 2*(b*c*e*x^2
 + 2*b*c*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(
c*x)) - (b*c*e*x^2 + b*c*d)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 -
 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*s
qrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - 2*(a*c*e*x^2 + 2*a*c
*d)*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), -1/4*(4*(b*c*e*x^2 + b*c*d)*sq
rt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*
sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2))
+ (b*e*x^2 + b*d)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^
4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)
*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) - 4*(b*c*e*x^2 + 2*b*c*...
```

3.160.6 Sympy [F]

$$\int \frac{x^3(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^3(a+b\operatorname{asech}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

3.160.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.160.8 Giac [F]

$$\int \frac{x^3 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{(b \operatorname{arsech}(cx) + a)x^3}{(ex^2 + d)^{\frac{3}{2}}} dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{x^3 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)}{(e x^2 + d)^{3/2}} dx$$

input
```
int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.161 $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx$

3.161.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=-\frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{d}e}$$

output
```
b*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c
*x+1)^(1/2)/e/d^(1/2)+(-a-b*arcsech(c*x))/e/(e*x^2+d)^(1/2)
```

3.161.2 Mathematica [A] (verified)

Time = 21.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.55

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=-\frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\sqrt{-d-ex^2}\arctan\left(\frac{\sqrt{d}\sqrt{1-c^2x^2}}{\sqrt{-d-ex^2}}\right)}{\sqrt{d}e(-1+cx)\sqrt{d+ex^2}}$$

input
```
Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-((a + b*ArcSech[c*x])/(e*Sqrt[d + e*x^2])) - (b*Sqrt[(1 - c*x)/(1 + c*x)]
*Sqrt[1 - c^2*x^2]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqr
t[-d - e*x^2]])/(Sqrt[d]*e*(-1 + c*x)*Sqrt[d + e*x^2])
```

3.161.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6853, 2036, 354, 104, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6853}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{ex^2+d}}dx}{e}-\frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 2036}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{e}-\frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2e}-\frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 104}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}}{e}-\frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{d}e}-\frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

input
```
Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-((a + b*ArcSech[c*x])/(e*Sqrt[d + e*x^2])) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt
[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(Sqrt[d]*e
)
```

3.161.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2036
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

rule 6853
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Simp[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x
^2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e
, p}, x] && NeQ[p, -1]
```

3.161.4 Maple [F]

$$\int \frac{x(a+b\ \mathrm{arcsech}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

3.161.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(57) = 114.

Time = 0.29 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.36

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx = \left[-\frac{4\sqrt{ex^2+d}\,bd\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+4\sqrt{ex^2+d}\,ad-(bex^2+bd)\sqrt{d}\log\left(\frac{(c\cdots}{\cdots}\right.}{4\left(de^2x^2+d\cdots\right.}\right.$$

$$-\frac{2\sqrt{ex^2+d}\,bd\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+2\sqrt{ex^2+d}\,ad-(bex^2+bd)\sqrt{-d}\arctan\left(-\frac{((c^3d-ce)x^3-2\,cdx)\sqrt{ex^2+d}\sqrt{-\cdots}}{2\,(c^2dex^4+(c^2d^2-de)x^2-\cdots}\right.}{2\left(de^2x^2+d^2e\right)}$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(4*sqrt(e*x^2 + d)*b*d*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/
(c*x)) + 4*sqrt(e*x^2 + d)*a*d - (b*e*x^2 + b*d)*sqrt(d)*log(((c^4*d^2 - 6
*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d
*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4))/
(d*e^2*x^2 + d^2*e), -1/2*(2*sqrt(e*x^2 + d)*b*d*log((c*x*sqrt(-(c^2*x^2 -
 1)/(c^2*x^2)) + 1)/(c*x)) + 2*sqrt(e*x^2 + d)*a*d - (b*e*x^2 + b*d)*sqrt(
-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqr
t(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)))/(d
*e^2*x^2 + d^2*e)]
```

3.161.6 Sympy [F]

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx = \int \frac{x(a+b\operatorname{asech}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

3.161.7 Maxima [F]

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
b*integrate(x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 +
d)^(3/2), x) - a/(sqrt(e*x^2 + d)*e)
```

3.161.8 Giac [F]

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.162 $\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx$

3.162.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx = \text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{3/2}},x\right)$$

output
```
Unintegrable((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x)
```

3.162.2 Mathematica [N/A]

Not integrable

Time = 11.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

3.162.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 6865}$$
$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}dx$$

input
```
Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.162.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.162.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b \,\operatorname{arcsech}\left(c x \right)}{x \left(e \,x^{2}+d \right)^{\frac{3}{2}}}d x$$

input
```
int((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x)
```

3.162.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsech}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x
), x)
```

3.162.6 Sympy [N/A]

Not integrable

Time = 18.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\text{asech}\left(cx\right)}{x\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asech(c*x))/x/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asech(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

3.162.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.162.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

3.162.9 Mupad [N/A]

Not integrable

Time = 4.95 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

3.163 $\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx$

3.163.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{a+b\,\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
Unintegrable((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

3.163.2 Mathematica [N/A]

Not integrable

Time = 15.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

3.163.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3 (d+ex^2)^{3/2}} \, dx$$

$$\Big\downarrow \ 6865$$

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3 (d+ex^2)^{3/2}}dx$$

input
```
Int[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.163.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.163.4 Maple [N/A] (verified)

Not integrable

Time = 0.72 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b \,\operatorname{arcsech}\left(c x \right)}{x^{3} \left(e \,x^{2}+d \right)^{\frac{3}{2}}}d x$$

input
```
int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

3.163.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x
^3), x)
```

3.163.6 Sympy [N/A]

Not integrable

Time = 77.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx=\int \frac{a+b\,\text{asech}\,(cx)}{x^3(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asech(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asech(c*x))/(x**3*(d + e*x**2)**(3/2)), x)
```

3.163.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.163.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx = \int \frac{b\,\text{arsech}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

3.163.9 Mupad [N/A]

Not integrable

Time = 4.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x^3\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)), x)
```

3.164 $\int \frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.164.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
Unintegrable(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

3.164.2 Mathematica [N/A]

Not integrable

Time = 19.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]
```

3.164.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 6865}$$

$$\int \frac{x^4 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

input
```
Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.164.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.164.4 Maple [N/A] (verified)

Not integrable

Time = 0.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4 (a + b \operatorname{arcsech}(cx))}{(e\, x^2 + d)^{\frac{3}{2}}} dx$$

input
```
int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

3.164.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^4 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsech(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
 + d^2), x)
```

3.164.6 Sympy [N/A]

Not integrable

Time = 40.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^4 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^4(a+b\,\text{asech}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**4*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

3.164.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.164.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)
```

3.164.9 Mupad [N/A]

Not integrable

Time = 4.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.165 $\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.165.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
Unintegrable(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

3.165.2 Mathematica [N/A]

Not integrable

Time = 7.97 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]
```

3.165.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 6865}$$

$$\int \frac{x^2 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}dx$$

input
```
Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.165.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.165.4 Maple [N/A] (verified)

Not integrable

Time = 0.87 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^{2} \left(a +b \,\operatorname{arcsech}\left(c x \right)\right)}{\left(e \,x^{2}+d \right)^{\frac{3}{2}}}d x$$

input
```
int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

3.165.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsech(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
 + d^2), x)
```

3.165.6 Sympy [N/A]

Not integrable

Time = 9.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^2(a+b\,\text{asech}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

3.165.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.165.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(b\operatorname{arsech}(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

3.165.9 Mupad [N/A]

Not integrable

Time = 4.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.166 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$

3.166.1 Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x(a+b\text{sech}^{-1}(cx))}{d\sqrt{d+ex^2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{cd\sqrt{d+ex^2}}$$

output
```
x*(a+b*arcsech(c*x))/d/(e*x^2+d)^(1/2)+b*EllipticF(c*x,(-e/c^2/d)^(1/2))*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d/(e*x^2+d)^(1/2)
```

3.166.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 50.61 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.63

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x(a+b\text{sech}^{-1}(cx))}{d\sqrt{d+ex^2}} + \frac{2ib\sqrt{\frac{1-cx}{1+cx}}\sqrt{\frac{(c\sqrt{d}+i\sqrt{e})(1+cx)}{(c\sqrt{d}-i\sqrt{e})(-1+cx)}}\left(-i\sqrt{d}+\sqrt{e}x\right)\sqrt{-\frac{-1+\frac{i\sqrt{e}x}{\sqrt{d}}+c\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{1-cx}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+\frac{ic\sqrt{d}}{\sqrt{e}}-cx+\frac{i\sqrt{e}x}{\sqrt{d}}}{2-2cx}}\right)\right)}{d\left(c\sqrt{d}+i\sqrt{e}\right)\sqrt{\frac{1+\frac{ic\sqrt{d}}{\sqrt{e}}-cx+\frac{i\sqrt{e}x}{\sqrt{d}}}{1-cx}}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSech[c*x]))/(d*Sqrt[d + e*x^2]) + ((2*I)*b*Sqrt[(1 - c*x)/(1
+ c*x)]*Sqrt[((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))/((c*Sqrt[d] - I*Sqrt[e])*
(-1 + c*x))]*((-I)*Sqrt[d] + Sqrt[e]*x)*Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d]
 + c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]*EllipticF[ArcSin[Sqrt[(1 + (I*
c*Sqrt[d])/Sqrt[e] - c*x + (I*Sqrt[e]*x)/Sqrt[d])/(2 - 2*c*x)]], ((-4*I)*c
*Sqrt[d]*Sqrt[e])/(c*Sqrt[d] - I*Sqrt[e])^2])/(d*(c*Sqrt[d] + I*Sqrt[e])*S
qrt[(1 + (I*c*Sqrt[d])/Sqrt[e] - c*x + (I*Sqrt[e]*x)/Sqrt[d])/(1 - c*x)]*S
qrt[d + e*x^2])
```

3.166.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6845, 27, 323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6845}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{d\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx+\frac{x(a+b\text{sech}^{-1}(cx))}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d}+\frac{x(a+b\text{sech}^{-1}(cx))}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{d\sqrt{d+ex^2}}+\frac{x(a+b\text{sech}^{-1}(cx))}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{x(a+b\text{sech}^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{cd\sqrt{d+ex^2}}$$

input
```
Int[(a + b*ArcSech[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSech[c*x]))/(d*Sqrt[d + e*x^2]) + (b*Sqrt[(1 + c*x)^(-1)]*Sqr
t[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*d*
Sqrt[d + e*x^2])
```

3.166.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 6845
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSech[c*x])   u
, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]   Int[SimplifyIntegrand[u/(x*
Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (
IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

3.166.4 Maple [F]

$$\int \frac{a + b\,\mathrm{arcsech}\,(cx)}{(e\,x^2 + d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex^2)^{3/2}}\,dx = \frac{\sqrt{ex^2 + d}bcdx \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) + \sqrt{ex^2 + d}acdx + (bex^2 + bd)\sqrt{d}F(\arcsin(cx)}{cd^2ex^2 + cd^3}$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(e*x^2 + d)*b*c*d*x*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x
)) + sqrt(e*x^2 + d)*a*c*d*x + (b*e*x^2 + b*d)*sqrt(d)*elliptic_f(arcsin(c
*x), -e/(c^2*d)))/(c*d^2*e*x^2 + c*d^3)
```

3.166.6 Sympy [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex^2)^{3/2}}\,dx = \int \frac{a + b\operatorname{asech}(cx)}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

3.166.7 Maxima [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex^2)^{3/2}}\,dx = \int \frac{b\operatorname{arsech}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)
^(3/2), x) + a*x/(sqrt(e*x^2 + d)*d)
```

3.166.8 Giac [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx=\int \frac{a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(3/2), x)
```

3.167 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

3.167.1 Optimal result

Integrand size = 23, antiderivative size = 249

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx = \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{d^2x} - \frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{d^2\sqrt{1+\frac{ex^2}{d}}} - \frac{b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{cd^2\sqrt{d+ex^2}}$$

output
```
(-a-b*arcsech(c*x))/d/x/(e*x^2+d)^(1/2)-2*e*x*(a+b*arcsech(c*x))/d^2/(e*x^
2+d)^(1/2)+b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^
(1/2)/d^2/x+b*c*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^
(1/2)*(e*x^2+d)^(1/2)/d^2/(1+e*x^2/d)^(1/2)-b*(c^2*d+2*e)*EllipticF(c*x,(-
e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d^2/(e
*x^2+d)^(1/2)
```

3.167.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.73 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.01

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx = \frac{\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(d+ex^2)}{x} - \frac{a(d+2ex^2)}{x} - \frac{b(d+2ex^2)\operatorname{sech}^{-1}(cx)}{x} + \frac{b\sqrt{\frac{1-cx}{1+cx}}\left(-c^2(d+ex^2)+\frac{(1+cx)\sqrt{\frac{c(\cdots}{(c\sqrt{d}\cdots}}\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
((b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2))/x - (a*(d + 2*e*x^2))
/x - (b*(d + 2*e*x^2)*ArcSech[c*x])/x + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c^
2*(d + e*x^2)) + ((1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] -
 I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*S
qrt[e])*(1 + c*x))]*((-I)*(c*Sqrt[d] - I*Sqrt[e])^2*EllipticE[I*ArcSinh[Sq
rt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqr
t[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + 2*(c*Sqrt[d] - (2*I)*Sqrt
[e])*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d]
+ I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt
[e])^2]))/Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt
[e])*(1 + c*x)))]))/c)/(d^2*Sqrt[d + e*x^2])
```

3.167.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.86, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {6855, 25, 27, 445, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 6855$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{2ex^2+d}{d^2x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx - \frac{2ex(a+b\operatorname{sech}^{-1}(cx))}{d^2\sqrt{d+ex^2}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{25}$$

$$-b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{2ex^2+d}{d^2x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx-\frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{2ex^2+d}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d^2}-\frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{445}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{de\left(2-c^2x^2\right)}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d}-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d^2}-\frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{de\left(2-c^2x^2\right)}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d}-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d^2}-\frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\int \frac{2-c^2x^2}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d^2}-\frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{399}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\left(\frac{\left(c^2d+2e\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{e}-\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}\right)-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d^2}-\frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{323}$$

3.167. $\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\left(\frac{(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}\right)-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d^2}-\frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 321$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\left(\frac{(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}\right)-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d^2}-\frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 330$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\left(\frac{(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c^2\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{\frac{ex^2}{d}+1}}\right)-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d^2}-\frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 327$$

$$-\frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(e\left(\frac{(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{\frac{ex^2}{d}+1}}\right)-\frac{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{x}\right)}{d^2}$$

input
```
Int[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
-((a + b*ArcSech[c*x])/(d*x*Sqrt[d + e*x^2])) - (2*e*x*(a + b*ArcSech[c*x]
))/(d^2*Sqrt[d + e*x^2]) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-((Sqrt[
1 - c^2*x^2]*Sqrt[d + e*x^2])/x) + e*(-((c*Sqrt[d + e*x^2]*EllipticE[ArcSi
n[c*x], -(e/(c^2*d))])/(e*Sqrt[1 + (e*x^2)/d])) + ((c^2*d + 2*e)*Sqrt[1 +
(e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*e*Sqrt[d + e*x^2]))))/
d^2
```

3.167. $\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

3.167.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.167. $\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.167.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arcsech}\left(cx\right)}{x^{2}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.96

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = -\frac{(2\,bcdex^2+bcd^2)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+\left(2\,acdex^2+acd^2-(bc^2dex^3+bc^2d^2x)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}\right)\sqrt{ex}}{cd}$$

input
```
integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
-((2*b*c*d*e*x^2 + b*c*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)) + 1)/(c*x)) + (2*a*c*d*e*x^2 + a*c*d^2 - (b*c^2*d*e*x^3 + b*c^2*
d^2*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d) - ((b*c^4*d*e*x^3 +
 b*c^4*d^2*x)*elliptic_e(arcsin(c*x), -e/(c^2*d)) - ((b*c^4*d*e + 2*b*e^2)
*x^3 + (b*c^4*d^2 + 2*b*d*e)*x)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(
d))/(c*d^3*e*x^3 + c*d^4*x)
```

3.167.6 Sympy [F]

$$\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\operatorname{asech}\left(cx\right)}{x^2\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asech(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asech(c*x))/(x**2*(d + e*x**2)**(3/2)), x)
```

3.167.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.167.8 Giac [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\text{arsech}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x^2\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)), x)
```

3.168 $\int \frac{x^5\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.168.1 Optimal result

Integrand size = 23, antiderivative size = 272

$$\int \frac{x^5(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e^2(c^2d+e)\sqrt{d+ex^2}} - \frac{d^2(a+b\text{sech}^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\text{sech}^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^3} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{ce^{5/2}} - \frac{8b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^3}$$

output
```
-1/3*d^2*(a+b*arcsech(c*x))/e^3/(e*x^2+d)^(3/2)-b*arctan(e^(1/2)*(-c^2*x^2
+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c/e^(5/2)-8/3
*b*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*d^(1/2)*(1/(c*x+1))
^(1/2)*(c*x+1)^(1/2)/e^3+2*d*(a+b*arcsech(c*x))/e^3/(e*x^2+d)^(1/2)-1/3*b*
d*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/e^2/(c^2*d+e)/(e*x^2+
d)^(1/2)+(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/e^3
```

3.168.2 Mathematica [A] (verified)

Time = 23.42 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.28

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx = \frac{-bde\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(d+ex^2\right)+a(c^2d+e)\left(8d^2+12dex^2+3e^2x^4\right)+b(c^2d+}{3e^3\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}}$$
$$+\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(3\sqrt{-c^2}\sqrt{-c^2d-e}\sqrt{e}\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+8c^3\sqrt{d}\sqrt{-d-ex^2}\arctan\left(\frac{\sqrt{d}}{\sqrt{}}\right.\right.}{3c^3e^3(-1+cx)\sqrt{d+ex^2}}$$

input
```
Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(-(b*d*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2)) + a*(c^2*d + e)*
(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4) + b*(c^2*d + e)*(8*d^2 + 12*d*e*x^2 + 3*e
^2*x^4)*ArcSech[c*x])/(3*e^3*(c^2*d + e)*(d + e*x^2)^(3/2)) + (b*Sqrt[(1 -
 c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(3*Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e
]*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])
/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])] + 8*c^3*Sqrt[d]*Sqrt[-d - e*x^2]*ArcTan[
(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(3*c^3*e^3*(-1 + c*x)*Sqrt
[d + e*x^2])
```

3.168.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6855, 27, 7282, 2117, 27, 175, 66, 104, 218, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{6855}$$
$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{3e^2x^4+12dex^2+8d^2}{3e^3x\sqrt{1-c^2x^2}\left(ex^2+d\right)^{3/2}}dx-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}$$
$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{3e^2x^4+12dex^2+8d^2}{x\sqrt{1-c^2x^2}(ex^2+d)^{3/2}}dx}{3e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \textbf{7282}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{3e^2x^4+12dex^2+8d^2}{x^2\sqrt{1-c^2x^2}(ex^2+d)^{3/2}}dx^2}{6e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \textbf{2117}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2\int \frac{d\left(dc^2+e\right)\left(3ex^2+8d\right)}{2x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{d(c^2d+e)}-\frac{2de\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\int \frac{3ex^2+8d}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2-\frac{2de\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \textbf{175}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(3e\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+8d\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2-\frac{2de\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \textbf{66}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(8d\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2+6e\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}-\frac{2de\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \textbf{104}$$

3.168. $\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}dx$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(6e\int\frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}+16d\int\frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}-\frac{2de\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(16d\int\frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}-\frac{6\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}-\frac{2de\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{ 220}$$

$$-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{6\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c}-16\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)-\frac{2de\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3}$$

input
```
Int[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(d^2*(a + b*ArcSech[c*x]))/(e^3*(d + e*x^2)^(3/2)) + (2*d*(a + b*ArcS
ech[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/
e^3 + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-2*d*e*Sqrt[1 - c^2*x^2])/((
c^2*d + e)*Sqrt[d + e*x^2]) - (6*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2]
)/(c*Sqrt[d + e*x^2])])/c - 16*Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sq
rt[1 - c^2*x^2])]))/(6*e^3)
```

3.168.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

3.168. $\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

3.168. $\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx$

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.168.4 Maple [F]

$$\int \frac{x^5(a + b\,\mathrm{arcsech}\,(cx))}{(e\,x^2 + d)^{\frac{5}{2}}}dx$$

input
```
int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

3.168.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(180) = 360$.

Time = 0.47 (sec) , antiderivative size = 2415, normalized size of antiderivative = 8.88

$$\int \frac{x^5(a + b\mathrm{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*
e + b*d*e^2)*x^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^
4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)
*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) - 4*(8*b*c^3*d^3 + 8*b*c*d
^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*s
qrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 8*(b*
c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d
*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e
)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2
*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - 4*(8*a*c^3*d^3 + 8*a*c*d^2*e + 3*(a*c
^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 - (b*c^2*d*e^2*
x^3 + b*c^2*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3
*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c^3*d^2*e^4 + c*d*e^5)
*x^2), -1/6*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2
*d^2*e + b*d*e^2)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sq
rt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d
*e - e^2)*x^2 - d*e)) - 2*(8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*
c*e^3)*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sq
rt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(b*c^3*d^3 + b*c*d^2*e + (b*c
^3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*log(...
```

3.168.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.168.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.168.8 Giac [F]

$$\int \frac{x^5(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^5}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{x^5\,\left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^{5/2}}\,dx$$

input
```
int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.169 $\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.169.1 Optimal result

Integrand size = 23, antiderivative size = 179

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3\sqrt{d}e^2}$$

output
```
1/3*d*(a+b*arcsech(c*x))/e^2/(e*x^2+d)^(3/2)+2/3*b*arctanh((e*x^2+d)^(1/2)
/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^2/d^(1/2)+(
-a-b*arcsech(c*x))/e^2/(e*x^2+d)^(1/2)+1/3*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/
2)*(-c^2*x^2+1)^(1/2)/e/(c^2*d+e)/(e*x^2+d)^(1/2)
```

3.169.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.22

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\frac{be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(d+ex^2\right)-a(c^2d+e)\left(2d+3ex^2\right)-b(c^2d+e)\left(2d+3ex^2\right)}{3e^2\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}}-\frac{2b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\sqrt{-d-ex^2}\,\arctan\left(\frac{\sqrt{d}\sqrt{1-c^2x^2}}{\sqrt{-d-ex^2}}\right)}{3\sqrt{d}e^2(-1+cx)\sqrt{d+ex^2}}$$

input
```
Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2) - a*(c^2*d + e)*(2*d
+ 3*e*x^2) - b*(c^2*d + e)*(2*d + 3*e*x^2)*ArcSech[c*x])/(3*e^2*(c^2*d + e
)*(d + e*x^2)^(3/2)) - (2*b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Sq
rt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]])/(3*Sq
rt[d]*e^2*(-1 + c*x)*Sqrt[d + e*x^2])
```

3.169.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6855, 27, 435, 169, 27, 104, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int -\frac{3ex^2+2d}{3e^2x\sqrt{1-c^2x^2}\left(ex^2+d\right)^{3/2}}dx-\frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{3ex^2+2d}{x\sqrt{1-c^2x^2}\left(ex^2+d\right)^{3/2}}dx}{3e^2}-\frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 435}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{3ex^2+2d}{x^2\sqrt{1-c^2x^2}\left(ex^2+d\right)^{3/2}}dx^2}{6e^2}-\frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 169}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2\int \frac{d\left(dc^2+e\right)}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{d(c^2d+e)}-\frac{2e\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^2}-\frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(2\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2-\frac{2e\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^2}-\frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(4\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}-\frac{2e\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^2}-\frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{220}$$

$$-\frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{4\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{d}}-\frac{2e\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^2}$$

input
```
Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(d*(a + b*ArcSech[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcSech[c*x])/
(e^2*Sqrt[d + e*x^2]) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-2*e*Sqrt[
1 - c^2*x^2])/((c^2*d + e)*Sqrt[d + e*x^2]) - (4*ArcTanh[Sqrt[d + e*x^2]/(
Sqrt[d]*Sqrt[1 - c^2*x^2])])/Sqrt[d]))/(6*e^2)
```

3.169.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

3.169. $\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.169.4 Maple [F]

$$\int \frac{x^3 (a + b \,\text{arcsech}\,(cx))}{(e\,x^2 + d)^{\frac{5}{2}}} dx$$

input
```
int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

3.169.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(117) = 234$.

Time = 0.35 (sec) , antiderivative size = 786, normalized size of antiderivative = 4.39

$$\int \frac{x^3(a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \left[-\frac{2\left(2\,bc^2d^3 + 2\,bd^2e + 3\left(bc^2d^2e + bde^2\right)x^2\right)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - (\right.$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(2*(2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e*
x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d^3
+ (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqr
t(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^
3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2
*x^2)) + 8*d^2)/x^4) + 2*(2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e
^2)*x^2 - (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sq
rt(e*x^2 + d))/(c^2*d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2
*d^3*e^3 + d^2*e^4)*x^2), 1/3*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*
d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)
*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c
^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (2*b*c^2*d^3 + 2*b*d^2*e + 3*(b
*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c
^2*x^2)) + 1)/(c*x)) - (2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2
)*x^2 - (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt
(e*x^2 + d))/(c^2*d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d
^3*e^3 + d^2*e^4)*x^2)]
```

3.169.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.169.7 Maxima [F]

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{(b\operatorname{arsech}(cx)+a)x^3}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d/((e*x^2 + d)^(3/2)*e^2)) + b*int
egrate(x^3*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^
(5/2), x)
```

3.169.8 Giac [F]

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{(b\operatorname{arsech}(cx)+a)x^3}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^3\,\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.170 $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx$

3.170.1 Optimal result

Integrand size = 21, antiderivative size = 154

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\,(c^2d+e)\,\sqrt{d+ex^2}} - \frac{a+b\text{sech}^{-1}(cx)}{3e\,(d+ex^2)^{3/2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3d^{3/2}e}$$

output
```
1/3*(-a-b*arcsech(c*x))/e/(e*x^2+d)^(3/2)+1/3*b*arctanh((e*x^2+d)^(1/2)/d^
(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d^(3/2)/e-1/3*b*
(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)^(
1/2)
```

3.170.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.32

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-ad(c^2d+e)-be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\,(d+ex^2)-bd(c^2d+e)\,\text{sech}^{-1}(cx)}{3de\,(c^2d+e)\,(d+ex^2)^{3/2}} - \frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\sqrt{-d-ex^2}\arctan\left(\frac{\sqrt{d}\sqrt{1-c^2x^2}}{\sqrt{-d-ex^2}}\right)}{3d^{3/2}e(-1+cx)\sqrt{d+ex^2}}$$

input
```
Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(-(a*d*(c^2*d + e)) - b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2)
- b*d*(c^2*d + e)*ArcSech[c*x])/(3*d*e*(c^2*d + e)*(d + e*x^2)^(3/2)) - (b
*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt
[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]])/(3*d^(3/2)*e*(-1 + c*x)*Sqrt[d +
 e*x^2])
```

3.170.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6853, 2036, 354, 107, 104, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{6853}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\left(ex^2+d\right)^{3/2}}dx}{3e}-\frac{a+b\text{sech}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{2036}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x\sqrt{1-c^2x^2}\left(ex^2+d\right)^{3/2}}dx}{3e}-\frac{a+b\text{sech}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{354}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(ex^2+d\right)^{3/2}}dx^2}{6e}-\frac{a+b\text{sech}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{107}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{d}+\frac{2e\sqrt{1-c^2x^2}}{d\left(c^2d+e\right)\sqrt{d+ex^2}}\right)}{6e}-\frac{a+b\text{sech}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2\int \frac{1}{x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}}{d}+\frac{2e\sqrt{1-c^2x^2}}{d(c^2d+e)\sqrt{d+ex^2}}\right)}{6e}-\frac{a+b\text{sech}^{-1}(cx)}{3e\,(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{ 220}$$

$$-\frac{a+b\text{sech}^{-1}(cx)}{3e\,(d+ex^2)^{3/2}}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2e\sqrt{1-c^2x^2}}{d(c^2d+e)\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{d^{3/2}}\right)}{6e}$$

input
```
Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(a + b*ArcSech[c*x])/(e*(d + e*x^2)^(3/2)) - (b*Sqrt[(1 + c*x)^(-1)]*
Sqrt[1 + c*x]*((2*e*Sqrt[1 - c^2*x^2])/(d*(c^2*d + e)*Sqrt[d + e*x^2]) - (
2*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/d^(3/2)))/(6*e)
```

3.170.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

3.170. $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2036
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

rule 6853
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Simp[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)]   Int[(d + e*x
^2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e
, p}, x] && NeQ[p, -1]
```

3.170.4 Maple [F]

$$\int \frac{x(a+b\,\mathrm{arcsech}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

3.170.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(94) = 188$.

Time = 0.34 (sec) , antiderivative size = 692, normalized size of antiderivative = 4.49

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \left[-\frac{4\,(bc^2d^3+bd^2e)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(bc^2d^3+(bc^2de^2+be^3)x^4+\cdots}{\cdots}\right.$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(4*(b*c^2*d^3 + b*d^2*e)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 -
1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2
*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^
2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^
2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(a*c^2*d^3
 + a*d^2*e + (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))
*sqrt(e*x^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(
c^2*d^4*e^2 + d^3*e^3)*x^2), 1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 +
 b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-d)*arctan(-1/2*((c^3*d - c
*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - 2*(b*c^2*d^3 + b*d^2*e)*sqrt
(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(a*c^2
*d^3 + a*d^2*e + (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^
2)))*sqrt(e*x^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 +
 2*(c^2*d^4*e^2 + d^3*e^3)*x^2)]
```

3.170.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.170.7 Maxima [F]

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
b*integrate(x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 +
d)^(5/2), x) - 1/3*a/((e*x^2 + d)^(3/2)*e)
```

3.170.8 Giac [F]

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.171 $\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$

3.171.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx = \text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{5/2}},x\right)$$

output
```
Unintegrable((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x)
```

3.171.2 Mathematica [N/A]

Not integrable

Time = 20.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

3.171.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{6865}$$

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}dx$$

input
```
Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

output
```
$Aborted
```

3.171.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.171.4 Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arcsech}\left(c x\right)}{x \left(e\,x^{2}+d \right)^{\frac{5}{2}}}d x$$

input
```
int((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x)
```

3.171.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{a + b\mathrm{sech}^{-1}(cx)}{x\left(d + ex^2\right)^{5/2}}\,dx = \int \frac{b\,\mathrm{arsech}\left(cx\right) + a}{\left(ex^2 + d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d
^2*e*x^3 + d^3*x), x)
```

3.171.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\mathrm{sech}^{-1}(cx)}{x\left(d + ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asech(c*x))/x/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.171.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\mathrm{sech}^{-1}(cx)}{x\left(d + ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.171.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\operatorname{arsech}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

3.171.9 Mupad [N/A]

Not integrable

Time = 4.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

3.172 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$

3.172.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx = \text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{5/2}},x\right)$$

output
```
Unintegrable((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

3.172.2 Mathematica [N/A]

Not integrable

Time = 24.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

3.172.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3 (d+ex^2)^{5/2}} \, dx$$
$$\Big\downarrow \text{6865}$$
$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3 (d+ex^2)^{5/2}}dx$$

input
```
Int[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

output
```
$Aborted
```

3.172.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.172.4 Maple [N/A] (verified)

Not integrable

Time = 0.84 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b \,\operatorname{arcsech}\left(c x\right)}{x^{3} \left(e \,x^{2}+d \right)^{\frac{5}{2}}}d x$$

input
```
int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

3.172.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\operatorname{arsech}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d
^2*e*x^5 + d^3*x^3), x)
```

3.172.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asech(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.172.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.172.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arsech}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

3.172.9 Mupad [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^3\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)), x)
```

3.173 $\int \frac{x^6\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.173.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^6\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\text{Int}\left(\frac{x^6\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

output
```
Unintegrable(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

3.173.2 Mathematica [N/A]

Not integrable

Time = 20.71 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^6\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^6\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

input
```
Integrate[(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]
```

3.173.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \left(a + b\text{sech}^{-1}(cx)\right)}{(d + ex^2)^{5/2}} \, dx$$

$$\Big\downarrow \; 6865$$

$$\int \frac{x^6 \left(a + b\text{sech}^{-1}(cx)\right)}{(d + ex^2)^{5/2}} dx$$

input
```
Int[(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.173.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.173.4 Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^6 (a + b \operatorname{arcsech}(cx))}{(e\,x^2 + d)^{\frac{5}{2}}} dx$$

input
```
int(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

3.173.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^6(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\, dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^6}{(ex^2 + d)^{\frac{5}{2}}}\, dx$$

input
```
integrate(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^6*arcsech(c*x) + a*x^6)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x
^4 + 3*d^2*e*x^2 + d^3), x)
```

3.173.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate(x**6*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.173.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.173.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^6\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^6}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

3.173.9 Mupad [N/A]

Not integrable

Time = 5.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^6\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^6\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((x^6*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^6*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.174 $\int \frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.174.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\text{Int}\left(\frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

output
```
Unintegrable(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

3.174.2 Mathematica [N/A]

Not integrable

Time = 19.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

input
```
Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]
```

3.174.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}} \, dx$$
$$\Big\downarrow \text{ 6865}$$
$$\int \frac{x^4 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}dx$$

input
```
Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.174.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.174.4 Maple [N/A] (verified)

Not integrable

Time = 0.90 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4 (a+b \text{ arcsech}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

3.174.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^4 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} \, dx = \int \frac{(b \,\text{arsech}\,(cx) + a)x^4}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsech(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x
^4 + 3*d^2*e*x^2 + d^3), x)
```

3.174.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.174.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.174.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^{5/2}} \, dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^4}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

3.174.9 Mupad [N/A]

Not integrable

Time = 4.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^{5/2}} \, dx = \int \frac{x^4 \left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^{5/2}} \, dx$$

input
```
int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.175 $\int \frac{x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.175.1 Optimal result

Integrand size = 23, antiderivative size = 246

$$\int \frac{x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{bx\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3de\left(c^2d+e\right)\sqrt{1+\frac{ex^2}{d}}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\mathrm{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3cde\sqrt{d+ex^2}}$$

output
```
1/3*x^3*(a+b*arcsech(c*x))/d/(e*x^2+d)^(3/2)-1/3*b*x*(1/(c*x+1))^(1/2)*(c*
x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)^(1/2)-1/3*b*c*Elliptic
E(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d/
e/(c^2*d+e)/(1+e*x^2/d)^(1/2)+1/3*b*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d/e/(e*x^2+d)^(1/2)
```

3.175.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.98

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{ax^3-\frac{b\sqrt{\frac{1-cx}{1+cx}}(-cd+ex)(d+ex^2)}{e(c^2d+e)}+bx^3\text{sech}^{-1}(cx)+\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\sqrt{\frac{c\left(\sqrt{d}+i\sqrt{e}x\right)}{\left(c\sqrt{d}+i\sqrt{e}\right)(1+cx)}}\sqrt{\frac{c}{(ic}}}{}$$

input
```
Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(a*x^3 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c*d) + e*x)*(d + e*x^2))/(e*(c^2*
d + e)) + b*x^3*ArcSech[c*x] + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*Sqrt
[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*
(I*Sqrt[d] + Sqrt[e]*x))/((I*c*Sqrt[d] + Sqrt[e])*(1 + c*x))]*(d + e*x^2)*
((I*c*Sqrt[d] + Sqrt[e])*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/
((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt
[d] - I*Sqrt[e])^2] - 2*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 -
 c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/
(c*Sqrt[d] - I*Sqrt[e])^2]))/(c*(c*Sqrt[d] + I*Sqrt[e])*e*Sqrt[((I*c*Sqrt[
d] + Sqrt[e])*(-1 + c*x))/(((-I)*c*Sqrt[d] + Sqrt[e])*(1 + c*x))]))/(3*d*(
d + e*x^2)^(3/2))
```

3.175.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6855, 27, 373, 326, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 6855$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x^2}{3d\sqrt{1-c^2x^2}\left(ex^2+d\right)^{3/2}}dx+\frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x^2}{\sqrt{1-c^2x^2}(ex^2+d)^{3/2}}dx}{3d}+\frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 373$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}dx}{c^2d+e}-\frac{x\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d}+\frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 326$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\left(c^2d+e\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{e}-\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}}{c^2d+e}-\frac{x\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d}+\frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 323$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}}{c^2d+e}-\frac{x\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d}+\frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 321$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e}}{c^2d+e}-\frac{x\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d}+\frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 330$$

3.175. $\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}dx$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c^2\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{\frac{ex^2}{d}+1}}}{c^2d+e}-\frac{x\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d}+\frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 327$$

$$\frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{d+ex^2}}-\frac{c\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{\frac{ex^2}{d}+1}}}{c^2d+e}-\frac{x\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d}$$

input
```
Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcSech[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (b*Sqrt[(1 + c*x)^(-1
)]*Sqrt[1 + c*x]*(-((x*Sqrt[1 - c^2*x^2])/((c^2*d + e)*Sqrt[d + e*x^2])) +
 (-((c*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[1 + (
e*x^2)/d])) + ((c^2*d + e)*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/
(c^2*d))])/(c*e*Sqrt[d + e*x^2]))/(c^2*d + e)))/(3*d)
```

3.175.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.175. $\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx$

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.175. $\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx$

3.175.4 Maple [F]

$$\int \frac{x^2(a+b\,\mathrm{arcsech}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

3.175.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(166) = 332.

Time = 0.10 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.36

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx = \frac{(bc^3d^2e+bcde^2)\sqrt{ex^2+d}x^3\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+\left((ac^3d^2e+acde^2)x^3-(bc\right.}{}$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((b*c^3*d^2*e + b*c*d*e^2)*sqrt(e*x^2 + d)*x^3*log((c*x*sqrt(-(c^2*x^2
 - 1)/(c^2*x^2)) + 1)/(c*x)) + ((a*c^3*d^2*e + a*c*d*e^2)*x^3 - (b*c^2*d*e
^2*x^4 + b*c^2*d^2*e*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d)
- ((b*c^4*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*elliptic_e(arcsin(c*x
), -e/(c^2*d)) - (b*c^4*d^3 + (b*c^4*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c
^4*d^2*e + b*d*e^2)*x^2)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(d))/(c^
3*d^5*e + c*d^4*e^2 + (c^3*d^3*e^3 + c*d^2*e^4)*x^4 + 2*(c^3*d^4*e^2 + c*d
^3*e^3)*x^2)
```

3.175.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.175.7 Maxima [F]

$$\int \frac{x^2 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} \, dx = \int \frac{(b \operatorname{arsech}(cx) + a)x^2}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + b*integrate(x
^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x
)
```

3.175.8 Giac [F]

$$\int \frac{x^2 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} \, dx = \int \frac{(b \operatorname{arsech}(cx) + a)x^2}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 (a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} \, dx = \int \frac{x^2 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)}{(e x^2 + d)^{5/2}} \, dx$$

input
```
int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.176 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

3.176.1 Optimal result

Integrand size = 20, antiderivative size = 266

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d^2(c^2d+e)\sqrt{d+ex^2}} + \frac{x(a+b\text{sech}^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{2x(a+b\text{sech}^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^2(c^2d+e)\sqrt{1+\frac{ex^2}{d}}} + \frac{2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3cd^2\sqrt{d+ex^2}}$$

output
```
1/3*x*(a+b*arcsech(c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arcsech(c*x))/d^2/(e
*x^2+d)^(1/2)+1/3*b*e*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)
/d^2/(c^2*d+e)/(e*x^2+d)^(1/2)+1/3*b*c*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*d+e)/(1+e*x^2/d)^(1/
2)+2/3*b*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(
1+e*x^2/d)^(1/2)/c/d^2/(e*x^2+d)^(1/2)
```

3.176.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.79 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.94

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{\frac{b\sqrt{\frac{1-cx}{1+cx}}(-cd+ex)(d+ex^2)}{c^2d+e} + ax(3d+2ex^2) + bx(3d+2ex^2)\,\text{sech}^{-1}(cx) - \frac{ib\sqrt{\frac{1-cx}{1+cx}}(1+cx)\sqrt{\cdots}}{\cdots}}{\cdots}$$

input
```
Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
((b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c*d) + e*x)*(d + e*x^2))/(c^2*d + e) + a*
x*(3*d + 2*e*x^2) + b*x*(3*d + 2*e*x^2)*ArcSech[c*x] - (I*b*Sqrt[(1 - c*x)
/(1 + c*x)]*(1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqr
t[e])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e]
)*(1 + c*x))]*(d + e*x^2)*((c*Sqrt[d] - I*Sqrt[e])*EllipticE[I*ArcSinh[Sqr
t[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt
[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] - 2*(3*c*Sqrt[d] + (2*I)*Sqr
t[e])*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqr
t[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]
))/(c*(c*Sqrt[d] + I*Sqrt[e])*Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/
((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x)))]))/(3*d^2*(d + e*x^2)^(3/2))
```

3.176.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.84, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6845, 27, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{6845}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{2ex^2+3d}{3d^2\sqrt{1-c^2x^2}\,(ex^2+d)^{3/2}}dx + \frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} + \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{2ex^2+3d}{\sqrt{1-c^2x^2}(ex^2+d)^{3/2}}dx}{3d^2}+\frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{ex\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\int -\frac{d\left(ex^2c^2+3dc^2+2e\right)}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d(c^2d+e)}\right)}{3d^2}+\frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{d\left(ex^2c^2+3dc^2+2e\right)}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d(c^2d+e)}+\frac{ex\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2}+\frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{ex^2c^2+3dc^2+2e}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{c^2d+e}+\frac{ex\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2}+\frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx+2(c^2d+e)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{c^2d+e}+\frac{ex\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2}+\frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx+\frac{2\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{\sqrt{d+ex^2}}}{c^2d+e}+\frac{ex\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2}+\frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

3.176. $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

$$\Big\downarrow \text{321}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx+\frac{2(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{d+ex^2}}}{c^2d+e}+\frac{ex\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2}+\frac{2x\left(a+b\,\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\,\text{sech}^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{c^2\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{\sqrt{\frac{ex^2}{d}+1}}+\frac{2(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{d+ex^2}}}{c^2d+e}+\frac{ex\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2}+\frac{2x\left(a+b\,\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\,\text{sech}^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{2x\left(a+b\,\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\,\text{sech}^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{2(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{d+ex^2}}+\frac{c\sqrt{d+ex^2}\,E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{\sqrt{\frac{ex^2}{d}+1}}}{c^2d+e}+\frac{ex\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2}$$

input
```
Int[(a + b*ArcSech[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcSech[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcSech[c*x
]))/(3*d^2*Sqrt[d + e*x^2]) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((e*x*
Sqrt[1 - c^2*x^2])/((c^2*d + e)*Sqrt[d + e*x^2]) + ((c*Sqrt[d + e*x^2]*Ell
ipticE[ArcSin[c*x], -(e/(c^2*d))])/Sqrt[1 + (e*x^2)/d] + (2*(c^2*d + e)*Sq
rt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*Sqrt[d + e*x^2]
))/(c^2*d + e)))/(3*d^2)
```

3.176. $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

3.176.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 6845
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSech[c*x])   u
, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]   Int[SimplifyIntegrand[u/(x*
Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (
IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

3.176.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arcsech}\left(cx\right)}{\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

3.176.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(182) = 364$.

Time = 0.11 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.49

$$\int \frac{a+b\mathrm{sech}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{(2\,(bc^3d^2e+bcde^2)x^3+3\,(bc^3d^3+bcd^2e)x)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+\Big(2\,(ac$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*(b*c^3*d^2*e + b*c*d*e^2)*x^3 + 3*(b*c^3*d^3 + b*c*d^2*e)*x)*sqrt(
e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + (2*(a*c^3
*d^2*e + a*c*d*e^2)*x^3 + 3*(a*c^3*d^3 + a*c*d^2*e)*x + (b*c^2*d*e^2*x^4 +
 b*c^2*d^2*e*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d) + ((b*c^
4*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*elliptic_e(arcsin(c*x), -e/(c
^2*d)) - (((b*c^4 - 3*b*c^2)*d*e^2 - 2*b*e^3)*x^4 + (b*c^4 - 3*b*c^2)*d^3
- 2*b*d^2*e + 2*((b*c^4 - 3*b*c^2)*d^2*e - 2*b*d*e^2)*x^2)*elliptic_f(arcs
in(c*x), -e/(c^2*d)))*sqrt(d))/(c^3*d^6 + c*d^5*e + (c^3*d^4*e^2 + c*d^3*e
^3)*x^4 + 2*(c^3*d^5*e + c*d^4*e^2)*x^2)
```

3.176.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.176.7 Maxima [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex^2)^{5/2}}\,dx = \int \frac{b\,\operatorname{arsech}(cx) + a}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(
log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

3.176.8 Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^{5/2}} \, dx = \int \frac{b \operatorname{arsech}(cx) + a}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^{5/2}} \, dx = \int \frac{a + b \operatorname{acosh}\left(\frac{1}{c x}\right)}{(e x^2 + d)^{5/2}} \, dx$$

input
```
int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(5/2), x)
```

3.177 $\int (fx)^m (d+ex^2)^3 (a+b\text{sech}^{-1}(cx)) \, dx$

3.177.1 Optimal result

Integrand size = 23, antiderivative size = 596

$$\int (fx)^m (d+ex^2)^3 (a+b\text{sech}^{-1}(cx)) \, dx =$$

$$-\frac{be\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4}{c^6f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)}$$

$$-\frac{be^2(e(5+m)^2+3c^2d(42+13m+m^2))(fx)^{3+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^4f^3(4+m)(5+m)(6+m)(7+m)}$$

$$-\frac{be^3(fx)^{5+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f^5(6+m)(7+m)}$$

$$+\frac{d^3(fx)^{1+m}(a+b\text{sech}^{-1}(cx))}{f(1+m)}+\frac{3d^2e(fx)^{3+m}(a+b\text{sech}^{-1}(cx))}{f^3(3+m)}$$

$$+\frac{3de^2(fx)^{5+m}(a+b\text{sech}^{-1}(cx))}{f^5(5+m)}+\frac{e^3(fx)^{7+m}(a+b\text{sech}^{-1}(cx))}{f^7(7+m)}$$

$$+\frac{b\left(\frac{c^6d^3(2+m)(4+m)(6+m)}{1+m}+\frac{e(1+m)\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{(3+m)(5+m)(7+m)}\right)(}{c^6f(1+m)(2+m)(4+m)(6+m)}$$

output
```
d^3*(f*x)^(1+m)*(a+b*arcsech(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arcsec
h(c*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arcsech(c*x))/f^5/(5+m)+e^3*(f*
x)^(7+m)*(a+b*arcsech(c*x))/f^7/(7+m)+b*(c^6*d^3*(2+m)*(4+m)*(6+m)/(1+m)+e
*(1+m)*(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+
22*m^3+179*m^2+638*m+840))/(m^3+15*m^2+71*m+105))*(f*x)^(1+m)*hypergeom([1
/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^6/f/
(1+m)/(2+m)/(4+m)/(6+m)-b*e*(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m)^2*(m^2+13*
m+42)+3*c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))*(f*x)^(1+m)*(1/(c*x+1))^(1
/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6/f/(6+m)/(m^2+6*m+8)/(m^3+15*m^2+7
1*m+105)-b*e^2*(e*(5+m)^2+3*c^2*d*(m^2+13*m+42))*(f*x)^(3+m)*(1/(c*x+1))^(
1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/f^3/(4+m)/(5+m)/(6+m)/(7+m)-b*e^
3*(f*x)^(5+m)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/f^5/(
6+m)/(7+m)
```

3.177.2 Mathematica [A] (verified)

Time = 4.67 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.74

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$
$$= x(fx)^m \left(\frac{ad^3}{1+m}+\frac{3ad^2ex^2}{3+m}+\frac{3ade^2x^4}{5+m}+\frac{ae^3x^6}{7+m}+\frac{bd^3\text{sech}^{-1}(cx)}{1+m}+\frac{3bd^2ex^2\text{sech}^{-1}(cx)}{3+m}+\frac{3bde^2x^4\text{sech}^{-1}(cx)}{5+m}+\frac{be^3x^6\text{sech}^{-1}(cx)}{7+m}-\frac{bd^3\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2(-1+cx)}-\frac{3bd^2ex^2\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(3+m)^2(-1+cx)}-\frac{3bde^2x^4\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},c^2x^2\right)}{(5+m)^2(-1+cx)}-\frac{be^3x^6\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7+m}{2},\frac{9+m}{2},c^2x^2\right)}{(7+m)^2(-1+cx)}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSech[c*x]),x]
```

output
```
x*(f*x)^m*((a*d^3)/(1 + m) + (3*a*d^2*e*x^2)/(3 + m) + (3*a*d*e^2*x^4)/(5
+ m) + (a*e^3*x^6)/(7 + m) + (b*d^3*ArcSech[c*x])/(1 + m) + (3*b*d^2*e*x^2
*ArcSech[c*x])/(3 + m) + (3*b*d*e^2*x^4*ArcSech[c*x])/(5 + m) + (b*e^3*x^6
*ArcSech[c*x])/(7 + m) - (b*d^3*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2
]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*(-1 +
c*x)) - (3*b*d^2*e*x^2*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hyperge
ometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/((3 + m)^2*(-1 + c*x)) - (
3*b*d*e^2*x^4*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hypergeometric2F
1[1/2, (5 + m)/2, (7 + m)/2, c^2*x^2])/((5 + m)^2*(-1 + c*x)) - (b*e^3*x^6
*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (7 + m
)/2, (9 + m)/2, c^2*x^2])/((7 + m)^2*(-1 + c*x)))
```

3.177.3 Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 527, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6855, 2340, 25, 1590, 25, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3 (fx)^m \left(a+b\text{sech}^{-1}(cx)\right) dx$$

$$\Big\downarrow \text{6855}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(fx)^m \left(\frac{e^3x^6}{m+7}+\frac{3de^2x^4}{m+5}+\frac{3d^2ex^2}{m+3}+\frac{d^3}{m+1}\right)}{\sqrt{1-c^2x^2}}dx+\frac{d^3(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{sech}^{-1}(cx)\right)}{f^7(m+7)}$$

$$\Big\downarrow \text{2340}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{(fx)^m\left(\frac{e^2\left(3d\left(m^2+13m+42\right)c^2+e(m+5)^2\right)x^4}{(m+5)(m+7)}+\frac{3c^2d^2e(m+6)x^2}{m+3}+\frac{c^2d^3(m+6)}{m+1}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+6)}-\frac{e^3\sqrt{1-c^2x^2}(fx)^{m+5}}{c^2f^5(m+6)(m+7)}\right)+\frac{d^3(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{sech}^{-1}(cx)\right)}{f^7(m+7)}$$

$$\Big\downarrow \text{25}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{(fx)^m\left(\frac{e^2\left(3d\left(m^2+13m+42\right)c^2+e(m+5)^2\right)x^4}{(m+5)(m+7)}+\frac{3c^2d^2e(m+6)x^2}{m+3}+\frac{c^2d^3(m+6)}{m+1}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+6)}-\frac{e^3\sqrt{1-c^2x^2}(fx)^{m+5}}{c^2f^5(m+6)(m+7)}\right)+\frac{d^3(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{sech}^{-1}(cx)\right)}{f^7(m+7)}$$

$$\Big\downarrow \text{1590}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{-\frac{\int -\frac{(fx)^m\left(\frac{d^3(m+4)(m+6)c^4}{m+1}+\frac{e\left(3d^2\left(m^4+22m^3+179m^2+638m+840\right)c^4+3de(m+3)^2\left(m^2+13m+42\right)c^2+e^2\left(m^2+8m+15\right)^2\right)x}{(m+3)(m+5)(m+7)}\right.}{\sqrt{1-c^2x^2}}}{c^2(m+4)}}{c^2(m+6)}\right.$$
$$+\frac{d^3(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{sech}^{-1}(cx)\right)}{f^7(m+7)}$$

$$\Big\downarrow \text{25}$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\int \frac{(fx)^m\left(\frac{d^3(m+4)(m+6)c^4}{m+1}+\frac{e\left(3d^2\left(m^4+22m^3+179m^2+638m+840\right)c^4+3de(m+3)^2\left(m^2+13m+42\right)c^2+e^2\left(m^2+8m+15\right)^2\right)x^2}{(m+3)(m+5)(m+7)}\right)}{\sqrt{1-c^2x^2}}}{c^2(m+4)}}{c^2(m+6)}\right.$$
$$+\frac{d^3(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{sech}^{-1}(cx)\right)}{f^7(m+7)}$$

$$\Big\downarrow \text{363}$$

3.177. $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{sech}^{-1}(cx)\right)\, dx$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\left(\frac{c^4d^3(m+4)(m+6)}{m+1}+\frac{e(m+1)\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)+3c^2de(m+3)^2\left(m^2+13m+42\right)+e^2\left(m^2+8m+15\right)^2\right)}{c^2(m+2)(m+3)(m+5)(m+7)}\right)\int \frac{}{\sqrt{}}}{c^2(m-}}{}\right.$$

$$\frac{d^3(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{sech}^{-1}(cx)\right)}{f^7(m+7)}$$

$$\Big\downarrow \; 278$$

$$\frac{d^3(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{sech}^{-1}(cx)\right)}{f^7(m+7)}+$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{(fx)^{m+1}\left(\frac{c^4d^3(m+4)(m+6)}{m+1}+\frac{e(m+1)\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)+3c^2de(m+3)^2\left(m^2+13m+42\right)+e^2\left(m^2+8m+15\right)^2\right)}{c^2(m+2)(m+3)(m+5)(m+7)}\right)}{f(m+1)}}{}\right.$$

input
```
Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSech[c*x]),x]
```

output
```
(d^3*(f*x)^(1 + m)*(a + b*ArcSech[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 +
 m)*(a + b*ArcSech[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*Ar
cSech[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcSech[c*x]))/(f^7
*(7 + m)) + b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-((e^3*(f*x)^(5 + m)*Sqr
t[1 - c^2*x^2])/(c^2*f^5*(6 + m)*(7 + m))) + (-((e^2*(e*(5 + m)^2 + 3*c^2*
d*(42 + 13*m + m^2))*(f*x)^(3 + m)*Sqrt[1 - c^2*x^2])/(c^2*f^3*(4 + m)*(5
+ m)*(7 + m))) + (-((e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 +
 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*(f*x)^(1
+ m)*Sqrt[1 - c^2*x^2])/(c^2*f*(2 + m)*(3 + m)*(5 + m)*(7 + m))) + (((c^4*
d^3*(4 + m)*(6 + m))/(1 + m) + (e*(1 + m)*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*
d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^
3 + m^4)))/(c^2*(2 + m)*(3 + m)*(5 + m)*(7 + m)))*(f*x)^(1 + m)*Hypergeome
tric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)))/(c^2*(4 + m)))/(
c^2*(6 + m)))
```

3.177. $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{sech}^{-1}(cx)\right)\, dx$

3.177.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.177.4 Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^3 (a+b\,\mathrm{arcsech}\,(cx))\,dx$$

input
```
int((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x)
```

3.177.5 Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\,\mathrm{sech}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^3 (b\,\mathrm{arsech}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsech(c*x))*(f*x)^m, x)
```

3.177.6 Sympy [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\,\mathrm{sech}^{-1}(cx)\right)\,dx = \int (fx)^m (a+b\,\mathrm{asech}\,(cx)) \left(d+ex^2\right)^3\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)**3*(a+b*asech(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*asech(c*x))*(d + e*x**2)**3, x)
```

3.177.7 Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^3 (b \operatorname{arsech}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*
x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) + (((m^3 + 9*m^2 + 23*m
+ 15)*b*e^3*f^m*x^7*x^m + 3*(m^3 + 11*m^2 + 31*m + 21)*b*d*e^2*f^m*x^5*x^m
 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*d^2*e*f^m*x^3*x^m + (m^3 + 15*m^2 + 71*m
 + 105)*b*d^3*f^m*x*x^m)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - ((m^3 + 9
*m^2 + 23*m + 15)*b*e^3*f^m*x^7*x^m + 3*(m^3 + 11*m^2 + 31*m + 21)*b*d*e^2
*f^m*x^5*x^m + 3*(m^3 + 13*m^2 + 47*m + 35)*b*d^2*e*f^m*x^3*x^m + (m^3 + 1
5*m^2 + 71*m + 105)*b*d^3*f^m*x*x^m)*log(x))/(m^4 + 16*m^3 + 86*m^2 + 176*
m + 105) - integrate((b*c^2*e^3*f^m*(m + 7)*x^2*log(c) - (e^3*f^m*(m + 7)*
log(c) - e^3*f^m)*b)*x^6*x^m/(c^2*(m + 7)*x^2 - m - 7), x) - integrate(3*(
b*c^2*d*e^2*f^m*(m + 5)*x^2*log(c) - (d*e^2*f^m*(m + 5)*log(c) - d*e^2*f^m
)*b)*x^4*x^m/(c^2*(m + 5)*x^2 - m - 5), x) - integrate(3*(b*c^2*d^2*e*f^m*
(m + 3)*x^2*log(c) - (d^2*e*f^m*(m + 3)*log(c) - d^2*e*f^m)*b)*x^2*x^m/(c^
2*(m + 3)*x^2 - m - 3), x) - integrate((b*c^2*d^3*f^m*(m + 1)*x^2*log(c) -
 (d^3*f^m*(m + 1)*log(c) - d^3*f^m)*b)*x^m/(c^2*(m + 1)*x^2 - m - 1), x) +
 integrate(((m^3 + 9*m^2 + 23*m + 15)*b*c^2*e^3*f^m*x^8*x^m + 3*(m^3 + 11*
m^2 + 31*m + 21)*b*c^2*d*e^2*f^m*x^6*x^m + 3*(m^3 + 13*m^2 + 47*m + 35)*b*
c^2*d^2*e*f^m*x^4*x^m + (m^3 + 15*m^2 + 71*m + 105)*b*c^2*d^3*f^m*x^2*x^m)
/((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^2*x^2 - m^4 - 16*m^3 + ((m^4 + 1
6*m^3 + 86*m^2 + 176*m + 105)*c^2*x^2 - m^4 - 16*m^3 - 86*m^2 - 176*m -...
```

3.177.8 Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^3 (b \operatorname{arsech}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (f\,x)^m \, \left(e\,x^2+d\right)^3 \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int((f*x)^m*(d + e*x^2)^3*(a + b*acosh(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^3*(a + b*acosh(1/(c*x))), x)
```

3.178 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

3.178.1 Optimal result

Integrand size = 23, antiderivative size = 372

$$\begin{aligned}
&\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx \\
&= -\frac{be\left(e(3+m)^2+2c^2d\left(20+9m+m^2\right)\right)(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^4f(2+m)(3+m)(4+m)(5+m)} \\
&\quad -\frac{be^2(fx)^{3+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f^3(4+m)(5+m)}+\frac{d^2(fx)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f(1+m)} \\
&\quad +\frac{2de(fx)^{3+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(3+m)}+\frac{e^2(fx)^{5+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(5+m)} \\
&\quad +\frac{b\left(c^4d^2(2+m)(3+m)(4+m)(5+m)+e(1+m)^2\left(e(3+m)^2+2c^2d\left(20+9m+m^2\right)\right)\right)(fx)^{1+m}\sqrt{\frac{1}{1+cx}}}{c^4f(1+m)^2(2+m)(3+m)(4+m)(5+m)}
\end{aligned}$$

output
```
d^2*(f*x)^(1+m)*(a+b*arcsech(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arcsech(
c*x))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arcsech(c*x))/f^5/(5+m)+b*(c^4*d^2*(2
+m)*(3+m)*(4+m)*(5+m)+e*(1+m)^2*(e*(3+m)^2+2*c^2*d*(m^2+9*m+20)))*(f*x)^(1
+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x
+1)^(1/2)/c^4/f/(1+m)^2/(2+m)/(3+m)/(4+m)/(5+m)-b*e*(e*(3+m)^2+2*c^2*d*(m^
2+9*m+20))*(f*x)^(1+m)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/
c^4/f/(4+m)/(5+m)/(m^2+5*m+6)-b*e^2*(f*x)^(3+m)*(1/(c*x+1))^(1/2)*(c*x+1)^
(1/2)*(-c^2*x^2+1)^(1/2)/c^2/f^3/(4+m)/(5+m)
```

3.178.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.87

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) dx$$
$$= x(fx)^m \left(\frac{ad^2}{1+m}+\frac{2adex^2}{3+m}+\frac{ae^2x^4}{5+m}+\frac{bd^2\text{sech}^{-1}(cx)}{1+m}+\frac{2bdex^2\text{sech}^{-1}(cx)}{3+m} + \frac{be^2x^4\text{sech}^{-1}(cx)}{5+m}-\frac{bd^2\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2(-1+cx)} - \frac{2bdex^2\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(3+m)^2(-1+cx)} - \frac{be^2x^4\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},c^2x^2\right)}{(5+m)^2(-1+cx)}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

output
```
x*(f*x)^m*((a*d^2)/(1 + m) + (2*a*d*e*x^2)/(3 + m) + (a*e^2*x^4)/(5 + m) +
 (b*d^2*ArcSech[c*x])/(1 + m) + (2*b*d*e*x^2*ArcSech[c*x])/(3 + m) + (b*e^
2*x^4*ArcSech[c*x])/(5 + m) - (b*d^2*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^
2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*(
-1 + c*x)) - (2*b*d*e*x^2*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hype
rgeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/((3 + m)^2*(-1 + c*x))
- (b*e^2*x^4*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hypergeometric2F1
[1/2, (5 + m)/2, (7 + m)/2, c^2*x^2])/((5 + m)^2*(-1 + c*x)))
```

3.178.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6855, 27, 1590, 25, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (fx)^m \left(a+b\text{sech}^{-1}(cx)\right) dx$$

$$\Big\downarrow \; 6855$$

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(fx)^m \left(e^2(m+1)(m+3)x^4+2de(m+1)(m+5)x^2+d^2(m+3)(m+5)\right)}{\left(m^3+9m^2+23m+15\right)\sqrt{1-c^2x^2}}dx+\frac{d^2(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 27$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(fx)^m\left(e^2(m+1)(m+3)x^4+2de(m+1)(m+5)x^2+d^2(m+3)(m+5)\right)}{\sqrt{1-c^2x^2}}dx}{m^3+9m^2+23m+15}+\frac{d^2(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 1590$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{(fx)^m\left(c^2(m+3)(m+4)(m+5)d^2+e(m+1)\left(2d\left(m^2+9m+20\right)c^2+e(m+3)^2\right)x^2\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+4)}-\frac{e^2(m+1)(m+3)\sqrt{1-c^2x^2}(fx)^{m+3}}{c^2f^3(m+4)}\right)}{m^3+9m^2+23m+15}+\frac{d^2(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 25$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{(fx)^m\left(c^2(m+3)(m+4)(m+5)d^2+e(m+1)\left(2d\left(m^2+9m+20\right)c^2+e(m+3)^2\right)x^2\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+4)}-\frac{e^2(m+1)(m+3)\sqrt{1-c^2x^2}(fx)^{m+3}}{c^2f^3(m+4)}\right)}{m^3+9m^2+23m+15}+\frac{d^2(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 363$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{\left(c^4d^2(m+3)(m+4)(m+5)+\frac{e(m+1)^2\left(2c^2d\left(m^2+9m+20\right)+e(m+3)^2\right)}{m+2}\right)\int \frac{(fx)^m}{\sqrt{1-c^2x^2}}dx}{c^2}-\frac{e(m+1)\sqrt{1-c^2x^2}(fx)^{m+1}\left(2c^2d\left(m^2+9m+\cdots\right.\right.}{c^2f(m+2)}}{c^2(m+4)}\cdots\right.}{m^3+9m^2+23m+15}+\frac{d^2(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 278$$

3.178. $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)\,dx$

$$\frac{d^2(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}+$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\frac{(fx)^{m+1}\left(c^4d^2(m+3)(m+4)(m+5)+\frac{e(m+1)^2\left(2c^2d\left(m^2+9m+20\right)+e(m+3)^2\right)}{m+2}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)}{c^2f(m+1)}-\frac{e(m+1)}{}}{c^2(m+4)}\right.}{m^3+9m^2+23m+15}$$

input
```
Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

output
```
(d^2*(f*x)^(1 + m)*(a + b*ArcSech[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m
)*(a + b*ArcSech[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcSech[
c*x]))/(f^5*(5 + m)) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-((e^2*(1 +
m)*(3 + m)*(f*x)^(3 + m)*Sqrt[1 - c^2*x^2])/(c^2*f^3*(4 + m))) + (-((e*(1
+ m)*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2))*(f*x)^(1 + m)*Sqrt[1 - c^2*x
^2])/(c^2*f*(2 + m))) + ((c^4*d^2*(3 + m)*(4 + m)*(5 + m) + (e*(1 + m)^2*(
e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2)))/(2 + m))*(f*x)^(1 + m)*Hypergeome
tric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(c^2*f*(1 + m)))/(c^2*(4 + m)
)))/(15 + 23*m + 9*m^2 + m^3)
```

3.178.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.178.4 Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^2 (a+b\,\mathrm{arcsech}\,(cx))\,dx$$

input
```
int((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x)
```

3.178.5 Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\mathrm{sech}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^2 (b\,\mathrm{arsech}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arcsech(c*x))*(f*x)^m, x)
```

3.178.6 Sympy [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (fx)^m (a+b\,\text{asech}\,(cx)) \left(d+ex^2\right)^2 \, dx$$

input
```
integrate((f*x)**m*(e*x**2+d)**2*(a+b*asech(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*asech(c*x))*(d + e*x**2)**2, x)
```

3.178.7 Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (ex^2+d)^2 (b\,\text{arsech}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*
d^2/(f*(m + 1)) + (((m^2 + 4*m + 3)*b*e^2*f^m*x^5*x^m + 2*(m^2 + 6*m + 5)*
b*d*e*f^m*x^3*x^m + (m^2 + 8*m + 15)*b*d^2*f^m*x*x^m)*log(sqrt(c*x + 1)*sq
rt(-c*x + 1) + 1) - ((m^2 + 4*m + 3)*b*e^2*f^m*x^5*x^m + 2*(m^2 + 6*m + 5)
*b*d*e*f^m*x^3*x^m + (m^2 + 8*m + 15)*b*d^2*f^m*x*x^m)*log(x))/(m^3 + 9*m^
2 + 23*m + 15) - integrate((b*c^2*e^2*f^m*(m + 5)*x^2*log(c) - (e^2*f^m*(m
 + 5)*log(c) - e^2*f^m)*b)*x^4*x^m/(c^2*(m + 5)*x^2 - m - 5), x) - integra
te(2*(b*c^2*d*e*f^m*(m + 3)*x^2*log(c) - (d*e*f^m*(m + 3)*log(c) - d*e*f^m
)*b)*x^2*x^m/(c^2*(m + 3)*x^2 - m - 3), x) - integrate((b*c^2*d^2*f^m*(m +
 1)*x^2*log(c) - (d^2*f^m*(m + 1)*log(c) - d^2*f^m)*b)*x^m/(c^2*(m + 1)*x^
2 - m - 1), x) + integrate(((m^2 + 4*m + 3)*b*c^2*e^2*f^m*x^6*x^m + 2*(m^2
 + 6*m + 5)*b*c^2*d*e*f^m*x^4*x^m + (m^2 + 8*m + 15)*b*c^2*d^2*f^m*x^2*x^m
)/((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 + ((m^3 + 9*m^2 + 23*m + 15)*c^
2*x^2 - m^3 - 9*m^2 - 23*m - 15)*sqrt(c*x + 1)*sqrt(-c*x + 1) - 9*m^2 - 23
*m - 15), x)
```

3.178.8 Giac [F]

$$\int (fx)^m (d+ex^2)^2 (a+b\operatorname{sech}^{-1}(cx))\,dx = \int (ex^2+d)^2 (b\operatorname{arsech}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^2)^2 (a+b\operatorname{sech}^{-1}(cx))\,dx = \int (f\,x)^m \left(e\,x^2+d\right)^2 \left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((f*x)^m*(d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)
```

3.179 $\int (fx)^m (d+ex^2) \left(a+b\text{sech}^{-1}(cx)\right) dx$

3.179.1 Optimal result

Integrand size = 21, antiderivative size = 206

$$\int (fx)^m (d+ex^2) \left(a+b\text{sech}^{-1}(cx)\right) dx = -\frac{be(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f(2+m)(3+m)} + \frac{d(fx)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(3+m)} + \frac{b(e(1+m)^2+c^2d(2+m)(3+m))(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{c^2f(1+m)^2(2+m)(3+m)}$$

output
```
d*(f*x)^(1+m)*(a+b*arcsech(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arcsech(c*x))/
f^3/(3+m)+b*(e*(1+m)^2+c^2*d*(2+m)*(3+m))*(f*x)^(1+m)*hypergeom([1/2, 1/2+
1/2*m],[3/2+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^2/f/(1+m)^2/
(2+m)/(3+m)-b*e*(f*x)^(1+m)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(
1/2)/c^2/f/(2+m)/(3+m)
```

3.179.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.92

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$
$$= x(fx)^m \left(-\frac{bd\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2(-1+cx)} + \frac{\frac{(3+m)(d(3+m)+e(1+m)x^2)\left(a+b\text{sech}^{-1}(cx)\right)}{1+m} - \frac{bex^2\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{-1+cx}}{(3+m)^2}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

output
```
x*(f*x)^m*(-((b*d*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hypergeometr
ic2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*(-1 + c*x))) + (((3
+ m)*(d*(3 + m) + e*(1 + m)*x^2)*(a + b*ArcSech[c*x]))/(1 + m) - (b*e*x^2*
Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (3 + m)
/2, (5 + m)/2, c^2*x^2])/(-1 + c*x))/(3 + m)^2)
```

3.179.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6855, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)(fx)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$
$$\Big\downarrow \; 6855$$
$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(fx)^m \left(e(m+1)x^2+d(m+3)\right)}{\left(m^2+4m+3\right)\sqrt{1-c^2x^2}}dx + \frac{d(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)} + \frac{e(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}$$
$$\Big\downarrow \; 27$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(fx)^m\left(e(m+1)x^2+d(m+3)\right)}{\sqrt{1-c^2x^2}}dx}{m^2+4m+3}+\frac{d(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}$$

$$\Big\downarrow \; 363$$

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\left(\frac{e(m+1)^2}{c^2(m+2)}+d(m+3)\right)\int \frac{(fx)^m}{\sqrt{1-c^2x^2}}dx-\frac{e(m+1)\sqrt{1-c^2x^2}(fx)^{m+1}}{c^2f(m+2)}\right)}{m^2+4m+3}+\frac{d(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}$$

$$\Big\downarrow \; 278$$

$$\frac{d(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{(fx)^{m+1}\left(\frac{e(m+1)^2}{c^2(m+2)}+d(m+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)}{f(m+1)}-\frac{e(m+1)\sqrt{1-c^2x^2}(fx)^{m+1}}{c^2f(m+2)}\right)}{m^2+4m+3}$$

input
```
Int[(f*x)^m*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

output
```
(d*(f*x)^(1 + m)*(a + b*ArcSech[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a +
 b*ArcSech[c*x]))/(f^3*(3 + m)) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-
((e*(1 + m)*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2])/(c^2*f*(2 + m))) + (((e*(1 +
m)^2)/(c^2*(2 + m)) + d*(3 + m))*(f*x)^(1 + m)*Hypergeometric2F1[1/2, (1 +
 m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m))))/(3 + 4*m + m^2)
```

3.179.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 6855
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcSech[c*x])   u, x] + Simp[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)]
Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] &&
 GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2
*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.179.4 Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right) \left(a+b\,\mathrm{arcsech}\left(cx\right)\right) dx$$

input
```
int((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x)
```

3.179.5 Fricas [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (ex^2+d)(b\,\mathrm{arsech}\left(cx\right)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*(f*x)^m, x)
```

3.179.6 Sympy [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (fx)^m (a+b\,\text{asech}\,(cx)) \left(d+ex^2\right) \, dx$$

input
```
integrate((f*x)**m*(e*x**2+d)*(a+b*asech(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*asech(c*x))*(d + e*x**2), x)
```

3.179.7 Maxima [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (ex^2+d)(b\,\text{arsech}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

output
```
a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1)) + ((b*e*f^m*(m + 1
)*x^3*x^m + b*d*f^m*(m + 3)*x*x^m)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) -
 (b*e*f^m*(m + 1)*x^3*x^m + b*d*f^m*(m + 3)*x*x^m)*log(x))/(m^2 + 4*m + 3)
 - integrate((b*c^2*e*f^m*(m + 3)*x^2*log(c) - (e*f^m*(m + 3)*log(c) - e*f
^m)*b)*x^2*x^m/(c^2*(m + 3)*x^2 - m - 3), x) - integrate((b*c^2*d*f^m*(m +
 1)*x^2*log(c) - (d*f^m*(m + 1)*log(c) - d*f^m)*b)*x^m/(c^2*(m + 1)*x^2 -
m - 1), x) + integrate((b*c^2*e*f^m*(m + 1)*x^4*x^m + b*c^2*d*f^m*(m + 3)*
x^2*x^m)/((m^2 + 4*m + 3)*c^2*x^2 + ((m^2 + 4*m + 3)*c^2*x^2 - m^2 - 4*m -
 3)*sqrt(c*x + 1)*sqrt(-c*x + 1) - m^2 - 4*m - 3), x)
```

3.179.8 Giac [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (ex^2+d)(b\,\text{arsech}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (f\,x)^m \left(e\,x^2+d\right) \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int((f*x)^m*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

3.180 $\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$

3.180.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x)
```

3.180.2 Mathematica [N/A]

Not integrable

Time = 3.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2), x]
```

3.180.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2} \, dx$$
$$\Big\downarrow \text{ 6865}$$
$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}dx$$

input
```
Int[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
```

output
```
$Aborted
```

3.180.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.180.4 Maple [N/A] (verified)

Not integrable

Time = 1.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b \,\operatorname{arcsech}\left(c x\right)\right)}{e \,x^{2}+d}d x$$

input
```
int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x)
```

output
```
int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x)
```

3.180.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a + b\operatorname{sech}^{-1}(cx)\right)}{d + ex^2} \, dx = \int \frac{(b \operatorname{arsech}(cx) + a)(fx)^m}{ex^2 + d} \, dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

3.180.6 Sympy [N/A]

Not integrable

Time = 8.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(fx)^m \left(a + b\operatorname{sech}^{-1}(cx)\right)}{d + ex^2} \, dx = \int \frac{(fx)^m (a + b \operatorname{asech}(cx))}{d + ex^2} \, dx$$

input
```
integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d),x)
```

output
```
Integral((f*x)**m*(a + b*asech(c*x))/(d + e*x**2), x)
```

3.180.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a + b\operatorname{sech}^{-1}(cx)\right)}{d + ex^2} \, dx = \int \frac{(b \operatorname{arsech}(cx) + a)(fx)^m}{ex^2 + d} \, dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

3.180.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)(fx)^m}{ex^2+d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

3.180.9 Mupad [N/A]

Not integrable

Time = 3.92 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(f\,x)^m\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

input
```
int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2),x)
```

output
```
int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2), x)
```

3.181 $\int \frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.181.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\operatorname{Int}\left(\frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

3.181.2 Mathematica [N/A]

Not integrable

Time = 7.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2, x]
```

3.181.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx$$

$$\Big\downarrow \text{ 6865}$$

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}dx$$

input
```
Int[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

3.181.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.181.4 Maple [N/A] (verified)

Not integrable

Time = 1.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^{m} \left(a+b \,\operatorname{arcsech}\left(c x \right)\right)}{\left(e \,x^{2}+d \right)^{2}}d x$$

input
```
int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

output
```
int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

3.181.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} dx = \int \frac{(b\,\text{arsech}\,(cx)+a)(fx)^m}{\left(ex^2+d\right)^2} dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsech(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.181.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.181.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} dx = \int \frac{(b\,\text{arsech}\,(cx)+a)(fx)^m}{\left(ex^2+d\right)^2} dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

3.181.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx = \int \frac{(b\operatorname{arsech}(cx)+a)(fx)^m}{(ex^2+d)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

3.181.9 Mupad [N/A]

Not integrable

Time = 4.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m \left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx = \int \frac{(f\,x)^m \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^2}\,dx$$

input
```
int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.182 $\int (fx)^m (d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx)) \, dx$

3.182.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m (d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx)) \, dx = \text{Int}\Big((fx)^m (d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx)), x\Big)$$

output
```
Unintegrable((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

3.182.2 Mathematica [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m (d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx)) \, dx = \int (fx)^m (d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx)) \, dx$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

output
```
Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

3.182.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2} (fx)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \ 6865$$

$$\int \left(d+ex^2\right)^{3/2} (fx)^m \left(a+b\text{sech}^{-1}(cx)\right) dx$$

input
```
Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

output
```
$Aborted
```

3.182.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.182.4 Maple [N/A] (verified)

Not integrable

Time = 1.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\ \text{arcsech}\left(cx\right)\right) dx$$

input
```
int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

3.182.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (ex^2+d)^{\frac{3}{2}}(b \operatorname{arsech}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas"
)
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)*(f
*x)^m, x)
```

3.182.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
```

output
```
Timed out
```

3.182.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (ex^2+d)^{\frac{3}{2}}(b \operatorname{arsech}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima"
)
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

3.182.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b \,\text{arsech}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

3.182.9 Mupad [N/A]

Not integrable

Time = 4.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (f\,x)^m \left(e\,x^2+d\right)^{3/2} \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

3.183 $\int (fx)^m \sqrt{d+ex^2} \left(a+b\text{sech}^{-1}(cx)\right) dx$

3.183.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \sqrt{d+ex^2} \left(a+b\text{sech}^{-1}(cx)\right) dx = \text{Int}\left((fx)^m \sqrt{d+ex^2} \left(a+b\text{sech}^{-1}(cx)\right),x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

3.183.2 Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m \sqrt{d+ex^2} \left(a+b\text{sech}^{-1}(cx)\right) dx = \int (fx)^m \sqrt{d+ex^2} \left(a+b\text{sech}^{-1}(cx)\right) dx$$

input
```
Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

output
```
Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

3.183.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(fx)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{6865}$$

$$\int \sqrt{d+ex^2}(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)dx$$

input
```
Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

output
```
$Aborted
```

3.183.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.183.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m \left(a+b \operatorname{arcsech}\left(cx\right)\right) \sqrt{e\,x^2+d}dx$$

input
```
int((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

3.183.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

3.183.6 Sympy [N/A]

Not integrable

Time = 8.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (fx)^m\,(a+b\,\text{asech}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate((f*x)**m*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral((f*x)**m*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

3.183.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

3.183.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))\ dx = \int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)(fx)^m\ dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

3.183.9 Mupad [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))\ dx = \int (f\,x)^m\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\ dx$$

input
```
int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

3.184 $\int \frac{(fx)^m\left(a+b\mathbf{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.184.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

3.184.2 Mathematica [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]
```

3.184.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}} \, dx$$

$$\Big\downarrow \text{ 6865}$$

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx$$

input
```
Int[((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.184.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.184.4 Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\left(f x \right)^{m} \left(a +b \,\operatorname{arcsech}\left(c x \right)\right)}{\sqrt{e \,x^{2}+d}}d x$$

input
```
int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

3.184.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral((b*arcsech(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

3.184.6 Sympy [N/A]

Not integrable

Time = 3.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m\,(a+b\,\text{asech}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((f*x)**m*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

3.184.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((b*arcsech(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

3.184.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arsech}\,(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

3.184.9 Mupad [N/A]

Not integrable

Time = 4.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(f\,x)^m\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.185 $\int \frac{(fx)^m\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.185.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

3.185.2 Mathematica [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(fx)^m\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]
```

3.185.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 6865}$$

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}dx$$

input
```
Int[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.185.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.185.4 Maple [N/A] (verified)

Not integrable

Time = 1.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\left(f x \right)^{m} \left(a +b \,\operatorname{arcsech}\left(c x \right)\right)}{\left(e \,x^{2}+d \right)^{\frac{3}{2}}}d x$$

input
```
int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

3.185.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{(fx)^m \left(a + b\text{sech}^{-1}(cx)\right)}{(d + ex^2)^{3/2}} \, dx = \int \frac{(b \operatorname{arsech}(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}} \, dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2
 + d^2), x)
```

3.185.6 Sympy [N/A]

Not integrable

Time = 22.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m \left(a + b\text{sech}^{-1}(cx)\right)}{(d + ex^2)^{3/2}} \, dx = \int \frac{(fx)^m \left(a + b \operatorname{asech}(cx)\right)}{(d + ex^2)^{\frac{3}{2}}} \, dx$$

input
```
integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((f*x)**m*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

3.185.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b\text{sech}^{-1}(cx)\right)}{(d + ex^2)^{3/2}} \, dx = \int \frac{(b \operatorname{arsech}(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}} \, dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

3.185.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)(fx)^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

3.185.9 Mupad [N/A]

Not integrable

Time = 4.66 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=\int \frac{(f\,x)^m \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.186 $\int \frac{x^{11}\left(a+b\,\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

3.186.1 Optimal result

Integrand size = 26, antiderivative size = 473

$$\begin{aligned}\int \frac{x^{11}\left(a+b\,\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = & -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}+\frac{7b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{3/2}}{90c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} \\ & -\frac{13b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{5/2}}{150c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}+\frac{3b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{7/2}}{70c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} \\ & -\frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{9/2}}{90c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}-\frac{\sqrt{1-c^4x^4}\left(a+b\,\text{sech}^{-1}(cx)\right)}{2c^{12}} \\ & +\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{3c^{12}} \\ & -\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{10c^{12}} \\ & +\frac{4b\sqrt{1-c^2x^2}\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{15c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}\end{aligned}$$

output
```
1/3*(-c^4*x^4+1)^(3/2)*(a+b*arcsech(c*x))/c^12-1/10*(-c^4*x^4+1)^(5/2)*(a+
b*arcsech(c*x))/c^12+7/90*b*(c^2*x^2+1)^(3/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(-
1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-13/150*b*(c^2*x^2+1)^(5/2)*(-c^2*x^2+1)^(1/
2)/c^13/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+3/70*b*(c^2*x^2+1)^(7/2)*(-c^2*
x^2+1)^(1/2)/c^13/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-1/90*b*(c^2*x^2+1)^(9
/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+4/15*b*arct
anh((c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^13/x/(-1+1/c/x)^(1/2)/(1+1/c/x
)^(1/2)-4/15*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^13/x/(-1+1/c/x)^(1/2
)/(1+1/c/x)^(1/2)-1/2*(a+b*arcsech(c*x))*(-c^4*x^4+1)^(1/2)/c^12
```

3.186.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.45

$$\int \frac{x^{11}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \frac{-105a\sqrt{1-c^4x^4}(8+4c^4x^4+3c^8x^8)+\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^4x^4}(768+36c^2x^2+78c^4x^4+5c^6x^6+35c^8x^8)}{-1+cx}-105b\sqrt{1-c^4x^4}(8+}{3150c^{12}}$$

input
```
Integrate[(x^11*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
(-105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*Sqrt[(1 - c*x)/
(1 + c*x)]*Sqrt[1 - c^4*x^4]*(768 + 36*c^2*x^2 + 78*c^4*x^4 + 5*c^6*x^6 +
35*c^8*x^8))/(-1 + c*x) - 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x
^8)*ArcSech[c*x] + 840*b*Log[x*(1 - c*x)] - 840*b*Log[1 - c*x - Sqrt[(1 -
c*x)/(1 + c*x)]*Sqrt[1 - c^4*x^4]])/(3150*c^12)
```

3.186.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.51, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6863, 27, 1388, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$$

$$\Big\downarrow \text{6863}$$

$$\frac{b\sqrt{1-c^2x^2}\int -\frac{\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)}{30c^{12}x\sqrt{1-c^2x^2}}dx}{cx\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)}{x\sqrt{1-c^2x^2}}dx}{30c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{1388}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}\left(3c^8x^8+4c^4x^4+8\right)}{x}dx}{30c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{2331}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}\left(3c^8x^8+4c^4x^4+8\right)}{x^2}dx^2}{60c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{2123}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \left(3c^2\left(c^2x^2+1\right)^{7/2}-9c^2\left(c^2x^2+1\right)^{5/2}+13c^2\left(c^2x^2+1\right)^{3/2}-7c^2\sqrt{c^2x^2+1}+\frac{8\sqrt{c^2x^2+1}}{x^2}\right)dx^2}{60c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{2009}$$

3.186. $\int \frac{x^{11}\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

$$-\frac{(1-c^4x^4)^{5/2}(a+b\text{sech}^{-1}(cx))}{10c^{12}}+\frac{(1-c^4x^4)^{3/2}(a+b\text{sech}^{-1}(cx))}{3c^{12}}-\frac{\sqrt{1-c^4x^4}(a+b\text{sech}^{-1}(cx))}{2c^{12}}-\frac{b\sqrt{1-c^2x^2}\left(-16\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{3}(c^2x^2+1)^{9/2}-\frac{18}{7}(c^2x^2+1)^{7/2}+\frac{26}{5}(c^2x^2+1)^{5/2}-\frac{14}{3}(c^2x^2+1)^{3/2}+}{60c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}$$

input
```
Int[(x^11*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcSech[c*x]))/c^12 + ((1 - c^4*x^4)^(3/2)*
(a + b*ArcSech[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)*(a + b*ArcSech[c*x])
)/(10*c^12) - (b*Sqrt[1 - c^2*x^2]*(16*Sqrt[1 + c^2*x^2] - (14*(1 + c^2*x^
2)^(3/2))/3 + (26*(1 + c^2*x^2)^(5/2))/5 - (18*(1 + c^2*x^2)^(7/2))/7 + (2
*(1 + c^2*x^2)^(9/2))/3 - 16*ArcTanh[Sqrt[1 + c^2*x^2]]))/(60*c^13*Sqrt[-1
 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)
```

3.186.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6863
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcSech[c*x])   v, x] + Simp[b*(Sqrt[1 - c^2*x^2]/(c*
x*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]))   Int[SimplifyIntegrand[v/(x*Sqrt[
1 - c^2*x^2]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}
, x]
```

3.186.4 Maple [F]

$$\int \frac{x^{11}(a+b\,\text{arcsech}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

input
```
int(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}(a+b\text{sech}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = -\frac{105\,(3\,bc^{10}x^{10}-3\,bc^8x^8+4\,bc^6x^6-4\,bc^4x^4+8\,bc^2x^2-8\,b)\sqrt{-c^4x^4+1}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(35\,bc^9x}{}$$

input
```
integrate(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/3150*(105*(3*b*c^10*x^10 - 3*b*c^8*x^8 + 4*b*c^6*x^6 - 4*b*c^4*x^4 + 8*
b*c^2*x^2 - 8*b)*sqrt(-c^4*x^4 + 1)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
) + 1)/(c*x)) - (35*b*c^9*x^9 + 5*b*c^7*x^7 + 78*b*c^5*x^5 + 36*b*c^3*x^3
+ 768*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 420*(b*c^
2*x^2 - b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*
x^2)) - 1)/(c^2*x^2 - 1)) - 420*(b*c^2*x^2 - b)*log(-(c^2*x^2 - sqrt(-c^4*
x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) + 105*(3*a
*c^10*x^10 - 3*a*c^8*x^8 + 4*a*c^6*x^6 - 4*a*c^4*x^4 + 8*a*c^2*x^2 - 8*a)*
sqrt(-c^4*x^4 + 1))/(c^14*x^2 - c^12)
```

3.186.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\text{Timed out}$$

input
```
integrate(x**11*(a+b*asech(c*x))/(-c**4*x**4+1)**(1/2),x)
```

output
```
Timed out
```

3.186.7 Maxima [F]

$$\int \frac{x^{11}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\int \frac{(b\operatorname{arsech}(cx)+a)x^{11}}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima"
)
```

output
```
-1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*s
qrt(-c^4*x^4 + 1)/c^12) + 1/30*b*((3*c^12*x^12 + c^8*x^8 + 4*c^4*x^4 - 8)*
log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqr
t(-c*x + 1)*c^12) - 30*integrate(1/30*(30*c^10*x^21*log(c) + 60*c^10*x^21*
log(sqrt(x)) + (60*c^10*x^21*log(sqrt(x)) + (3*c^10*x^10*(10*log(c) + 1) +
 3*c^8*x^8 + 4*c^6*x^6 + 4*c^4*x^4 + 8*c^2*x^2 + 8)*x^11)*e^(1/2*log(c*x +
 1) + 1/2*log(-c*x + 1)))/((c^10*x^10*e^(log(c*x + 1) + log(-c*x + 1)) + c
^10*x^10*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))*sqrt(c^2*x^2 + 1)), x))
```

3.186.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^{11}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}\left(a+b\mathrm{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\int \frac{x^{11}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((x^11*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^11*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.187 $\int \frac{x^7\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

3.187.1 Optimal result

Integrand size = 26, antiderivative size = 316

$$\int \frac{x^7\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}+\frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{18c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{30c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8} + \frac{(1-c^4x^4)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8} + \frac{b\sqrt{1-c^2x^2}\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{3c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

output
```
1/6*(-c^4*x^4+1)^(3/2)*(a+b*arcsech(c*x))/c^8+1/18*b*(c^2*x^2+1)^(3/2)*(-c
^2*x^2+1)^(1/2)/c^9/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-1/30*b*(c^2*x^2+1)^
(5/2)*(-c^2*x^2+1)^(1/2)/c^9/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/3*b*arct
anh((c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^9/x/(-1+1/c/x)^(1/2)/(1+1/c/x)
^(1/2)-1/3*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^9/x/(-1+1/c/x)^(1/2)/(
1+1/c/x)^(1/2)-1/2*(a+b*arcsech(c*x))*(-c^4*x^4+1)^(1/2)/c^8
```

3.187.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.56

$$\int \frac{x^7(a+b\operatorname{sech}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$
$$= \frac{-15a\sqrt{1-c^4x^4}(2+c^4x^4)+\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^4x^4}(28+c^2x^2+3c^4x^4)}{-1+cx}-15b\sqrt{1-c^4x^4}(2+c^4x^4)\operatorname{sech}^{-1}(cx)+30b\log(}{90c^8}$$

input
```
Integrate[(x^7*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
(-15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt
[1 - c^4*x^4]*(28 + c^2*x^2 + 3*c^4*x^4))/(-1 + c*x) - 15*b*Sqrt[1 - c^4*x
^4]*(2 + c^4*x^4)*ArcSech[c*x] + 30*b*Log[x*(1 - c*x)] - 30*b*Log[1 - c*x
- Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^4*x^4]])/(90*c^8)
```

3.187.3 Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.53, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6863, 27, 1388, 1579, 517, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(a+b\operatorname{sech}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$
$$\Big\downarrow \text{6863}$$
$$\frac{b\sqrt{1-c^2x^2}\int -\frac{\sqrt{1-c^4x^4}(c^4x^4+2)}{6c^8x\sqrt{1-c^2x^2}}dx}{cx\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{(1-c^4x^4)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\operatorname{sech}^{-1}(cx))}{2c^8}$$
$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{1-c^2x^2}\int\frac{\sqrt{1-c^4x^4}(c^4x^4+2)}{x\sqrt{1-c^2x^2}}dx}{6c^9x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8}$$

$$\Big\downarrow \text{1388}$$

$$-\frac{b\sqrt{1-c^2x^2}\int\frac{\sqrt{c^2x^2+1}(c^4x^4+2)}{x}dx}{6c^9x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8}$$

$$\Big\downarrow \text{1579}$$

$$-\frac{b\sqrt{1-c^2x^2}\int\frac{\sqrt{c^2x^2+1}(c^4x^4+2)}{x^2}dx^2}{12c^9x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8}$$

$$\Big\downarrow \text{517}$$

$$-\frac{b\sqrt{1-c^2x^2}\int-\frac{x^4(c^4x^8-2c^4x^4+3c^4)}{1-x^4}d\sqrt{c^2x^2+1}}{6c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\sqrt{1-c^2x^2}\int\frac{x^4(c^4x^8-2c^4x^4+3c^4)}{1-x^4}d\sqrt{c^2x^2+1}}{6c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8}$$

$$\Big\downarrow \text{1584}$$

$$\frac{b\sqrt{1-c^2x^2}\int\left(-c^4x^8+c^4x^4-2c^4+\frac{2c^4}{1-x^4}\right)d\sqrt{c^2x^2+1}}{6c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8}$$

$$\Big\downarrow \text{2009}$$

3.187. $\int\frac{x^7\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}dx$

$$\frac{(1-c^4x^4)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8}-\frac{b\sqrt{1-c^2x^2}\left(-2c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{c^4x^{10}}{5}-\frac{c^4x^6}{3}+2c^4\sqrt{c^2x^2+1}\right)}{6c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}$$

input
```
Int[(x^7*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcSech[c*x]))/c^8 + ((1 - c^4*x^4)^(3/2)*(
a + b*ArcSech[c*x]))/(6*c^8) - (b*Sqrt[1 - c^2*x^2]*(-1/3*(c^4*x^6) + (c^4
*x^10)/5 + 2*c^4*Sqrt[1 + c^2*x^2] - 2*c^4*ArcTanh[Sqrt[1 + c^2*x^2]]))/(6
*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)
```

3.187.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.187. $\int \frac{x^7\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6863
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcSech[c*x])   v, x] + Simp[b*(Sqrt[1 - c^2*x^2]/(c*
x*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]))   Int[SimplifyIntegrand[v/(x*Sqrt[
1 - c^2*x^2]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}
, x]
```

3.187.4 Maple [F]

$$\int \frac{x^7 (a + b \operatorname{arcsech}(cx))}{\sqrt{-c^4 x^4 + 1}} dx$$

input
```
int(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.06

$$\int \frac{x^7 (a + b\operatorname{sech}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx =$$

$$-\frac{15 (bc^6 x^6 - bc^4 x^4 + 2 bc^2 x^2 - 2 b)\sqrt{-c^4 x^4 + 1} \log\left(\frac{cx\sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} + 1}{cx}\right) - (3 bc^5 x^5 + bc^3 x^3 + 28 bcx)\sqrt{-c^4 x^4}}{}$$

input
```
integrate(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/90*(15*(b*c^6*x^6 - b*c^4*x^4 + 2*b*c^2*x^2 - 2*b)*sqrt(-c^4*x^4 + 1)*l
og((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (3*b*c^5*x^5 + b*c^3*
x^3 + 28*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 15*(b*
c^2*x^2 - b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)) - 1)/(c^2*x^2 - 1)) - 15*(b*c^2*x^2 - b)*log(-(c^2*x^2 - sqrt(-c^4
*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) + 15*(a*c
^6*x^6 - a*c^4*x^4 + 2*a*c^2*x^2 - 2*a)*sqrt(-c^4*x^4 + 1))/(c^10*x^2 - c^
8)
```

3.187.6 Sympy [F]

$$\int \frac{x^7\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\int \frac{x^7(a+b\,\text{asech}\,(cx))}{\sqrt{-\left(cx-1\right)\left(cx+1\right)\left(c^2x^2+1\right)}}\,dx$$

input
```
integrate(x**7*(a+b*asech(c*x))/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral(x**7*(a + b*asech(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)
), x)
```

3.187.7 Maxima [F]

$$\int \frac{x^7\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^7}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) + 1/6*b*((c^8*
x^8 + c^4*x^4 - 2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(c^2*x^2 + 1
)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8) - 6*integrate(1/6*(6*c^6*x^13*log(c) +
 12*c^6*x^13*log(sqrt(x)) + (12*c^6*x^13*log(sqrt(x)) + (c^6*x^6*(6*log(c)
 + 1) + c^4*x^4 + 2*c^2*x^2 + 2)*x^7)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x +
 1))/((c^6*x^6*e^(log(c*x + 1) + log(-c*x + 1)) + c^6*x^6*e^(1/2*log(c*x
+ 1) + 1/2*log(-c*x + 1)))*sqrt(c^2*x^2 + 1)), x))
```

3.187.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^7(a+b\operatorname{sech}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\operatorname{sech}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^7\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((x^7*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^7*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.188 $\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

3.188.1 Optimal result

Integrand size = 26, antiderivative size = 159

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=-\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^4}+\frac{b\sqrt{1-c^2x^2}\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{2c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

output
```
1/2*b*arctanh((c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^5/x/(-1+1/c/x)^(1/2)
/(1+1/c/x)^(1/2)-1/2*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^5/x/(-1+1/c/
x)^(1/2)/(1+1/c/x)^(1/2)-1/2*(a+b*arcsech(c*x))*(-c^4*x^4+1)^(1/2)/c^4
```

3.188.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.88

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=-\frac{a\sqrt{1-c^4x^4}+\frac{b\sqrt{1-c^4x^4}}{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}+b\sqrt{1-c^4x^4}\,\text{sech}^{-1}(cx)-b\log(x(1-cx))+b\log\left(1-cx-\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c\right.}{2c^4}$$

input
```
Integrate[(x^3*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(a*Sqrt[1 - c^4*x^4] + (b*Sqrt[1 - c^4*x^4])/(Sqrt[(1 - c*x)/(1 + c*x
)]*(1 + c*x)) + b*Sqrt[1 - c^4*x^4]*ArcSech[c*x] - b*Log[x*(1 - c*x)] + b*
Log[1 - c*x - Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^4*x^4]])/c^4
```

3.188.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.71, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {6863, 27, 1388, 243, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}dx$$

$$\Big\downarrow \text{6863}$$

$$\frac{b\sqrt{1-c^2x^2}\int -\frac{\sqrt{1-c^4x^4}}{2c^4x\sqrt{1-c^2x^2}}dx}{cx\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{1-c^4x^4}}{x\sqrt{1-c^2x^2}}dx}{2c^5x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{1388}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}}{x}dx}{2c^5x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{243}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}}{x^2}dx^2}{4c^5x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{60}$$

$$-\frac{b\sqrt{1-c^2x^2}\left(\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2+2\sqrt{c^2x^2+1}\right)}{4c^5x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{b\sqrt{1-c^2x^2}\left(\frac{2\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c^2}+2\sqrt{c^2x^2+1}\right)}{4c^5x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^4}-\frac{b\sqrt{1-c^2x^2}\left(2\sqrt{c^2x^2+1}-2\operatorname{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{4c^5x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}$$

input
```
Int[(x^3*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcSech[c*x]))/c^4 - (b*Sqrt[1 - c^2*x^2]*(
2*Sqrt[1 + c^2*x^2] - 2*ArcTanh[Sqrt[1 + c^2*x^2]]))/(4*c^5*Sqrt[-1 + 1/(c
*x)]*Sqrt[1 + 1/(c*x)]*x)
```

3.188.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 6863
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcSech[c*x])   v, x] + Simp[b*(Sqrt[1 - c^2*x^2]/(c*
x*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]))   Int[SimplifyIntegrand[v/(x*Sqrt[
1 - c^2*x^2]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}
, x]
```

3.188.4 Maple [F]

$$\int \frac{x^3 (a + b \operatorname{arcsech}(cx))}{\sqrt{-c^4x^4+1}} dx$$

input
```
int(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

3.188.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(135) = 270$.

Time = 0.27 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.75

$$\int \frac{x^3 \left(a + b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}} dx$$
$$= \frac{2\sqrt{-c^4x^4+1}bcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 2\sqrt{-c^4x^4+1}(bc^2x^2-b)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - (bc^2x^2-b)\log\left(\frac{c^2x^2+\sqrt{-c^4 \cdots}}{\cdots}\right.}{4\left(c^6x^2-c \cdots\right.}$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(-c^4*x^4 + 1)*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*sqrt(-c
^4*x^4 + 1)*(b*c^2*x^2 - b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(
c*x)) - (b*c^2*x^2 - b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) + (b*c^2*x^2 - b)*log(-(c^2*x^2 - s
qrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) -
 2*sqrt(-c^4*x^4 + 1)*(a*c^2*x^2 - a))/(c^6*x^2 - c^4)
```

3.188.6 Sympy [F]

$$\int \frac{x^3(a + b\text{sech}^{-1}(cx))}{\sqrt{1 - c^4x^4}} dx = \int \frac{x^3(a + b\,\text{asech}\,(cx))}{\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}} dx$$

input
```
integrate(x**3*(a+b*asech(c*x))/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral(x**3*(a + b*asech(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)
), x)
```

3.188.7 Maxima [F]

$$\int \frac{x^3(a + b\text{sech}^{-1}(cx))}{\sqrt{1 - c^4x^4}} dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^3}{\sqrt{-c^4x^4+1}} dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*((c^4*x^4 - 1)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(c^2*x^2 +
 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^4) - 2*integrate(1/2*(2*c^2*x^5*log(c)
+ 4*c^2*x^5*log(sqrt(x)) + (4*c^2*x^5*log(sqrt(x)) + (c^2*x^2*(2*log(c) +
1) + 1)*x^3)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))/((c^2*x^2*e^(log(c*
x + 1) + log(-c*x + 1)) + c^2*x^2*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))
)*sqrt(c^2*x^2 + 1)), x)) - 1/2*sqrt(-c^4*x^4 + 1)*a/c^4
```

3.188.8 Giac [F]

$$\int \frac{x^3(a+b\text{sech}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x^3}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{sech}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^3\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((x^3*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^3*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.189 $\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$

3.189.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}},x\right)$$

output
```
Unintegrable((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

3.189.2 Mathematica [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

output
```
Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

3.189.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$
$$\Big\downarrow \text{ 6865}$$
$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}dx$$

input
```
Int[(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

output
```
$Aborted
```

3.189.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.189.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\,\operatorname{arcsech}\left(cx\right)}{x\sqrt{-c^{4}x^{4}+1}}dx$$

input
```
int((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

output
```
int((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

3.189.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^4*x^4 + 1)*(b*arcsech(c*x) + a)/(c^4*x^5 - x), x)
```

3.189.6 Sympy [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{a + b\,\text{asech}\,(cx)}{x\sqrt{-\left(cx - 1\right)\left(cx + 1\right)\left(c^2x^2 + 1\right)}}\,dx$$

input
```
integrate((a+b*asech(c*x))/x/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral((a + b*asech(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)))
, x)
```

3.189.7 Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.85

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*int
egrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(sqrt(-(c^2*x^2
+ 1)*(c*x + 1)*(c*x - 1))*x), x)
```

3.189.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)
```

3.189.9 Mupad [N/A]

Not integrable

Time = 5.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{1 - c^4\,x^4}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)), x)
```

3.190 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$

3.190.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}},x\right)$$

output
```
Unintegrable((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

3.190.2 Mathematica [N/A]

Not integrable

Time = 9.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

input
```
Integrate[(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
```

output
```
Integrate[(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

3.190.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6865}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^5\sqrt{1 - c^4x^4}} \, dx$$

$$\Big\downarrow \; 6865$$

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^5\sqrt{1 - c^4x^4}} dx$$

input
```
Int[(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
```

output
```
$Aborted
```

3.190.3.1 Defintions of rubi rules used

rule 6865
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSech[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.190.4 Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a + b \,\text{arcsech}\,(cx)}{x^5\sqrt{-c^4x^4 + 1}} dx$$

input
```
int((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

output
```
int((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

3.190.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^4*x^4 + 1)*(b*arcsech(c*x) + a)/(c^4*x^9 - x^5), x)
```

3.190.6 Sympy [N/A]

Not integrable

Time = 24.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\text{asech}\,(cx)}{x^5\sqrt{-\left(cx-1\right)\left(cx+1\right)\left(c^2x^2+1\right)}}\,dx$$

input
```
integrate((a+b*asech(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral((a + b*asech(c*x))/(x**5*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1
))), x)
```

3.190.7 Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 124, normalized size of antiderivative = 4.77

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) +
2*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*
x) - 1) + 1/(c*x))/(sqrt(-(c^2*x^2 + 1)*(c*x + 1)*(c*x - 1))*x^5), x)
```

3.190.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

input
```
integrate((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsech(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)
```

3.190.9 Mupad [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x^5\,\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((a + b*acosh(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)),x)
```

output
```
int((a + b*acosh(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
                finalresult={"A"," "}
            ,(*ELSE*)
                finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
      return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])   #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
      print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```